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CALCULEMUS!
NEW RESEARCH RESULTS AND FUNCTIONAL MODELS REGARDING LEIBNIZ’ FOUR FUNCTION CALCULATING MACHINE AND BINARY CALCULATING MACHINE

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The decimal carries of Leibniz’ Four Function Calculating Machine have mainly been discussed herein. This machine may not complete the decimal carries which is indicated by the pentagonal discs in skew positions. This can be overcome by setting the input number to zero and turning the crank in idle mode at the end of a calculation. N. J. Lehmann, Dresden, built several replicas in the 1980s with decreasing spreading angels of the twin-horn wheels from right to left in order to complete the decimal carries without post-turning the crank which, however, is not sufficient in general. F. O. Kopp, K. Popp\textsuperscript{†} and E. Stein additionally reduced the pitch angle of the step drums, based on analytical and numerical optimizations, to get the correct decimal carries and built a fully functioning Hannover model in 2004/05, scale 2:1. Moreover, a new construction and functional model of the “machina arithmeticae dyadicae” (a binary calculating machine for adding and multiplying with down-rolling metal balls) after Leibniz’ description has been presented.

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1. LEIBNIZ’ INVENTION OF FOUR FUNCTION CALCULATING MACHINES

The creative history of counting and calculating devices goes back to the pre-Christian centuries. The Chinese Abacus with lengthwise relocatable knobs is probably among the oldest. The ancient Greeks already used complex gear mechanisms, see e.g. [1,2].

Wilhelm Schickard, a professor of theology, built a 6-place calculating machine for multiplication and dividing in Tuebingen in 1623. This machine had rotatable cylinders in which the small multiplication tables were embedded (as in Napier’s Bones). The original machine got lost during the Thirty Years’ War, but it could have been replicated in Tuebingen after World War II owing to a description of its design in a letter to Johannes Kepler from 20 September 1623.

The philosopher and mathematician Blaise Pascal commissioned the construction of an 8-place adding and subtracting machine- the Pascaline- in 1644.

Gottfried Wilhelm Leibniz, however, was not aware of these two machines when he set work on the design of a novel decimal Four Function Calculating Machine (for all four basic arithmetic operations) in Mainz in 1670/71. He had been improving it from 1672, during his stay in Paris abundant in discoveries and inventions, and there he had watchmakers build the machine with 4/7/3 places. Leibniz presented it to the Royal Society London in 1673. Unfortunately, the machine was not in working order and thus earned Leibniz the disapproval of the Society president Robert Hooke. Nevertheless, he became a member.

Leibniz’ first Four Function Calculating Machine had pin wheels for the number input with pins shifted radially outwards. These were gripped by the reception gear-wheel.

The new 8/16/1-place machines, developed after 1682 and built in Hannover, Braunschweig, Wolfenbuettel and Zeitz from 1693 on, had stepped drums (with cogs of varying length which were axially shifted under the reception gear-wheel). These machines represented another fundamental leap in the development. A stepped drum had nine cogs with decreasing length for the figures 1 to 9. These cogs were positioned on half the circumference with a pitch angle of $\alpha = 22.5^\circ$.

This ingenious invention was a result of Leibniz’ “ars inveniendi” in terms of synthesis, combinatorics in particular. Its validity lasted well into the 20th century. Moreover, this machine was a product of a logically abstract
thinking process, and Leibniz was not afraid of realizing its high technical complexity in order to systematically allow for all four basic arithmetic operations with homogenous, simple mechanically executable algorithms.

This was achieved by the subdivision into two main assembly groups: (i) the input device, which was located in a carriage axially shiftable by means of a crank, with a maximum of 8 places, and (ii) the 16-place result device for adding and subtracting, including the necessary decimal carries, while multiplication and dividing were achieved by multiple additions and subtractions respectively in different carriage positions. The two-step decimal carries with mechanical amplifier, next to the logically related pin wheels and stepped drums, made up the machine’s ingenious core. The single-horn wheel on the reception gear-wheel’s shaft rotated the five-bay wheel by 18°. This is “bearing ‘one’ in mind”, so to speak. Due to the rotation, the five-horn wheel, which was located on the same shaft (but 18° in advance), got into the twin-horn wheel’s sphere. The twin-horn wheels were located between the stepped drums and were primarily driven by the Magna Rota crank ("Magna-Rota-Kurbel", MRK). Turning the MRK further up to 360° caused the twin-horn wheel to carry along the respective five-horn wheel by 360°/5-18°=54°. By this, the five-bay wheel also rotated by 54° and, as the actual decimal carry, carried the counting wheel of the next left place along by an angle that should ideally be 36°. However, in Leibniz’ machine, as well as in Lehmann’s replicas, this angle was only 21.5°. Leibniz’ calculating machine had a cycloid and special gearing, but it could not be a gear unit with only one actual degree of freedom. Instead, when the MRK was turned with a certain amount of speed, it allowed the counting wheels’ free flight, possibly by 360° or even more, due to the possible impulsive contact by the five-bay wheels.

A geometrically and technically elegant realization of a compulsory guide for the five-bay wheels would have been possible, as shown by F. O. Kopp, but was not part of Leibniz’ design.

Leibniz invented the impressive new construction of the dwelt-notch wheels, in order to achieve a counting wheel rotation of 36° (instead of 21.5°) even with a slow MRK rotation. These dwelt-notch wheels (located on the shafts of the five-bay wheels and five-horn wheels) and the counting wheels had spring-mounted rolls that enforced the difference rotation of 36°-21.5°=14.5° by means of flank pressure on the tooth flanks. The rolls would stop in the notches.

Moreover, Leibniz had large pentagon discs attached to the extended shafts of the five-bay wheels (as well as five-horn wheels and dwelt-notch wheels) on the back of the machine. The pentagon discs upper sides had to be horizontal at the beginning of a calculation.

Leibniz’ secretary Johann Georg Eckhart wrote him on 1 December 1699: “Adam asks Your Excellency to report if the little main shafts, to which the
pentagonal wheels, showing if the gear performed well, will be attached, shall be lengthened."

Apparently, Leibniz had realized that the non-completion of the decimal carries (after full rotations of the Magna Rota crank) was indicated by the tangential deviation of the pentagon discs’ upper edges; hence, these discs served merely as an indicator, not as a device for corrections.

The following pictures show Leibniz’ last and only remaining „large“ Four Function Calculating Machine, which was built and improved from shortly before 1700 till his death in 1716. It is now in the possession of the GWLB (Gottfried Wilhelm Leibniz Library) Hannover.

Fig. 1. Photos of Leibniz’ last large Four Function Calculating Machine, built from ~1700 till 1716: a) view from above, b) view from beneath

A virtually authentic replica of Leibniz’ machine was built by Klaus Badur and Wolfgang Rottstedt, Garbsen, in 2003 and 2004. As shown with this machine, non-completed decimal carries can be corrected subsequently. After the execution of all the additions and subtractions, the input number (multiplier or divisor) has to be set to zero, and as many additional full MRK rotations as necessary have to be carried out, running idle, causing the slanted pentagon discs to move into horizontal basic position again. After this, the number input is repeated and the calculation can be continued. With this clearly enhanced mechanical algorithm, Leibniz’ machine operates correctly in the whole available range of numbers. This, however, depends on the maximum admissible number of input places, which has resulted from our research as n = 8 – exactly the number of places Leibniz used.

The spreading angles of the twin-horn wheels are crucial for the two-step decimal carry. The twin-horn wheels are directly driven by and located between the stepped drums. According to measurements by F. O. Kopp, K. Popp and E. Stein from January 2004, the spreading angles of the 8 twin-horn wheels in Leibniz’ original machine are between 86° and 94°, thus averaging out at 90°.
2. DRESDEN REPLICAS BY NIKOLAUS J. LEHMANN

Nikolaus Joachim Lehmann designed and built three replicas of Leibniz „large“ machine by order of the Council of Ministers of the former German Democratic Republic in the 1980s. He had done detailed preliminary inspections together with the mechanists M. Göbel and K. Rühle from the Institute of Scientific Computing of the Technische Universität Dresden. The replicas are as close to the original as possible but have evolvent cogwheels instead of regular cycloid cogwheels. Lehmann’s goal was the realization of the complete decimal carry in the normal calculation process by means of twin-horn spreading angles decreasing from left to right, [3,4]. It is uncertain whether Lehmann knew the alternative solution of an enhanced algorithm due to the slanted pentagons, as described in chapter 1.

Lehmann’s improvement of Leibniz’ calculating machines was based on the correct conclusion that a certain difference angle between two adjoining twin-horn wheels (from right to left) is needed to pass all incoming decimal carries completely on to the left; Lehmann realized in his replicas $30^\circ \leq \Delta \alpha \leq 33^\circ$, according to the following twin-horn spreading angles, decreasing from right to left: $\max(2\alpha_{ZH}) = \alpha_1 = 132^\circ$, $\alpha_2 = 102^\circ$, $\alpha_3 = 69^\circ$, $\alpha_4 = 36^\circ$, $\alpha_5 = -39^\circ$, $\alpha_6 = -69^\circ$, $\alpha_7 = -102^\circ$, $\min(2\alpha_{ZH}) = \alpha_8 = -135^\circ$. These angles are not given in Lehmann’s publications.

Investigations by the authors in chapter 3 have revealed that not only the largest/smallest spreading angles are bounded above/below, but also that the medial angles $\alpha_4, 5 = 36^\circ/-39^\circ$ are bounded below/above to avoid blockages and ensure operational reliability.

As stated in chapter 3, Lehmann’s decreasing twin-horn wheel angles, however, are insufficient for a correct completion of all decimal carries with the normal algorithm. One of Lehmann’s replicas showed that adding 99,999,999 + 1 results in the carry of only the four “nines” on the right, but not the remaining ones. This is indicated by the slanted pentagon discs (as perceived by Leibniz). Thus, it is obvious that even Lehmann’s replicas require a further rotation of the Magna Rota crank under restrictive conditions. The authors have analyzed these conditions in detail in the Hannover Model 2005 of Leibniz’ machine and have realized them constructively.

3. THE HANNOVER MODEL BY KARL POPP†, ERWIN STEIN UND FRANZ OTTO KOPP

Karl Popp† and Erwin Stein built a functional model of Leibniz’ machine, supported by the German Research Foundation (DFG) in the years 2003 - 2005, at
the Institutes of Mechanics and Gear Mechanics for the Leibniz Exhibition. This functional model is as close to the authentic as possible, with doubled interspace between the stepped drums, [5].

The exhibits are supposed to meet the demand of the exhibition “living, comprehensible Leibniz”. They do not only have to be clearly comprehensible and accessible but also must fulfill several reliability requirements, e.g. automatic locking of the carriage crank during Magna Rota rotations (and vice versa) to prevent damage by improper handling. The main goals of this project were: constructive and mathematical optimization of the decimal carry and determination of the maximum number of input places for Leibniz’ construction principle. This was combined with the construction of a sturdy, inexpensive functional model, scale 2:1, and large scale (8:1) acrylic models of the stepped drum and the decimal carry.

The following perspective drawings show the working structure of the number input, Fig. 2 a), and the two-step decimal carry, Fig. 2 b). The number input uses the stepped drum with nine cogs of gradually varying length. It is axially shifted beneath the input wheel to the position of the desired number.

During the first step in the left part of the decimal carry, the single-horn wheel rotates the (odd-numbered necessarily) five-bay wheel by 18° (in Leibniz’ and Lehmann’s construction; 14° in the optimized Hannover Model). Consequently, the five-horn wheel, which is located 18° in advance of the five-bay wheel, gets into the sphere of the respective twin-horn wheel and is further rotated by 72° - 18° = 54° (in the Hannover Model: 72° - 14° = 58°), [6].

In the second step, the five-bay wheel rotates the next left counting wheel by only 21.5° – instead of the desired 36° (in the Hannover Model it is rotated by 31.5°). As stated above, the desired rotation of the counting wheel by 36° per decimal place is achieved by flank pressure of the spring-mounted rolls. This also moves the dwelt-notch wheel, which is located on the same shaft as the five-bay wheel and five-horn wheel, and the counting wheel into the correct “digital” position.

From gear kinematic studies and mathematical analysis, chapter 4, follows for the Hannover model that the largest/smallest twin-horn wheel spreading angles are bounded by max(2αZH) = |min(2αZH)| = 2·68.65° = 137.3° to avoid feedback (access from behind) of an input wheel with the longest cog of a stepped drum, i.e. to avoid blockage. The smallest twin-horn wheel spreading angle min(α4) = max(-α5) is bounded by 40° to prevent the five-horn wheels from perpetually accessing the twin-horn wheels’ spheres. The larger the twin-horn wheel radius, the larger becomes the max(α4).

An important result of the research in our Hannover DFG project is the fact that a decimal carry that has not been completed (indicated by a slanted pentagon disc) after a full rotation of the MRK can be correctly finished by a further rotation of the MRK (and, by this, of the twin-horn wheels). The whole
necessary rotation angle of the twin-horn wheel is nec $\Phi_{ZH} = 360° + \Delta \Phi_{ZH}$, nec $\Delta \Phi_{ZH} = \max(2\alpha_{ZH}) + \alpha^*_{FH}$, with the maximum half twin-horn wheel rotation angle $\alpha_{ZH}$ and the switching angle $\alpha^*_{FH} = 16°$ of the decimal carry from the twin-horn wheel to the five-horn wheel; $\alpha^*_{FH} = \frac{1}{2} \Delta \alpha_{ZH}$ (different twin-horn wheel spreading angle).

For the Hannover Model (HM) and the Dresden Replica (DR) result: (HM) nec $\Delta \Phi_{ZH} = 68.65° + 16° = 84.65°$ and (DR) nec $\Delta \Phi_{ZH} = 67.5° + 16° = 83.5°$.

Fig. 2. Perspective drawings of the number input and the decimal carry: a) number input by means of a stepped drum, b) decimal carry of Leibniz’ calculating machine

Obviously, this further rotation has to be performed with the input number still set. The admissible rotation angle $\Phi_{Stw}$ of the stepped drum from the starting position to the beginning of a new adding or subtraction sequence thus depends on the pitch angle $\alpha_K$ of the stepped drum cogs (Leibniz and Lehmann: $\alpha_{K,DR} = 22.5°$) and the complete angle $8 \cdot \alpha_{K,DR} = 180°$, chapter 1. In the Hannover Model these are optimized to $\alpha_{K,HM} = 21°$ and $8 \cdot \alpha_{K,HM} = 168°$.

Consequently, $\Phi_{Stw} = 90° + (180° - 8 \cdot \alpha_K)/2 - \varphi_3$, with $\varphi_3 = 9°$ (the sum of the half cog-width angles of the stepped drum and the input wheel) follows. For the Hannover Model and Dresden Replica the results are: (HM) $\Phi_{Stw} = 90° + 12°/2 - 9° = 87°$ and (DR) $\Phi_{Stw} = 90° - 9° = 81°$.

With this, the difference between the admissible and the necessary further rotation angles of the MRK beyond 360° until the completion of the decimal carries, $87.0° - 84.65° = 2.35°$, is positive. Thus, a completion of the decimal carry is possible, in this case constructively realized by means of a disc behind the MRK with a stopper at ± 87°. On the other hand, the difference angle in Leibniz’ and Lehmann’s machines, $81.0° - 83.5° = -2.5°$, is negative. Here, a further rotation of the MRK in order to complete the decimal carries is not possible, in principle, without setting the input number to zero.

From the difference between the largest and the smallest admissible half spreading angles of the twin-horn wheels and the necessary difference angles
follows the maximum number of input places for Leibniz’ construction concept. For the Hannover Model the difference angle is \( \max(\alpha_k) - \min(\alpha_k) = 137.3^\circ - 43.1^\circ = 94.2^\circ \), although it would be possible to further decrease \( \min(\alpha_k) \) by 3°.

With the chosen \( n = 8 \) places, the difference angle \( \Delta \alpha_{HM} = 94.2^\circ / 3 = 31.4^\circ \) results. This difference angle is equally and sufficiently large, and for every 4 input places symmetrical to the middle; in the HM the angle \( \Delta \alpha_{HM} = 2 \cdot 16^\circ = 32^\circ \) was realized. With ten input places there would result a difference angle \( \Delta \alpha^* = 94.2^\circ / 4 = 23.1^\circ \); this would be much too small for safe decimal carries.

The result is similar in the Dresden Replica, but here the difference angles vary between 30° and 33°.

All in all, it has been shown that only 8 input places are possible, as, in fact, used by Leibniz in his ingenious construction. This result is proven analytically in the following chapter.

4. MATHEMATICAL OPTIMIZATION OF THE DECIMAL CARRY IN THE HANNOVER MODEL BY KARIN WIECHMANN, ERWIN STEIN AND FRANZ OTTO KOPP

A mathematical multi-objective optimization has been carried out on the basis of the complete analytical description of Leibniz’ machine’s kinematics in line with plane trigonometry. Five equality side conditions \( h = [GNB1, GNB2, GNB3, GNB4, GNB5] = 0 \) and six inequality side conditions \( g = [UNB1, UNB2, UNB3, UNB4, UNB5, UNB6] \leq 0 \) have to be fulfilled; these will be described in detail, including illustrations, in a projected contribution to the Studia Leibnitiana, and they also can be found in [7].
Fig. 3. Mathematical optimization of the Hannover Model and objective function

Objective function of the optimization

\[ ZF = (\alpha_{GMR} - 18^\circ)^2 + (\alpha_{GZR} - 36^\circ)^2 + (\alpha_{ZH \text{ max}} - 90^\circ)^2 \]  \hspace{1cm} (4.1)

with the optimal angles

\[ \alpha_{GMR} = 13.84^\circ, \] \hspace{1cm} (4.2)
\[ \alpha_{GZR} = 31.55^\circ, \] \hspace{1cm} (4.3)
\[ \alpha_{ZH \text{ max}} = 68.66^\circ \] \hspace{1cm} (4.4)

and the minimum of the objective function

\[ \sqrt{f(x_{\text{HM}})} = 22.19^\circ. \] \hspace{1cm} (4.5)

The 8 design variables \( x_i \), \( i = 1 \) to 8, are: the radii of the five-bay wheels, single-horn wheels, five-horn wheels, twin-horn wheels and counting wheels, furthermore the rotation angles of the five-bay wheels and counting wheels, and the maximum half twin-horn wheel spreading angle, summed up in the vector \( x = [R_{MR}, R_{EZ}, R_{FH}, R_{ZH}, R_{ZR}, \alpha_{GMR}, \alpha_{GZR}, \alpha_{ZH \text{ max}}] \). The square objective function consists of three objectives: five-bay wheel rotation by single-horn wheel close
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to 18°, counting wheel rotation by five-horn wheel (and five-bay wheel) close to 36° and half twin-horn spreading angle as close as possible to 90°.

From these the superordinate optimization objective, the maximum number of input places, can be determined.

The respective exact penalty function

\[ P(x; \varepsilon) = f(x) + \sum_{j=1}^{5} \varepsilon | h_j(x) | + \sum_{i=1}^{6} \varepsilon \max(0, g_i(x)) \]  

(4.6)

with the penalty parameter \(10^3 \leq \varepsilon \leq 10^4\) has been optimized, using the program “Mathlab”, by means of a genetic algorithm with a downstream descent procedure.

The optimization results are to be found in the following chart:

Table 1. Optimization of the gear radii and angles of the decimal carry

<table>
<thead>
<tr>
<th>Lauf</th>
<th>(R_{MR})</th>
<th>(R_{EZ})</th>
<th>(R_{FH})</th>
<th>(R_{ZH})</th>
<th>(R_{ZR})</th>
<th>(\alpha_{GMR})</th>
<th>(\alpha_{GZR})</th>
<th>(\alpha_{ZH\text{max}})</th>
<th>(\alpha_{ZH\text{min}})</th>
<th>(\sqrt{ZF})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.78</td>
<td>11.00</td>
<td>18.22</td>
<td>31.51</td>
<td>12.57</td>
<td>13.79°</td>
<td>29.26°</td>
<td>72.32°</td>
<td>19.84°</td>
<td>19.39°</td>
</tr>
<tr>
<td>2</td>
<td>26.86</td>
<td>10.83</td>
<td>18.46</td>
<td>31.23</td>
<td>12.52</td>
<td>13.58°</td>
<td>29.79°</td>
<td>72.37°</td>
<td>19.45°</td>
<td>19.21°</td>
</tr>
<tr>
<td>4</td>
<td>26.78</td>
<td>10.96</td>
<td>18.38</td>
<td>31.24</td>
<td>12.11</td>
<td><strong>14.42°</strong></td>
<td>27.01°</td>
<td>73.32°</td>
<td>19.13°</td>
<td>19.28°</td>
</tr>
<tr>
<td>HM</td>
<td><strong>26.50</strong></td>
<td><strong>11.13</strong></td>
<td><strong>19.00</strong></td>
<td><strong>31.45</strong></td>
<td><strong>13.00</strong></td>
<td><strong>13.84°</strong></td>
<td><strong>31.55°</strong></td>
<td><strong>68.66°</strong></td>
<td><strong>21.50°</strong></td>
<td><strong>22.19°</strong></td>
</tr>
</tbody>
</table>

The Hannover Model with the counting wheel angle of \(\alpha_{GZR} = 31.55°\) comes closest to the goal of 36°. However, the maximum twin-horn spreading angle of \(2 \cdot 73.32° = 146.64°\) (as in run 4) is not achieved with this, but only \(2 \cdot 68.66° = 137.32°\), which is sufficient for 8 input places.

The maximum number \(n\) of input places for Leibniz’ way of construction results from this as

\[ n = 2(\text{Integer}[(\alpha_{ZH\text{max}} - \alpha_{ZH\text{min}})/\Delta\alpha_{ZH}] + 1). \]  

(4.7)

The biggest difference \(\alpha_{ZH\text{max}} - \alpha_{ZH\text{min}} = 73.32° - 19.13°\) results in run 4. From this follows

\[ n = 2(\text{Integer}[(73.32° - 19.13°)/15.70°] + 1) = 2(\text{Integer}[3.45] + 1). \]  

(4.8)
Fig. 4. Hannover Model 2005 of Leibniz’ Four Function Calculating Machine with doubled stepped drum interspace, 6 input places, 12 resulting places and constructional optimizations; K. Popp†, E. Stein and F. O. Kopp (2005)

Fig. 5. Hannover Model 2004 of a 7/12/5-place binary calculating machine for addition and multiplication according to a description by G. W. Leibniz in 1679; E. Stein, G. Weber (2003/04)
5. THE HANNOVER MODEL OF LEIBNIZ’ DYADIC CALCULATING MACHINE BY ERWIN STEIN AND GERHARD WEBER

Leibniz described a binary calculating machine (working with the numbers 0 and 1) for addition and multiplication, the “Machina Arithmeticae Dyadicae”, in the handwritten Latin text “De Progressione Dyadica” in 1679. Instead of a gear mechanism (as in the decimal Four Function Machine), it uses small metal spheres that, because of gravity, roll out of the moveable carriage (“box”), which contains the binary input device, down a doubly-sloped layer into the stationary calculating and result device, which are, together with the binary carries, the most important elements of the construction. This machine thus has the same two main groups as the decimal gear machine: the shiftable input device and the calculating device.

Ludolf von Mackensen translated the Latin text in 1968 and designed a 7/12/5-place binary calculating machine, which was built by the Deutsches Museum in Munich in 1971 and by the Hessisches Landesmuseum in Kassel in 1985.

In the decimal carry of the binary addition 01 + 01 = 10 (decimal: 1 + 1 = 2), the second sphere released from the input device rolls over a small spring-mounted trigger plate at a binary place. This causes the first sphere, located at the respective place in the result device, to be released, rolling down into a reservoir at the bottom on the left. The second sphere is slowed down by a small guiding plate, which is slightly sloped towards it, and then guided into the next left result place. However, this transmission is not reliable due to the sensitive slope combined with both slants of the doubly sloped layer of the whole machine, allowing the spheres to roll beyond the intended next left place. Moreover, the spheres rolling to the left can cause blockages with the spheres following from above during the binary carries, if the carriage is shifted downward to the front too fast. The installation of the board with the calculating device into a wooden cabinet, modeled on other historical examples, unfortunately does not allow an insight into the calculation process.

After borrowing Mackensen’s machine for the Leibniz Exhibitions several times, the desire to have a reliable machine with an insight into the calculating process (according to the demand of the exhibition “living, comprehensible Leibniz”) arose. A three-place game model of the binary machine was built by Gerhard Weber, Staufenberg-Speele, in 2000. In this functional model, right-angled hooks (composing sticks) with torsion springs and stoppers were used for the binary carries instead of the above described small guiding plates. These hooks reliably guide the spheres rolling down out of the binary carry each into the next left binary place in the result device.
Erwin Stein and Gerhard Weber then designed, computed and built a new 7/12/5-place binary machine with the assistance of Franz Otto Kopp, Hannover, and Jürgen Anton, Institute of Mechanics, University of Hannover, in 2003/2004, Fig. 5. The cabinet of the calculating device and carriage are made of acrylic. The pitch of the doubly-sloped layer has been optimized, and the carriage guiding has been dissected in both directions (to realize the controlled digital steps at an admissible speed). Return of the steel spheres through a hose into the reservoir of the input device by means of a knurled head screw (thus in a closed system) allows for safe handling. A significant innovation of this machine is the binary carry with catching hooks made of synthetics and an indicator for the number “1” that is swiveled in above, and furthermore the shaft with a balance-wheel torsion spring and stoppers beneath the board, Fig. 6.

Fig. 6. New construction of the binary carry in the binary calculating machine according to Leibniz: a) catching hook with indicator swiveled above a sphere in the result device, b) sphere guiding from the input device and shaft of the binary carry with catching hook, reset torsion spring and stoppers

All elements have been optimized to ensure the fastest possible reset of the catching hooks and, thus, a free roll off of the spheres. For this reason, small high-quality bearings and torsion springs had to be used. Logically, the binary calculating machine described by Leibniz can be considered as the predecessor of binary computers. The first mechanical one, the Zuse Z1, was built and patented by Konrad Zuse in 1936.
BIBLIOGRAPHY


It is our great pleasure to devote this paper to Professor Andrzej Garstecki, the esteemed colleague and friend of Professor Erwin Stein, especially regarding his noteworthy work and interest in structural optimization. Therefore, we are pleased to present, for the first time in English, our recent research results concerning the analytical and numerical optimization of Leibniz’ Four Function Calculating Machine here.