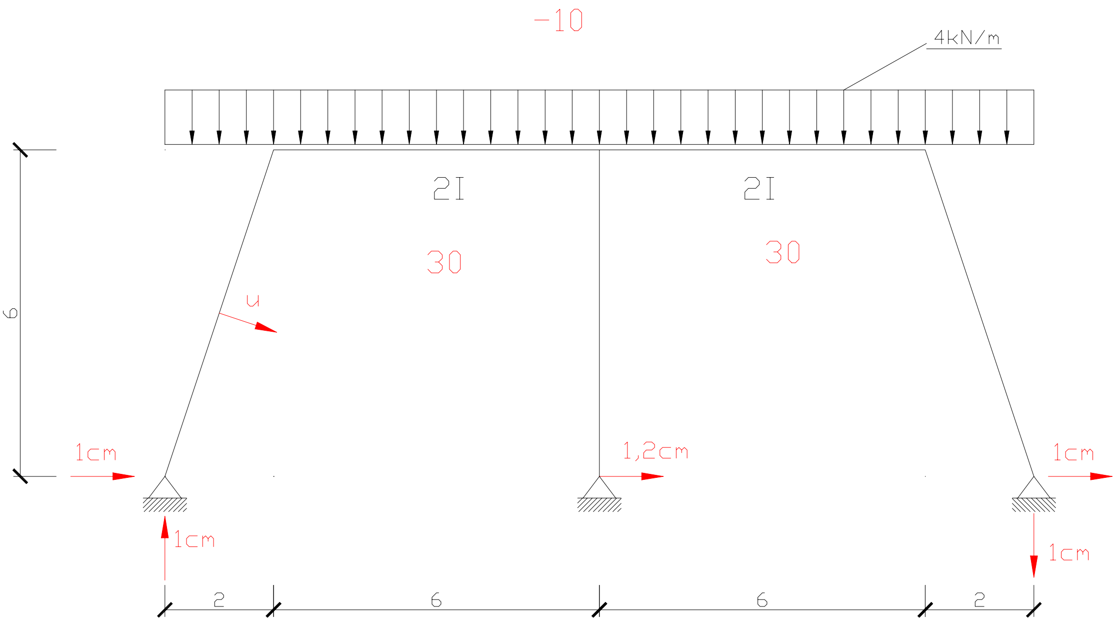
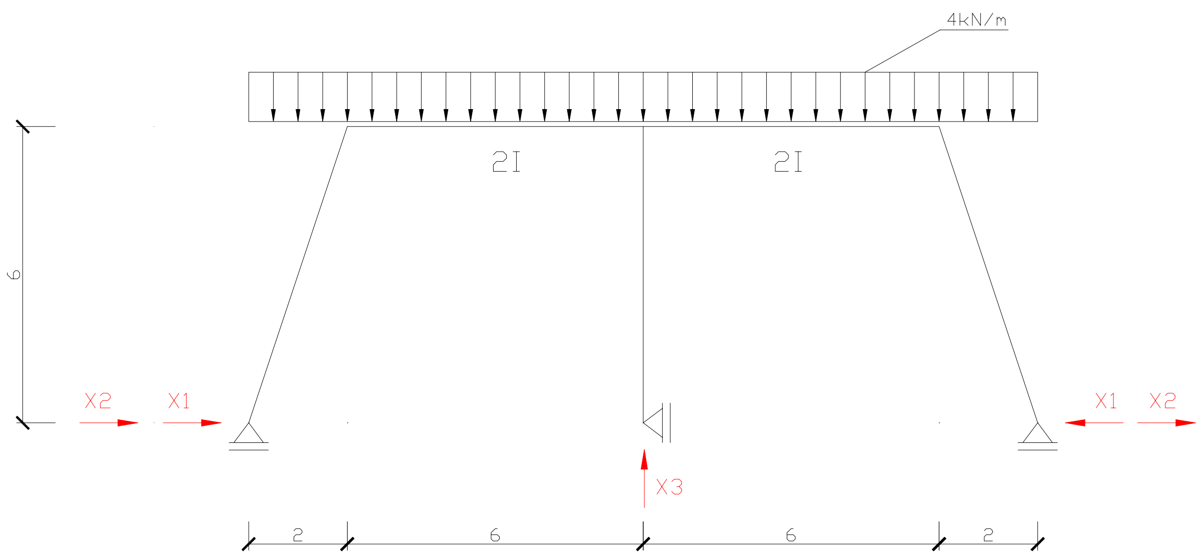
OBLICZANIE UKŁADÓW STATYCZNIE NIEWYZNACZALNYCH METODĄ SIŁ.

Zadana rama wygląda następująco:



Siły wewnętrzne od obciążenia zewnętrznego. Dobieram układ podstawowy w ten sposób aby zachować symetrię:



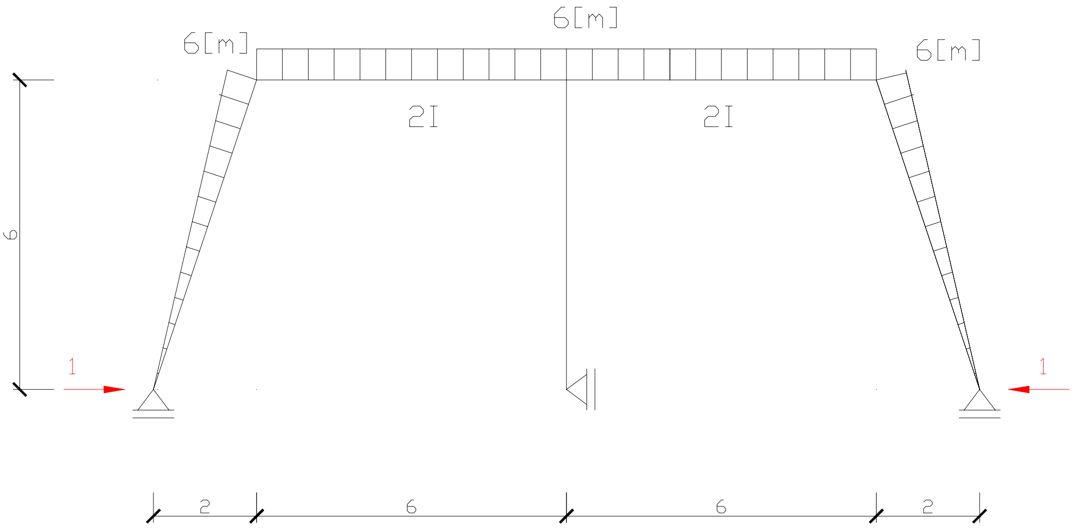
Zapisuję układ równań kanonicznych:

$$\begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 + \Delta_{1P} = 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 + \Delta_{2P} = 0 \\ \delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 + \Delta_{3P} = 0 \end{cases}$$

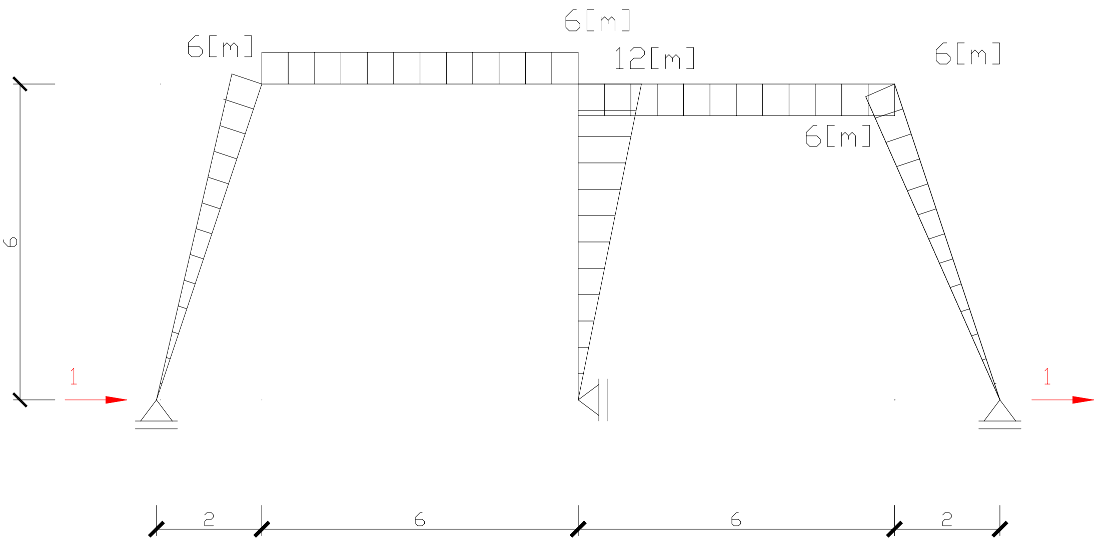
$$\delta_{ik} = \int \frac{M_i \cdot M_k}{EI} ds \qquad \Delta_{iP} = \int \frac{M_P \cdot M_i}{EI} ds$$

Rysuję wykresy momentów od poszczególnych sił jednostkowych:

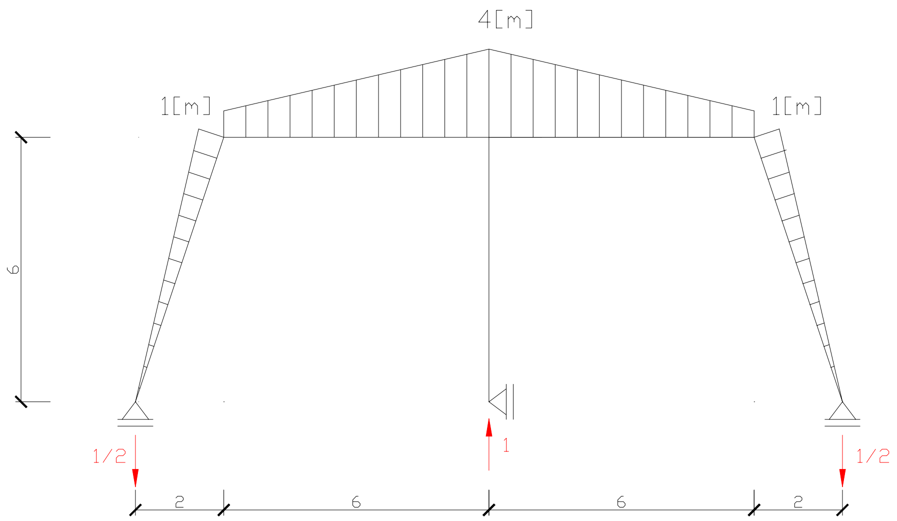
M_1



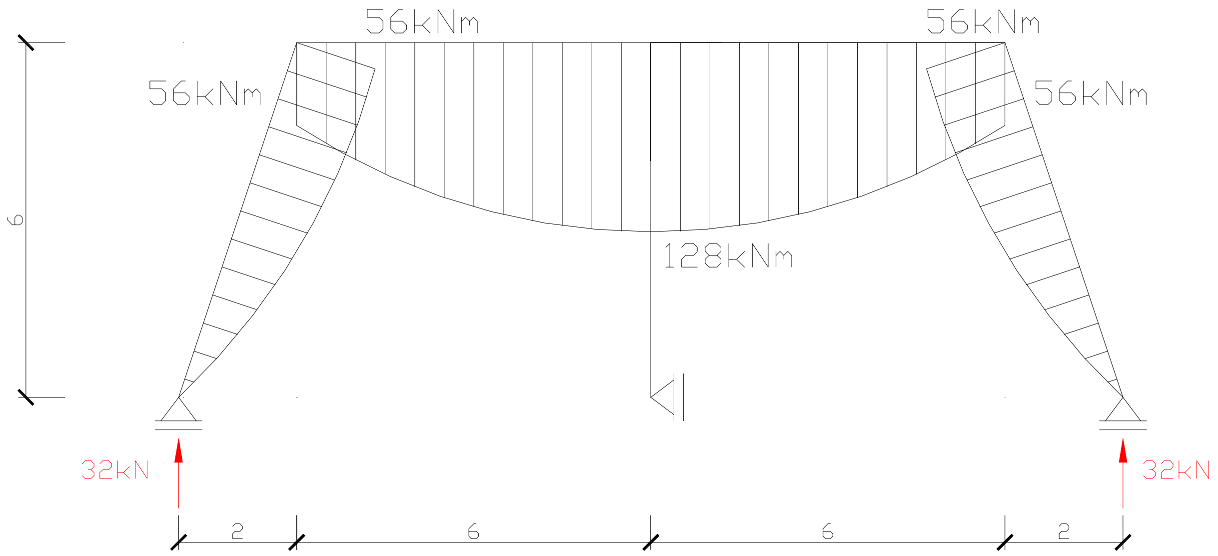
M_2



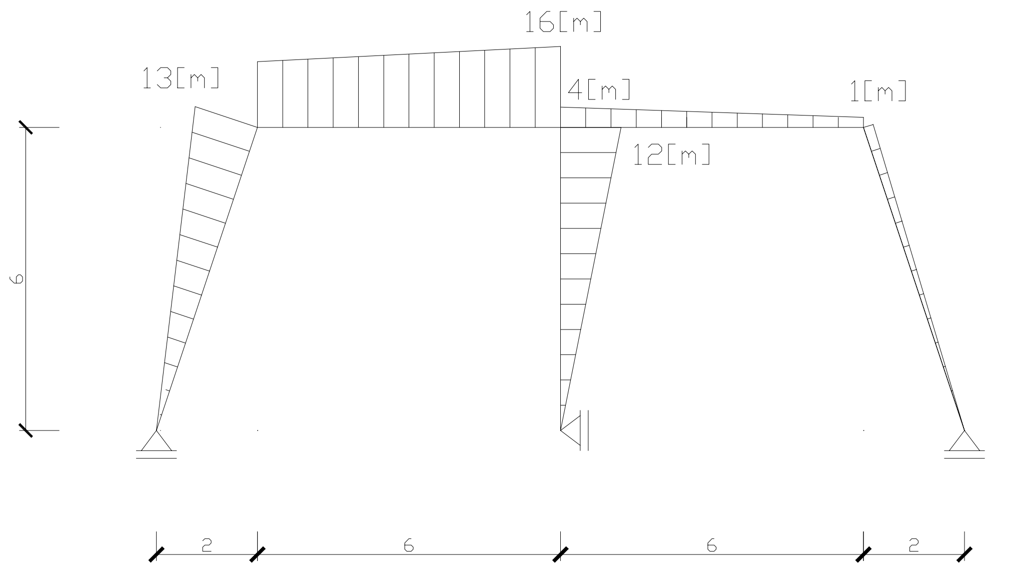
M_3



M_P


56kNm
56kNm
56kNm
56kNm
128kNm
6
32kN
32kN
2
6
6
2

M_S


16[m]
13[m]
4[m]
1[m]
12[m]
6
2
6
6
2

Korzystając z metody Wereszczegina- Mohra całkowania iloczynu dwóch funkcji (w tym jednej prostoliniowej) otrzymuje się:

$$\delta_{21} = \int \frac{M_2 \cdot M_1}{EI} ds = 0$$

$$\delta_{22} = \int \frac{M_2 \cdot M_2}{EI} ds = \frac{1}{EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot 2 \cdot \sqrt{10} \cdot 6 \cdot \left(\frac{2}{3} \cdot 6 \right) \right] + \frac{1}{2EI} \cdot 2 \cdot [6 \cdot 6 \cdot 6] + \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 12 \cdot 6 \cdot \left(\frac{2}{3} \cdot 12 \right) \right] =$$

$$= \frac{1}{EI} \left[48\sqrt{10} + 504 \right]$$

$$\delta_{23} = \int \frac{M_2 \cdot M_3}{EI} ds = 0$$

$$\delta_{31} = \int \frac{M_3 \cdot M_1}{EI} ds = \frac{1}{EI} \left[8\sqrt{10} + 90 \right]$$

$$\delta_{32} = \int \frac{M_3 \cdot M_2}{EI} ds = 0$$

$$\delta_{33} = \int \frac{M_3 \cdot M_3}{EI} ds = \frac{1}{EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot 2 \cdot \sqrt{10} \cdot \left(\frac{2}{3} \cdot 1 \right) \right] + \frac{1}{2EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot 6 \cdot 1 \cdot \left(\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 6 \cdot 4 \cdot \left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 1 \right) \right] =$$

$$= \frac{1}{EI} \left[\frac{4}{3} \sqrt{10} + 42 \right]$$

$$\Delta_{1P} = \int \frac{M_P \cdot M_1}{EI} ds = -\frac{1}{EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot 2 \cdot \sqrt{10} \cdot 56 \cdot \left(\frac{2}{3} \cdot 6 \right) + \frac{2}{3} \cdot 2 \cdot \sqrt{10} \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{1}{2} \cdot 6 \right) \right] -$$

$$-\frac{1}{EI} \cdot \left[12 \cdot 56 \cdot 6 + \frac{2}{3} \cdot 12 \cdot \frac{4 \cdot 12^2}{8} \cdot 6 \right] = -\frac{1}{EI} \left[464\sqrt{10} + 3744 \right]$$

$$\Delta_{2P} = \int \frac{M_P \cdot M_2}{EI} ds = 0$$

$$\Delta_{3P} = \int \frac{M_P \cdot M_3}{EI} ds = -\frac{1}{EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot 2 \cdot \sqrt{10} \cdot 56 \cdot \left(\frac{2}{3} \cdot 1 \right) + \frac{2}{3} \cdot 2 \cdot \sqrt{10} \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{1}{2} \cdot 1 \right) \right] -$$

$$-\frac{1}{2EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot 56 \cdot 6 \cdot \left(\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 128 \cdot 6 \cdot \left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 1 \right) + \frac{2}{3} \cdot 6 \cdot \frac{4 \cdot 6^2}{8} \cdot \left(\frac{5}{2} \right) \right] = -\frac{1}{EI} \left[\frac{232}{3} \sqrt{10} + 1668 \right]$$

Sprawdzenie globalne delt:

$$\int \frac{{M_S}^2}{EI} ds = \sum_i \sum_k \delta_{ik}$$

$$\int \frac{{M_S}^2}{EI} ds = \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 13 \cdot 2 \cdot \sqrt{10} \cdot \left(\frac{2}{3} \cdot 13 \right) \right] + \frac{1}{2EI} \cdot \left[\frac{1}{2} \cdot 6 \cdot 13 \cdot \left(\frac{2}{3} \cdot 13 + \frac{1}{3} \cdot 16 \right) + \frac{1}{2} \cdot 6 \cdot 16 \cdot \left(\frac{2}{3} \cdot 16 + \frac{1}{3} \cdot 13 \right) \right] +$$

$$+\frac{1}{2EI} \cdot \left[\frac{1}{2} \cdot 4 \cdot 6 \cdot \left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 1 \right) + \frac{1}{2} \cdot 1 \cdot 6 \cdot \left(\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 4 \right) \right] + \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 12 \cdot 6 \cdot \left(\frac{2}{3} \cdot 12 \right) \right] = \frac{1}{EI} \cdot \left[\frac{340}{3} \cdot \sqrt{10} + 942 \right]$$

$$\sum_i \sum_k \delta_{ik} = \delta_{11} + \delta_{12} + \delta_{13} + \delta_{21} + \delta_{22} + \delta_{23} + \delta_{31} + \delta_{32} + \delta_{33} = \frac{1}{EI} \cdot \left[\frac{340}{3} \cdot \sqrt{10} + 942 \right]$$

$$\int \frac{M_P \cdot M_S}{EI} ds = \sum \Delta_{iP}$$

$$\int \frac{M_P \cdot M_S}{EI} ds = -\frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 56 \cdot 2 \cdot \sqrt{10} \cdot \left(\frac{2}{3} \cdot 13 \right) + \frac{2}{3} \cdot 2 \cdot \sqrt{10} \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{1}{2} \cdot 13 \right) \right] -$$

$$-\frac{1}{2EI} \cdot \left[\frac{1}{2} \cdot 56 \cdot 6 \cdot \left(\frac{2}{3} \cdot 13 + \frac{1}{3} \cdot 16 \right) + \frac{1}{2} \cdot 128 \cdot 6 \cdot \left(\frac{2}{3} \cdot 16 + \frac{1}{3} \cdot 13 \right) + \frac{2}{3} \cdot 6 \cdot \frac{4 \cdot 6^2}{8} \cdot \left(\frac{39}{2} \right) \right] -$$

$$-\frac{1}{2EI} \cdot \left[\frac{1}{2} \cdot 56 \cdot 6 \cdot \left(\frac{1}{3} \cdot 4 + \frac{2}{3} \cdot 1 \right) + \frac{1}{2} \cdot 128 \cdot 6 \cdot \left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 1 \right) + \frac{2}{3} \cdot 6 \cdot \frac{4 \cdot 6^2}{8} \cdot \left(\frac{5}{2} \right) \right] -$$

$$-\frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 56 \cdot 2 \cdot \sqrt{10} \cdot \left(\frac{2}{3} \cdot 1 \right) + \frac{2}{3} \cdot 2 \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{1}{2} \right) \right] = -\frac{1}{EI} \cdot \left[\frac{1624}{3} \cdot \sqrt{10} + 5412 \right]$$

$$\sum \Delta_{iP} = \Delta_{1P} + \Delta_{2P} + \Delta_{3P} = -\frac{1}{EI} \cdot \left[\frac{1624}{3} \cdot \sqrt{10} + 5412 \right]$$

Mając dane wszystkie wielkości podstawiam je do układu równań i rozwiązuje go:

$$\begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 + \Delta_{1P} = 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 + \Delta_{2P} = 0 \\ \delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 + \Delta_{3P} = 0 \end{cases}$$

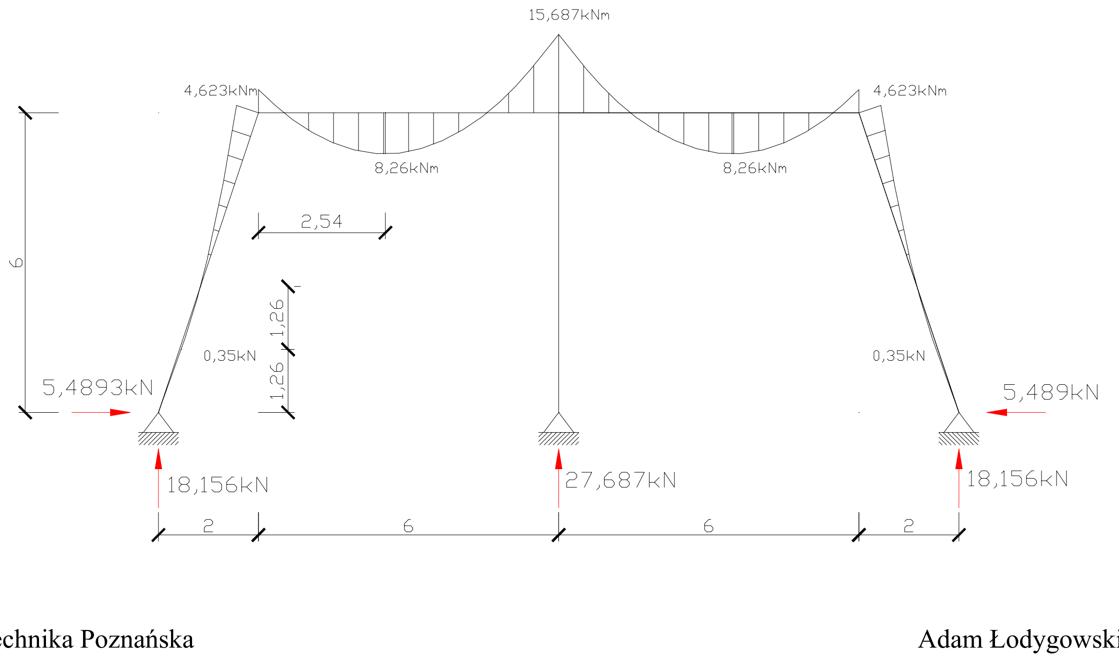
$$\begin{cases} \left(48 \cdot \sqrt{10} + 216\right) \cdot X_1 + 0 \cdot X_2 + \left(8 \cdot \sqrt{10} + 90\right) \cdot X_3 - \left(464 \cdot \sqrt{10} + 3744\right) = 0 \\ 0 \cdot X_1 + \left(48 \cdot \sqrt{10} + 504\right) \cdot X_2 + 0 \cdot X_3 + 0 = 0 \\ \left(8 \cdot \sqrt{10} + 90\right) \cdot X_1 + 0 \cdot X_2 + \left(\frac{4}{3} \cdot \sqrt{10} + 42 \right) \cdot X_3 + \left(\frac{232}{3} \cdot \sqrt{10} + 1668 \right) = 0 \end{cases}$$

$$X_1 = 5{,}489344[kN]$$

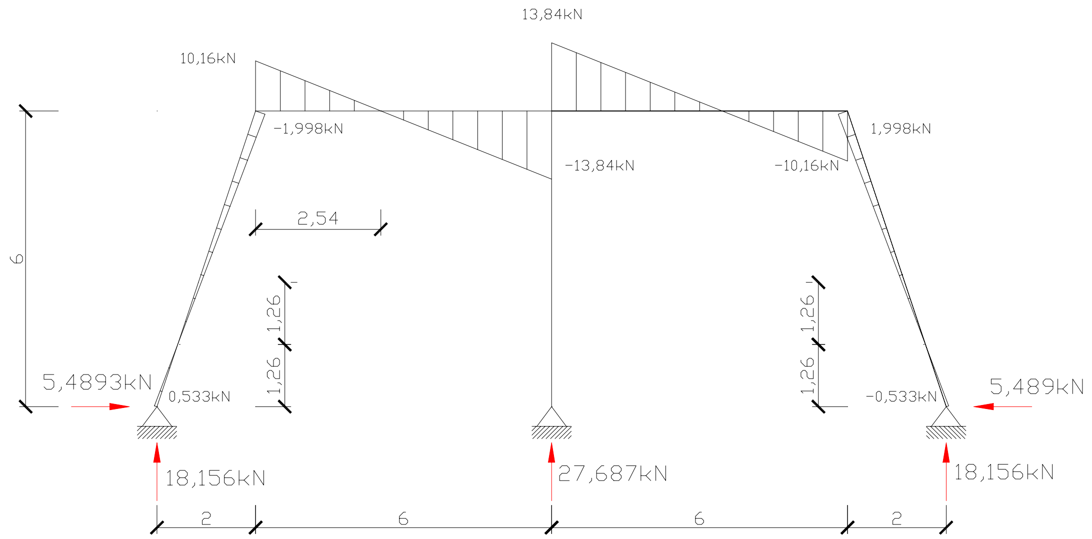
$$X_2 = 0$$

$$X_3 = 27{,}687978[kN]$$

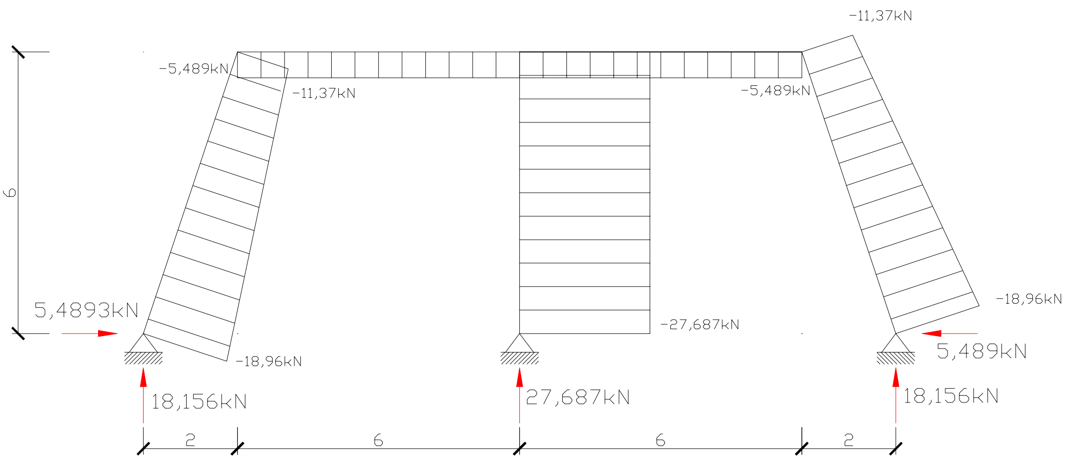
M_P



T_P

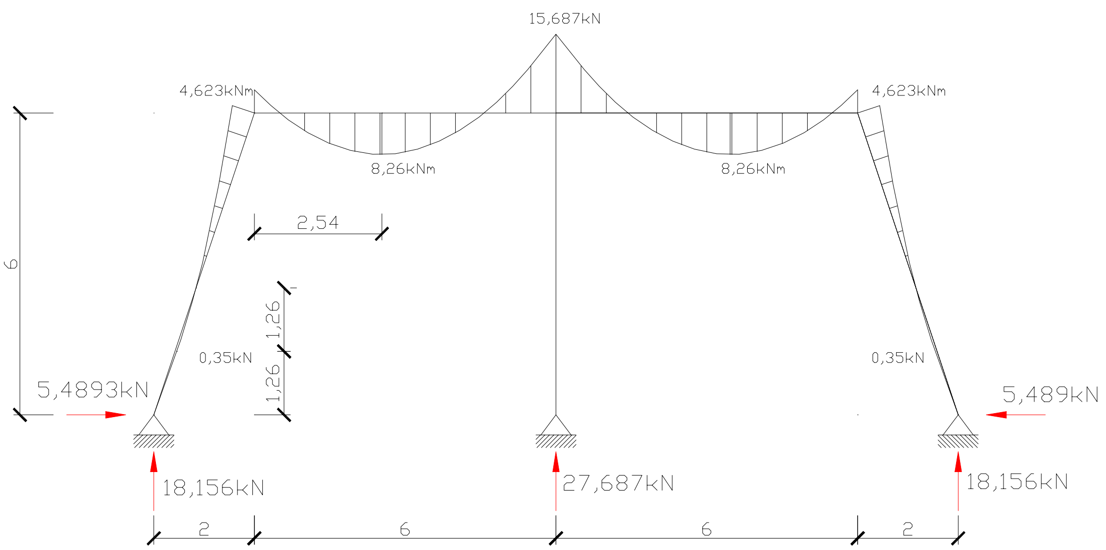


N_P

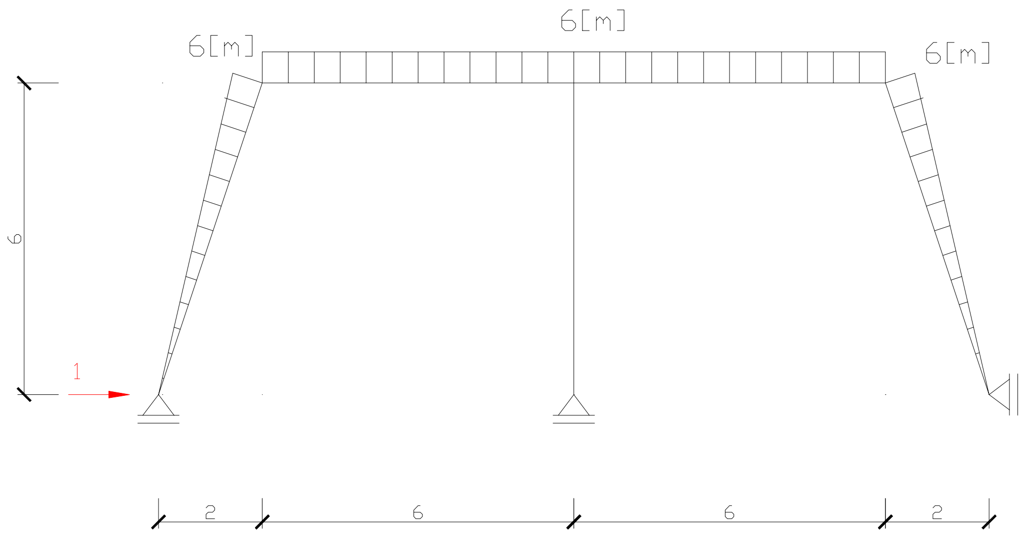


Sprawdzenie kinematyczne:

M_P



M_i



$$u_i = \int \frac{M_n \cdot M_i}{EI} ds$$

$$u_i = \frac{1}{EI} \cdot \left[-\frac{2}{3} \cdot \sqrt{40} \cdot \frac{4 \cdot 2^2}{8} \cdot 3 + \frac{1}{2} \cdot \sqrt{40} \cdot 4{,}623 \cdot 4 \right] + \frac{1}{EI} \cdot \left[\frac{4{,}623 + 15{,}687}{2} \cdot 6 \cdot 6 - \frac{2}{3} \cdot 6 \cdot \frac{4 \cdot 6^2}{8} \cdot 6 \right] = \frac{0{,}031}{EI}$$

Dobieram odpowiedni przekrój dwuteowy:

$$\frac{1{,}2 \cdot M}{W} \le \sigma_{dop}$$

$$\frac{1{,}2 \cdot 1567 kNcm}{W} \le 19{,}5 \frac{kN}{cm^2}$$

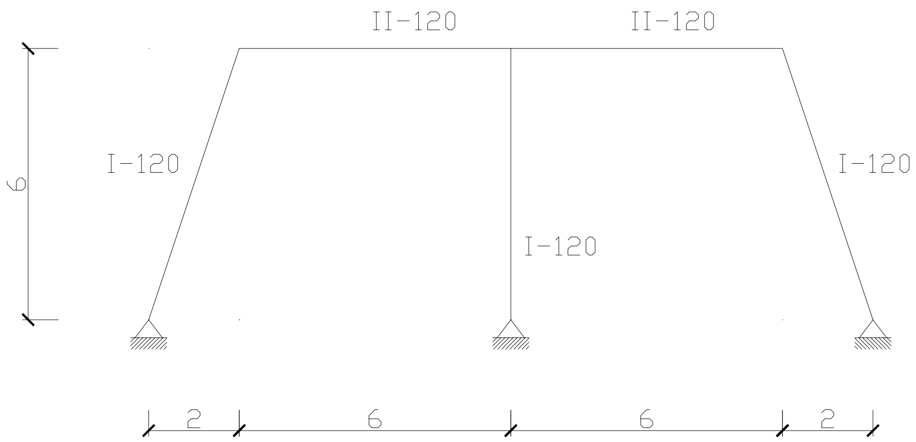
$$W \ge 96{,}43$$

Dwuteownik 120:

$$I = 328 cm^4$$

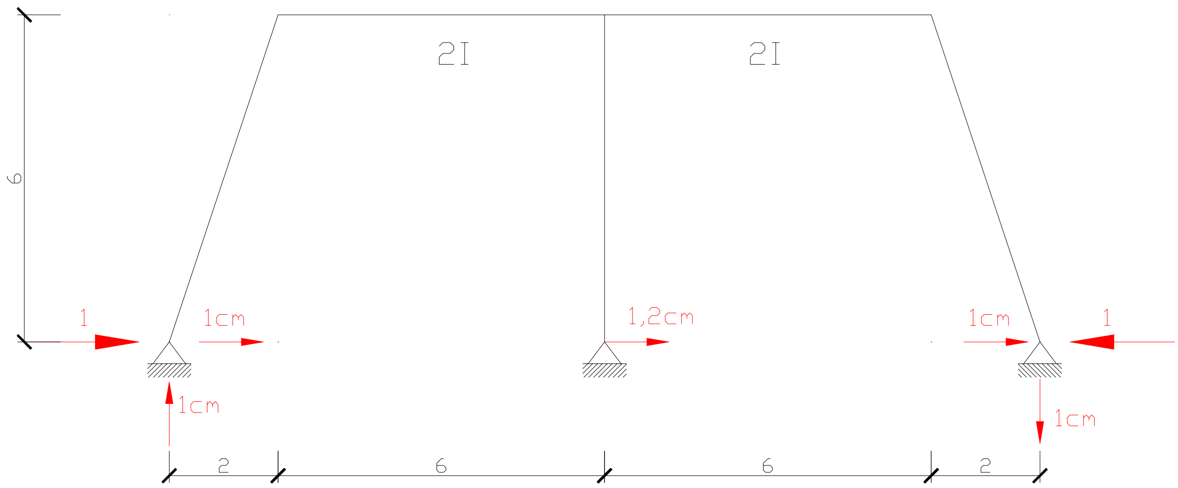
$$W = 54{,}7 cm^3 \quad EI = 672{,}4 \left[kNm^2 \right]$$

$$h = 12{,}0 cm$$

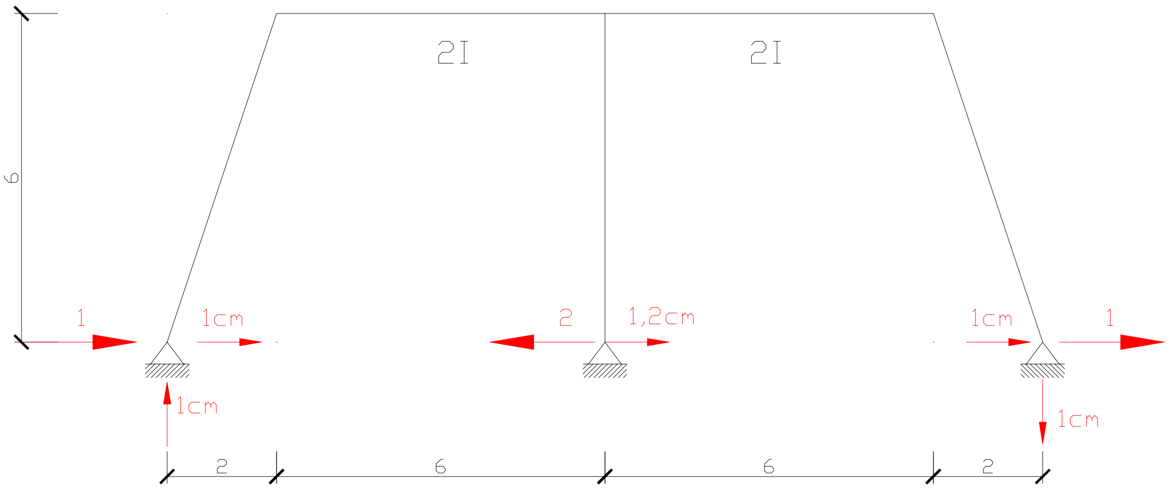


Siły wewnętrzne od osiadania podpór.

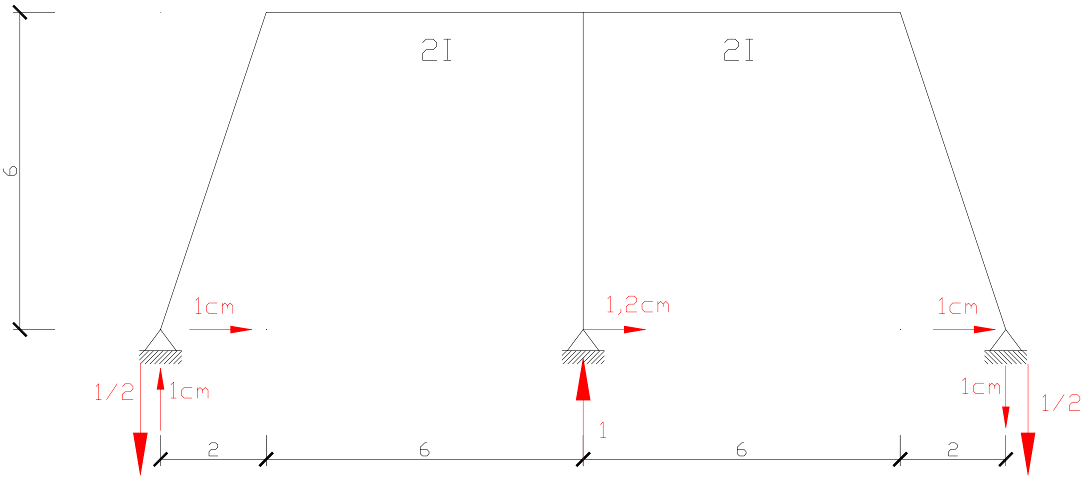
Układ podstawowy przyjmuję podobnie jak w poprzednio:



$$\Delta_{1\Delta} = -\sum R_i \Delta = \Delta_{1\Delta} = -[1 \cdot (0{,}01) - 1 \cdot (0{,}01)] = 0$$



$$\Delta_{2\Delta} = -\sum R_i \Delta = \Delta_{2\Delta} = -[1 \cdot (0{,}01) + 1 \cdot (0{,}01) - 2 \cdot (0{,}012)] = 0{,}004$$



$$\Delta_{3\Delta} = -\sum R_i \Delta = \Delta_{2\Delta} = -\left[\frac{1}{2}\cdot(0{,}01) - \frac{1}{2}\cdot(0{,}01) + 1\cdot 0\right] = 0$$

Delty wykorzystuję z obliczonego wcześniej układu podstawowego:

$$\begin{cases} \delta_{11}\cdot X_1 + \delta_{12}\cdot X_2 + \delta_{13}\cdot X_3 + \Delta_{1\Delta} = 0 \\ \delta_{21}\cdot X_1 + \delta_{22}\cdot X_2 + \delta_{23}\cdot X_3 + \Delta_{2\Delta} = 0 \\ \delta_{31}\cdot X_1 + \delta_{32}\cdot X_2 + \delta_{33}\cdot X_3 + \Delta_{3\Delta} = 0 \end{cases}$$

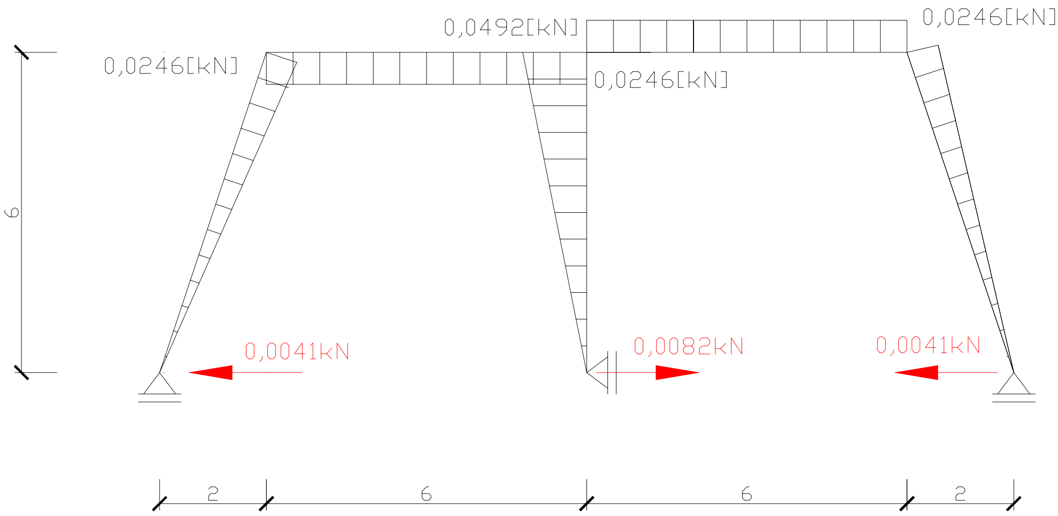
$$\begin{cases} \left(48\cdot\sqrt{10} + 216\right)\cdot X_1 + 0\cdot X_2 + \left(8\cdot\sqrt{10} + 90\right)\cdot X_3 + EI\cdot(0) = 0 \\ 0\cdot X_1 + \left(48\cdot\sqrt{10} + 504\right)\cdot X_2 + 0\cdot X_3 + EI\cdot(0{,}004) = 0 \\ \left(8\cdot\sqrt{10} + 90\right)\cdot X_1 + 0\cdot X_2 + \left(\frac{4}{3}\cdot\sqrt{10} + 42\right)\cdot X_3 + EI\cdot(0) = 0 \end{cases}$$

$X_1 = 0[kN]$

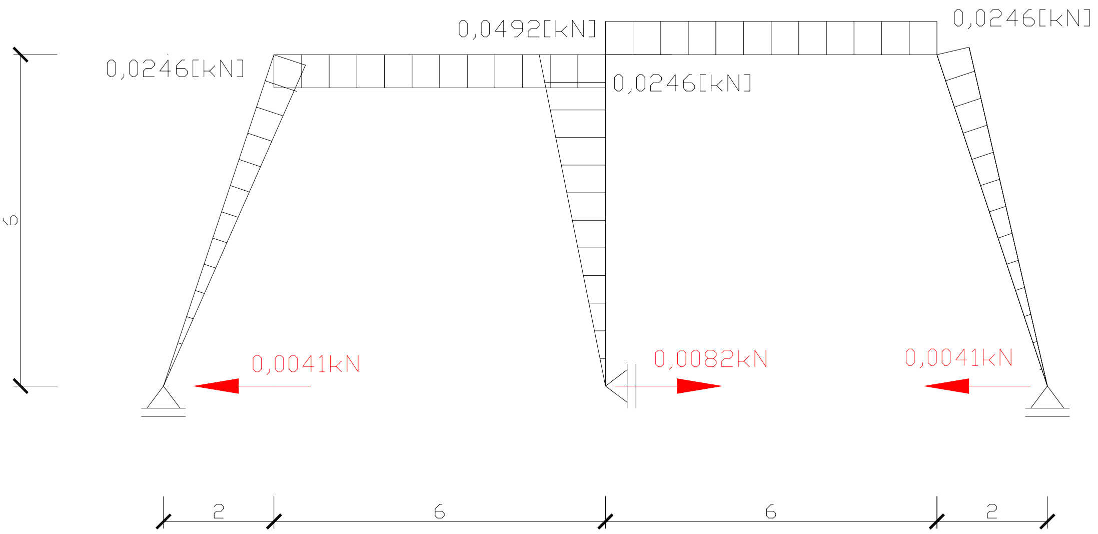
$X_2 = -0{,}0041[kN]$

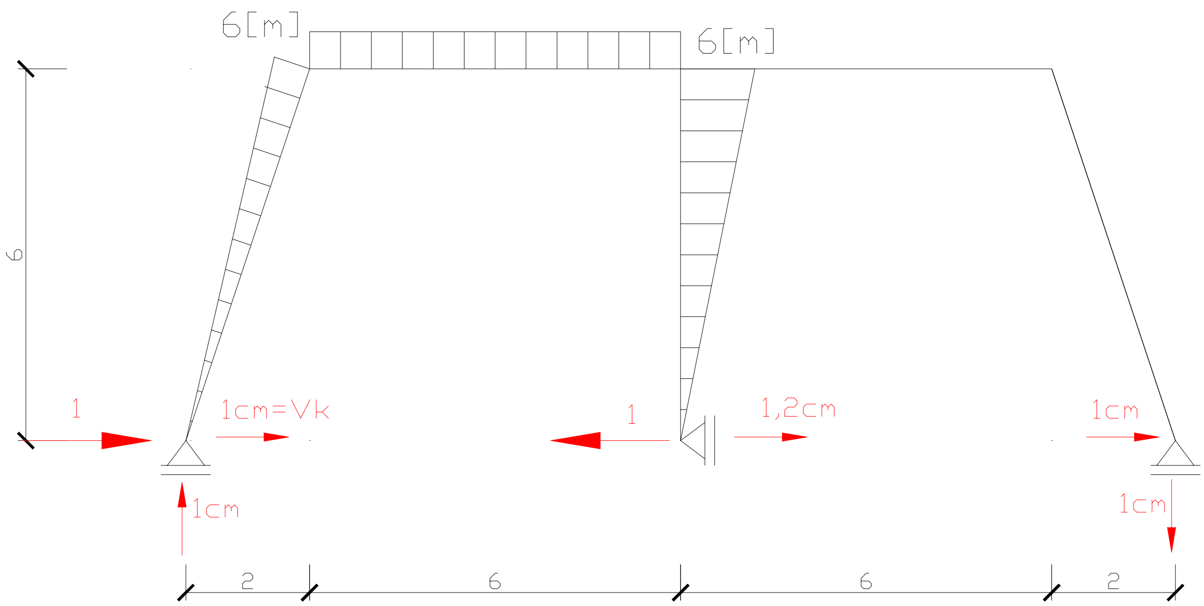
$X_3 = 0[kN]$

$\mathbf{M}_\Delta^{\,n}$



Sprawdzenie:





$$1 \cdot V_K + \sum R_i \Delta = \int \frac{M_\Delta{}^n \cdot M_i}{EI} ds$$

$$1 \cdot V_K - 0{,}012 = -\frac{1}{EI} \cdot \left[\frac{1}{2} \cdot \sqrt{40} \cdot 0{,}0246 \cdot \left(\frac{2}{3} \cdot 6\right)\right] - \frac{1}{2 \cdot EI} \cdot [0{,}0246 \cdot 6 \cdot 6] - \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 0{,}0492 \cdot 6 \cdot \left(\frac{2}{3} \cdot 6\right)\right]$$

$$V_K = 0{,}01000074[m] \approx 0{,}01[m]$$

Siły wewnętrzne od wpływu temperatur:

Schemat podstawowy przyjęto jak w poprzednim zadaniu:

$t_d = 30^0 C$ $\quad \Delta t' = 40^0 C$ $\quad \alpha_t = 1{,}2 \cdot 10^{-5}$

$t_g = -10^0 C$ $\quad \Delta t'' = 0^0 C$ $\quad h = 0{,}12 m$

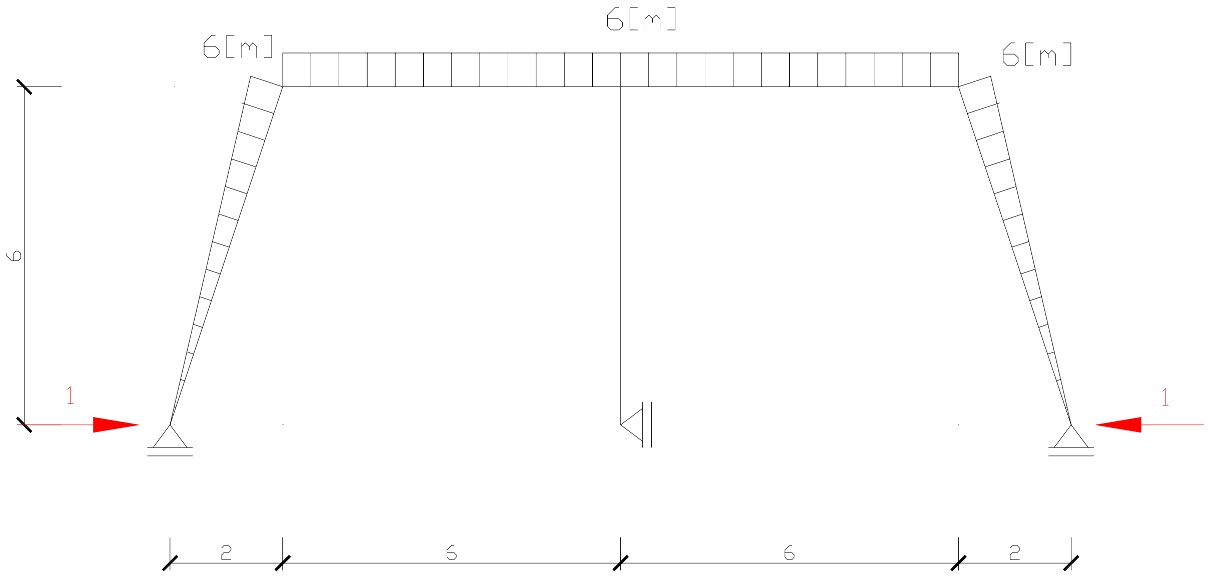
$t_m = 10^0 C$ $\quad t_0' = 0^0 C$

$EI = 672{,}4 [kNm^2]$ $\quad t_0'' = 20^0 C$

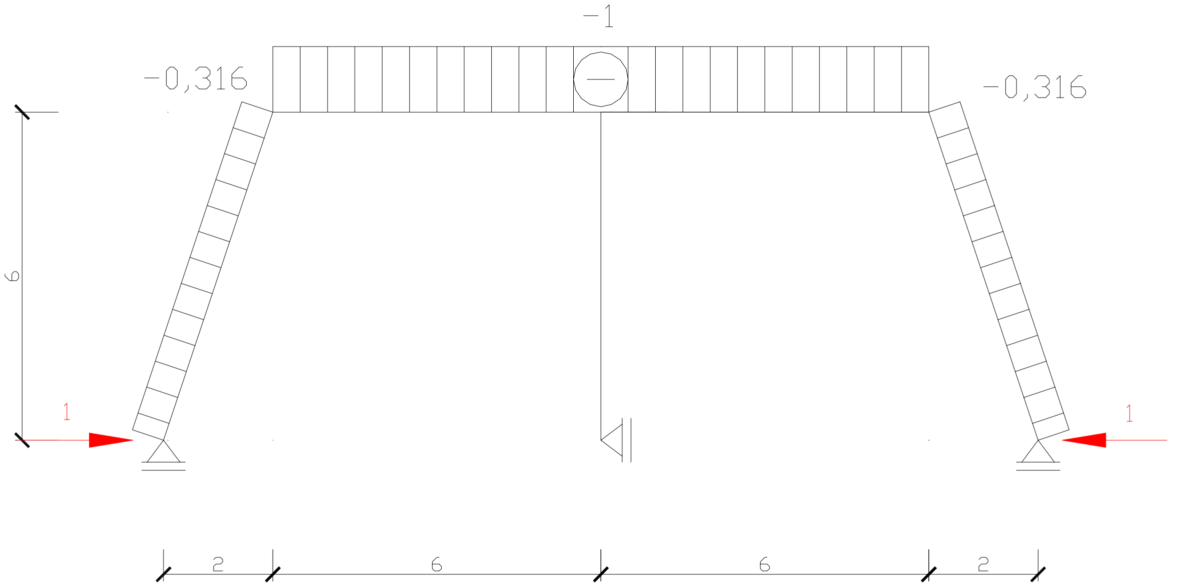
Delty od temperatur obliczam według wzoru:

$$\Delta_{it} = \int M_i \frac{\alpha_t \Delta t}{h} ds + \int N_i \alpha_t t_0 ds$$

M₁

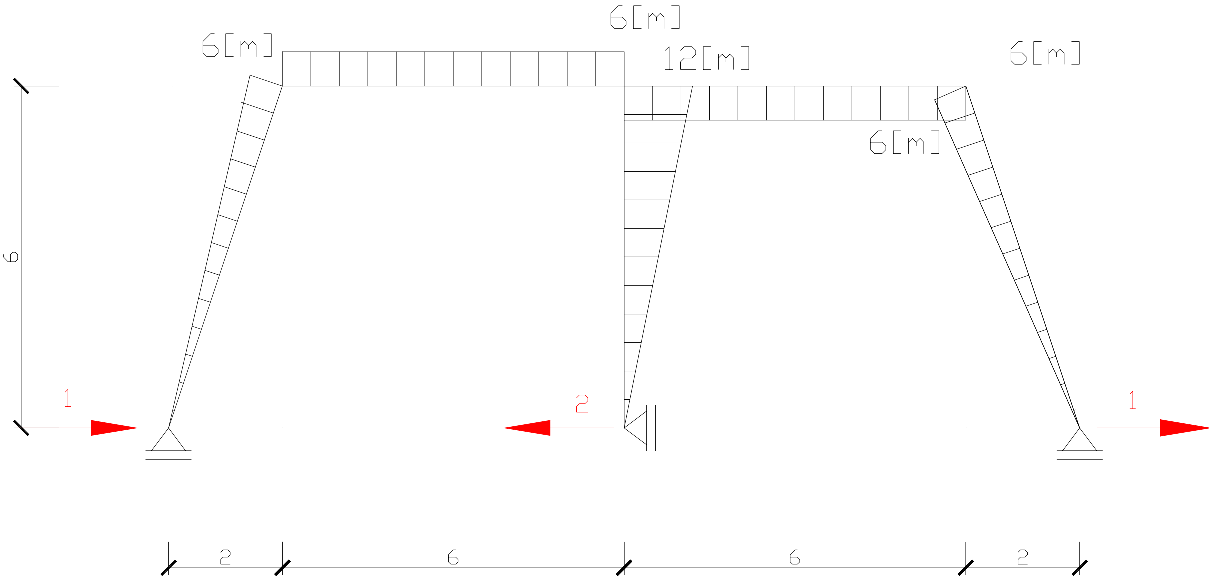


N₁

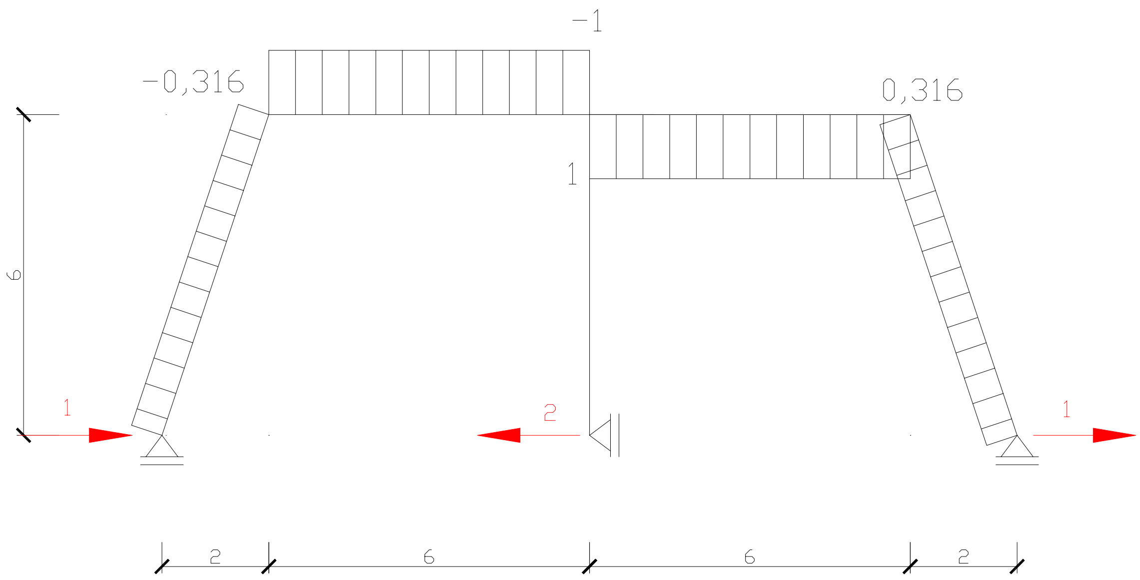


$$\Delta_{it} = \int M_i \frac{\alpha_t \Delta t}{h} ds + \int N_i \alpha_t t_0 ds = -\left[2 \cdot \left(\frac{1}{2} \cdot \sqrt{40} \cdot 6 \cdot \frac{40}{0{,}12} \cdot 12 \cdot 10^{-6}\right) + \left(6 \cdot 12 \cdot \frac{40}{0{,}12} \cdot 12 \cdot 10^{-6}\right)\right] = -0{,}4397893$$

M₂

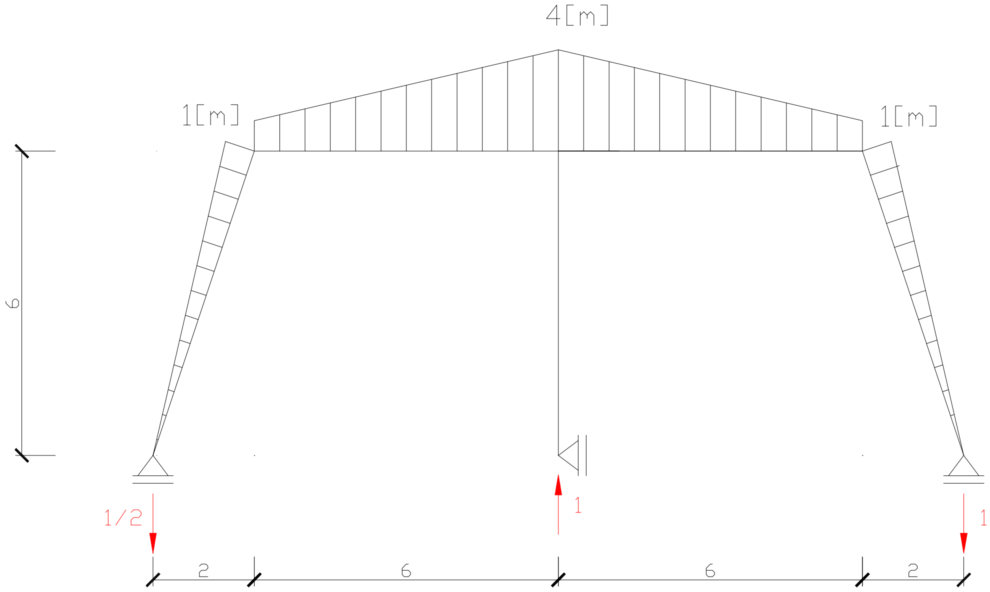


N₂

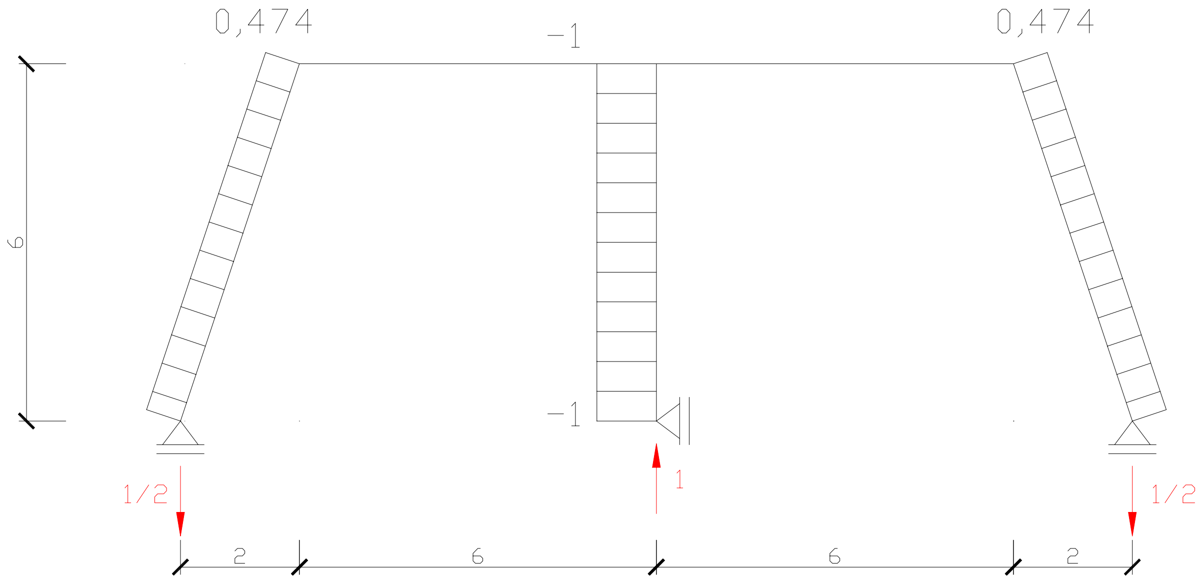


$$\Delta_{it} = \int M_i \frac{\alpha_t \Delta t}{h} ds + \int N_i \alpha_t t_0 ds = (symetria\ i\ \Delta t = 0) = 0$$

M_3



N_3



$$\Delta_{it}=\int M_i\frac{\alpha_t\Delta t}{h}ds+\int N_i\alpha_t t_0 ds=-\left[2\cdot\left(\frac{1}{2}\cdot\sqrt{40}\cdot1\cdot\frac{40}{0{,}12}\cdot12\cdot10^{-6}\right)-2\cdot\left(\frac{5}{2}\cdot6\cdot\frac{40}{0{,}12}\cdot12\cdot10^{-6}\right)\right]-\left[1\cdot6\cdot1{,}2\cdot10^{-5}\cdot20\right]=-0{,}146738$$

Układ równań kanonicznych:

$$\begin{cases}\delta_{11}\cdot X_1+\delta_{12}\cdot X_2+\delta_{13}\cdot X_3+\Delta_{1t}=0\\ \delta_{21}\cdot X_1+\delta_{22}\cdot X_2+\delta_{23}\cdot X_3+\Delta_{2t}=0\\ \delta_{31}\cdot X_1+\delta_{32}\cdot X_2+\delta_{33}\cdot X_3+\Delta_{3t}=0\end{cases}$$

Podstawiamy obliczone delty od wpływu temperatur:

$$\begin{cases} \left(48 \cdot \sqrt{10} + 216\right) \cdot X_1 + 0 \cdot X_2 + \left(8 \cdot \sqrt{10} + 90\right) \cdot X_3 - EI \cdot (0{,}439789) = 0 \\ 0 \cdot X_1 + \left(48 \cdot \sqrt{10} + 504\right) \cdot X_2 + 0 \cdot X_3 + EI \cdot (0) = 0 \\ \left(8 \cdot \sqrt{10} + 90\right) \cdot X_1 + 0 \cdot X_2 + \left(\frac{4}{3} \cdot \sqrt{10} + 42\right) \cdot X_3 - EI \cdot (0{,}146738) = 0 \end{cases}$$

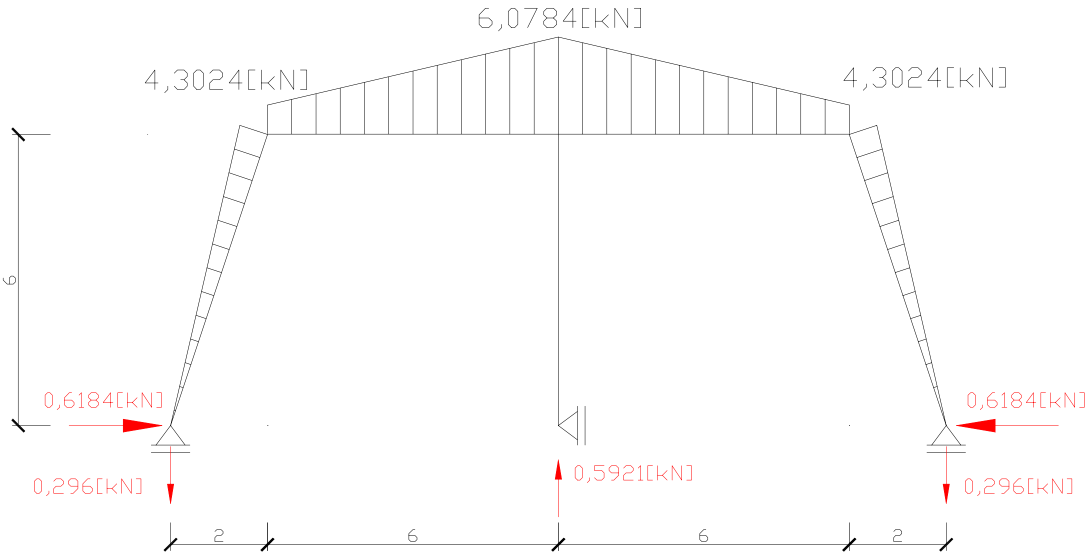
$X_1 = 0{,}6184[kN]$

$X_2 = 0[kN]$

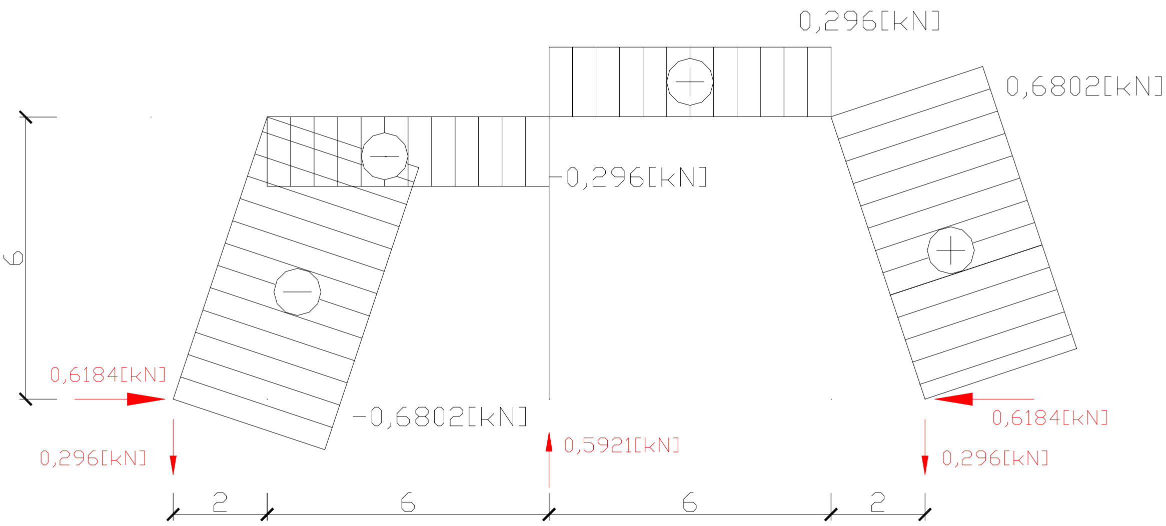
$X_3 = 0{,}5921[kN]$

Wykres końcowy od wpływu temperatury:

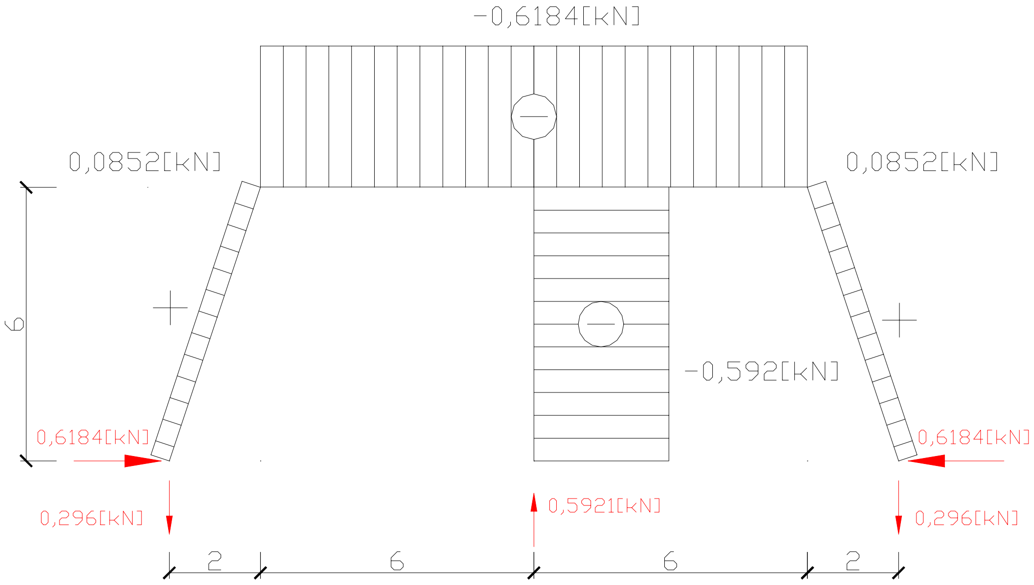
M$_t$



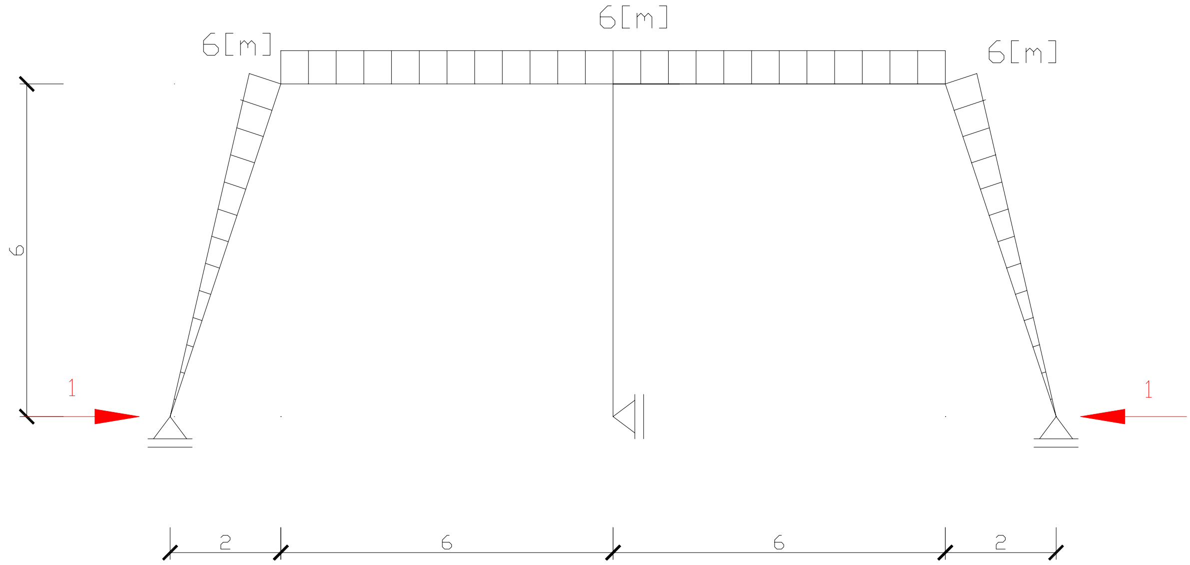
T$_t$



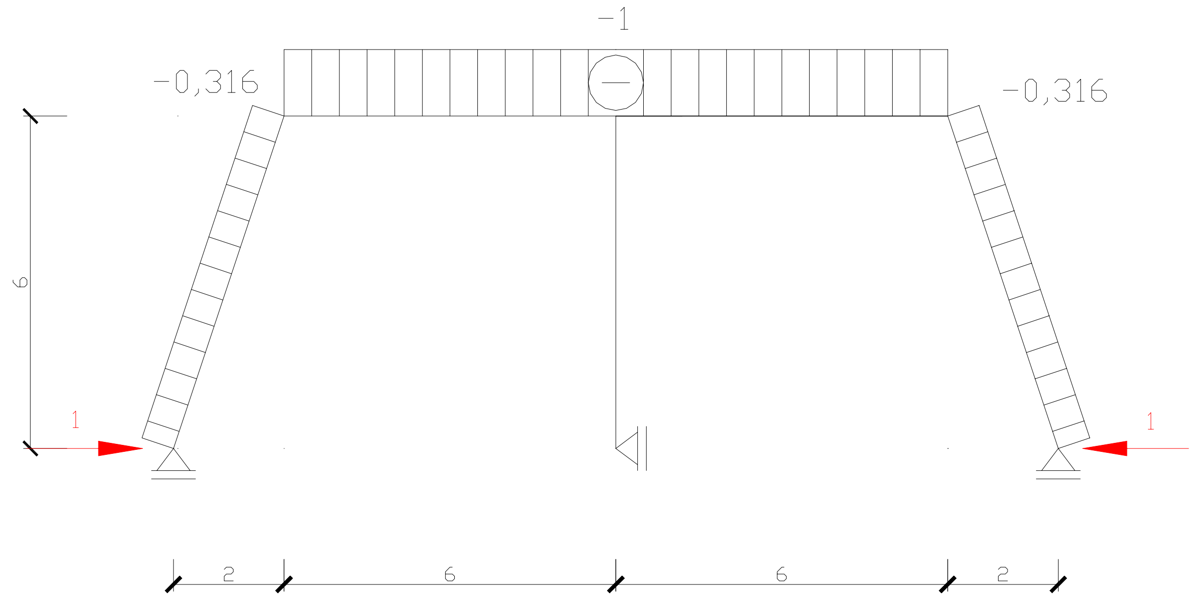
T_t


-0,6184[kN]
0,0852[kN]
0,0852[kN]
-0,592[kN]
0,6184[kN]
0,6184[kN]
0,296[kN]
0,5921[kN]
0,296[kN]
6
2
6
6
2

M_i


6[m]
6[m]
6[m]
6
1
1
2
6
6
2

N$_i$



$$V_k = \int \frac{M_t \cdot M_i}{EI} ds + \int M_i \frac{\alpha_t \Delta t}{h} ds + \int N_i \alpha_t t_0 ds = 0$$

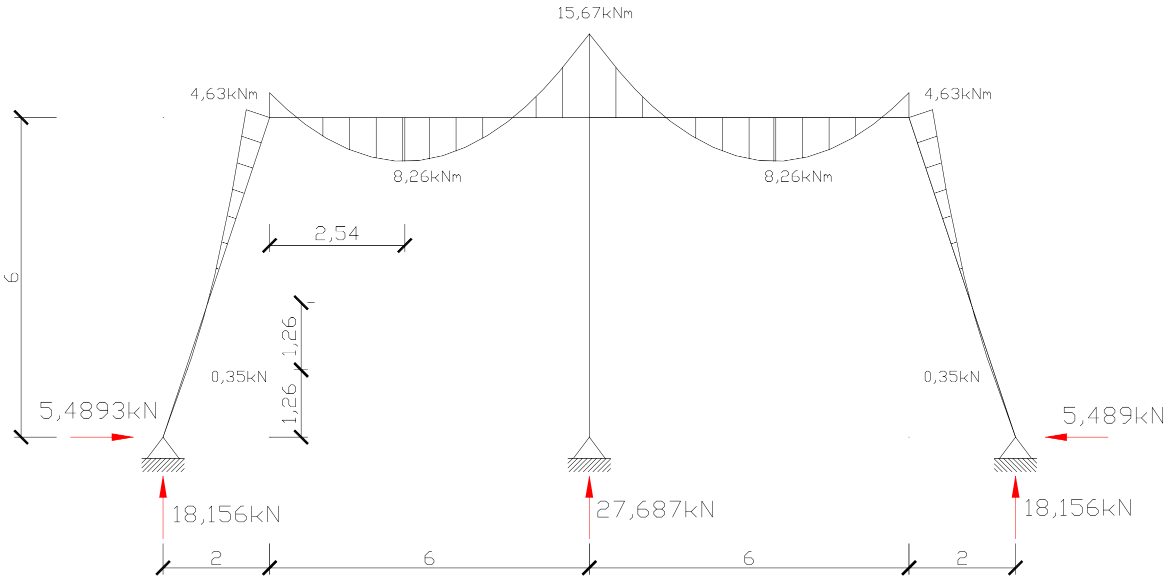
$$V_k = \frac{1}{EI} \cdot 2 \cdot \left[\frac{1}{2} \cdot \sqrt{40} \cdot 4{,}3024 \cdot 4 \right] + \frac{1}{2EI} \cdot 2 \cdot \left[\frac{6{,}0784 + 4{,}3024}{2} \cdot 6 \cdot 6 \right] - \frac{\alpha_t \cdot \Delta t}{h} \cdot \left[\frac{1}{2} \cdot \sqrt{40} \cdot 6 + 12 \cdot 6 \right]$$

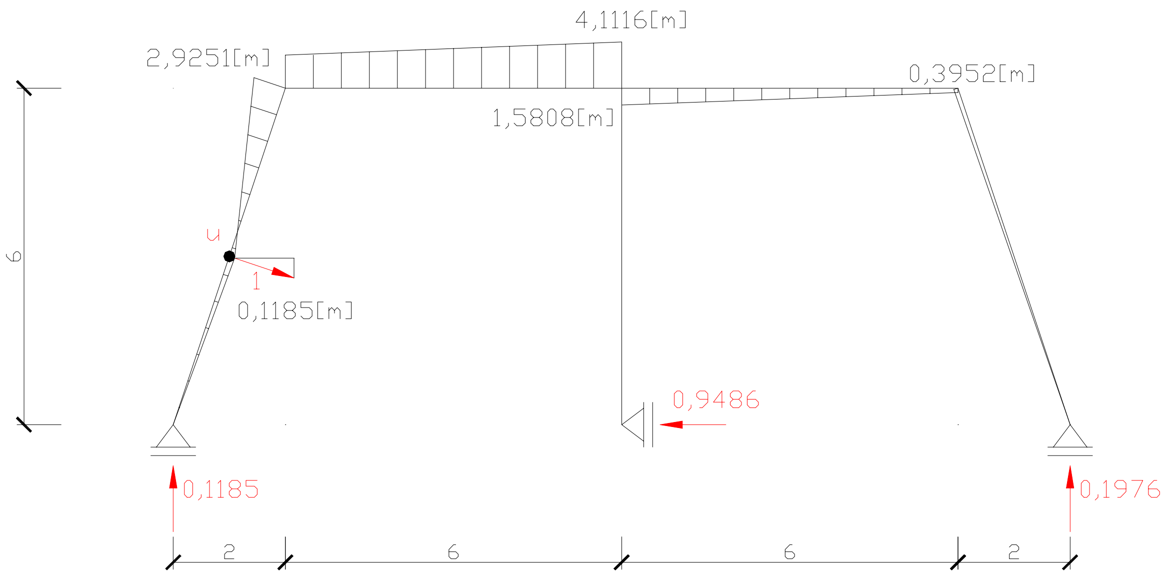
$$V_k = \frac{1}{672{,}4} \cdot 2 \cdot \left[\frac{1}{2} \cdot \sqrt{40} \cdot 4{,}3024 \cdot 4 \right] + \frac{1}{672{,}4} \cdot \left[\frac{6{,}0784 + 4{,}3024}{2} \cdot 6 \cdot 6 \right] - 0{,}004 \cdot \left[\frac{1}{2} \cdot \sqrt{40} \cdot 6 + 12 \cdot 6 \right]$$

$$V_k = 0{,}000025099 [m]$$

Obliczam zadane przemieszczenie

Korzystam z twierdzenia redukcyjnego. Wykorzystuję końcowy wykres momentów dla układu statycznie niewyznaczalnego i rysuję wykres momentów od przyłożonej jednostkowej siły wirtualnej dla schematu zastępczego.





$$V_u = \int \frac{M_n \cdot \overline{M}}{EI} ds$$

$$V_k = \frac{1}{EI} \cdot \left[\begin{aligned} &\frac{1}{2} \cdot \frac{2}{3} \cdot \sqrt{40} \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{5}{8} \cdot 0{,}1185 \right) + \frac{1}{2} \cdot \frac{1}{2} \cdot \sqrt{40} \cdot 2{,}315 \cdot \left(\frac{2}{3} \cdot 0{,}1185 \right) + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{5}{8} \cdot 0{,}1185 - \frac{3}{8} \cdot 2{,}9251 \right) + \\ &+ \frac{1}{2} \cdot \frac{1}{2} \cdot \sqrt{40} \cdot 2{,}315 \cdot \left(-\frac{2}{3} \cdot 0{,}1185 + \frac{1}{3} \cdot 2{,}9251 \right) + \frac{1}{2} \cdot 4{,}63 \cdot \frac{1}{2} \cdot \sqrt{40} \cdot \left(\frac{2}{3} \cdot 2{,}9251 - \frac{1}{3} \cdot 0{,}1185 \right) \end{aligned} \right] +$$

$$+ \frac{1}{2EI} \cdot \left[\frac{1}{2} \cdot 4{,}63 \cdot 6 \cdot \left(\frac{2}{3} \cdot 2{,}9251 + \frac{1}{3} \cdot 4{,}1116 \right) + \frac{1}{2} \cdot 15{,}67 \cdot 6 \cdot \left(\frac{1}{3} \cdot 2{,}9251 + \frac{2}{3} \cdot 4{,}1116 \right) + \frac{2}{3} \cdot 6 \cdot \frac{4 \cdot 6^2}{8} \cdot \left(-\frac{2{,}9251 + 4{,}1116}{2} \right) \right] +$$

$$+ \frac{1}{2EI} \cdot \left[\frac{1}{2} \cdot 15{,}67 \cdot 6 \cdot \left(-\frac{2}{3} \cdot 1{,}5808 - \frac{1}{3} \cdot 0{,}3952 \right) + \frac{1}{2} \cdot 4{,}63 \cdot 6 \cdot \left(-\frac{1}{3} \cdot 1{,}5808 - \frac{2}{3} \cdot 0{,}3952 \right) + \frac{2}{3} \cdot 6 \cdot \frac{4 \cdot 6^2}{8} \cdot \left(\frac{1{,}5808 + 0{,}3952}{2} \right) \right] +$$

$$+ \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot \sqrt{40} \cdot 4{,}63 \cdot \left(-\frac{2}{3} \cdot 0{,}3952 \right) + \frac{2}{3} \cdot \sqrt{40} \cdot \frac{4 \cdot 2^2}{8} \cdot \left(\frac{0{,}3952}{2} \right) \right] = -0{,}0093[m]$$