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DETERMINING OF THE SOIL STRENGTH CHARACTERISTICS THROUGH THE PLATE BEARING TEST

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In the international literature there is often a confusion between the terms Modulus of Elasticity, E_{Young}, and Modulus of Deformation, E_{Def} which is sometimes plainly referred to as Modulus E. In this paper the definitions of both the E_{Young} and the E_{Def} have been clarified. Moreover, a soil classification diagram has been derived based on numerical analysis through a finite element software where soils are classified according to their shear strength parameters (c, φ) and their behavior under loads generated using a rigid circular bearing plate. Finally, a method for the determination of shear strength parameters, as well as the E_{Young} of soils has been proposed using the in-situ Plate Bearing Test and solving a 3x3 mathematical system. The study and documentation of the proposed method is supported through three theoretical examples.

Key words: plate bearing test, modulus of elasticity, modulus of deformation, cohesion, internal friction angle

1. INTRODUCTION

In highway earthworks the knowledge of the Modulus of Elasticity of soils is of major importance, whether the projects are in the stage of design, construction or compaction check. However, due to the linear nature of highways where different geologies follow one another, the overall procedure for the determination of the modulus of elasticity has to be done in an easy and

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reliable manner. As the laboratory Triaxial Test is both costly and time-consuming, the less elaborated Plate Bearing Test is usually performed to obtain the Modulus of Deformation rather than the Modulus of Elasticity. It should be mentioned that, although soils are elastoplastic middles, the determination of the Modulus of Deformation is based on the elastic theory of Boussinesq.

2. BOUSSINESQ'S THEORY

The issue of the vertical displacement under a circular area carrying a uniform pressure on the surface of a semi-infinite, homogenous, isotropic mass is a classical problem in highway engineering. For the solution of this problem, analytical and numerical methods have been proposed. Boussinesq (1885) gave the first complete solution for the settlement of elastic masses. Under a concentrated vertical load acting in its centerline, the rigid circular plate will settle uniformly (Fig.1). The distribution of vertical stresses in the centerline is found by integration, whence the compression stress along z axis of a cylindrical element in the centerline can be calculated. Thus

$$\sigma_z = \frac{1}{E} \cdot (\sigma_z - 2 \cdot \nu \cdot \sigma_r), \tag{2.1}$$

and the displacement of a layer extending from z=0 to z=z becomes

$$\Delta z = \frac{1}{\mathrm{E}} \int_0^z (\sigma_z - 2\nu\sigma_r) dz. \tag{2.2}$$

Since in the case of rigid, circular plate (Kezdi και Rethati, 1988)

$$\sigma_z - 2 \cdot \nu \cdot \sigma_r = \frac{1}{2}(1+\nu) p \sin^2 \omega \cdot (1 - 2\nu + 2\cos^2 \omega) \tag{2.3}$$

$$z = a \cdot \cot \omega, \tag{2.4}$$

$$dz = -a \cdot d\omega / \sin^2 \omega. \tag{2.5}$$

Hence,

$$\Delta z = \frac{1}{2}(1+\nu) \frac{pa}{E} \int_a^{\pi/2} (1 - 2\nu + 2\cos^2 \omega) d\omega = \tag{2.6}$$

$$(1+\nu) \frac{pa}{2\mathrm{E}} \left[2(1-\nu)(\frac{\pi}{2} - \omega) - \sin \omega \cos \omega \right]. \tag{2.7}$$

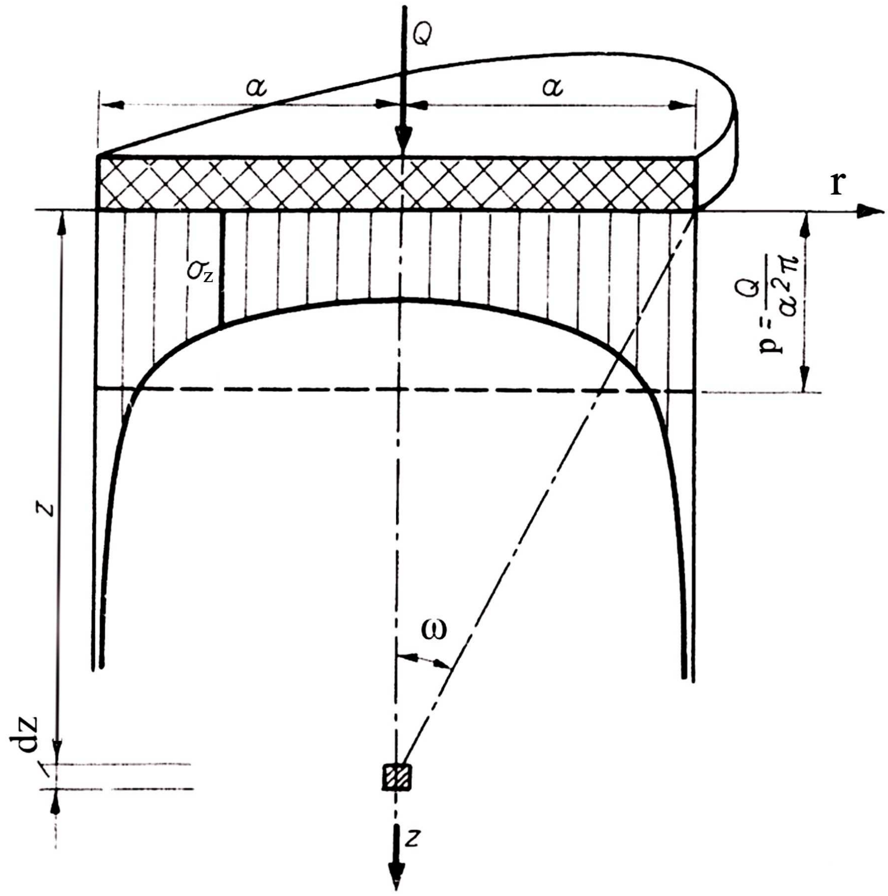

Fig. 1. Contact pressure distribution below a rigid circular plate loaded by a vertical singular force (Q) at the center (Kezdi και Rethati 1988) [1]

For semi-infinite, elastic mass, $z \rightarrow \infty$ also $\omega \rightarrow 0$, the total settlement becomes

$$\Delta z_o = \frac{\pi \cdot p \cdot a}{2 \cdot E}(1-\nu^2), \tag{2.8}$$

where p: uniform pressure that is applied on the circular rigid plate ($p=Q/\pi\alpha^2$), α: the radius of the plate, ν: Poisson ratio and E: the Modulus of Elasticity

3. MODULUS OF ELASTICITY (E_{Young}) AND MODULUS OF DEFORMATION (E_{Def})

The modulus of elasticity, for homogenous, isotropic masses, is given by the Hook's linear law of elasticity. The law in question states that the stress applied to a material is proportional to the strain on that material ($\sigma = E_{Young} \cdot \varepsilon$). Hooke's law only holds for some materials under certain loading conditions. Generally, the stress-strain behavior of materials, including soils, is elastoplastic.

The Modulus of Elasticity, which is an index of the material stiffness and a fundamental material constant, can be graphically defined by the slope of the tangent passing through the origin point O(0, 0) of a stress-strain diagram (tangent modulus). During the initial (small) loading increments materials exhibit elastic behavior and the displacements are resilient. An important remark is that the Modulus of Deformation, as it is defined and represented in Figure 2, is dependent on the loading pressure (σ). Graphically it is defined according to the slope of line OA (secant modulus).

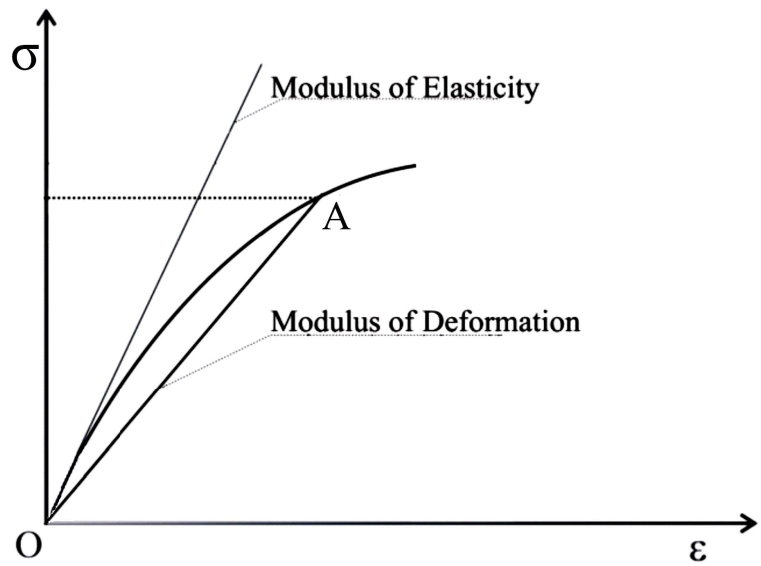

Fig. 2. The Modulus of Elasticity and the Modulus of Deformation [2]

4. PLATE BEARING TEST

The plate bearing test is intended for use in earthworks and foundation engineering, however, its main application is the compaction check of road and airfield earthworks. In other words, the objective of the test is the deformability assessment of soils or rocks. The test allows the determination of the relationship between the applied pressure and the displacements (pressure- displacement curve). According to the test procedure, a hydraulic device transfers pressure in stepwise through a circular rigid plate onto the surface of the earth or rock half-space, until the displacement or pressure criterion is satisfied. Rigid plates with diameter 300, 420, 600 and 760mm are the most common used on site.

Due to the fact that soils under loading exhibit elastoplastic behavior, the Modulus E that is derived through the plate bearing test, is the Modulus of Deformation (E_{Def}) and not the Modulus of Elasticity. The two moduli are approximately equal in the case of small loading values applied on strong middles (e.g. strong rock mass). According to the geotechnical textbooks, the determination of the modulus of deformation is done using the Boussinesq's mathematical formula - Equation (2.8) – which was initially proposed for elastic middles.

Fig 3. The Plate Bearing Test

5. CLASSIFICATION OF SOILS ACCORDING TO THEIR BEHAVIOR UNDER SPECIFIC LOADING CONDITIONS

As known, the deformability of soil subjected to loading depends on soil's shear strength parameters, that is, the cohesion c and the angle of internal friction φ. Therefore, depending on the applied stresses, soils may be deformed either elastically or elastoplastically.

In Figure 4 (cohesion – internal friction diagram) soils are classified in three categories for the specific loading of 250kPa distributed by a rigid bearing plate having 300mm diameter. Regarding the diagram in question, soils with c-φ pair of values that belong to “area A or Ø” exhibits elastoplastic behavior, whilst soils with c- φ pair of values that belong in “area B”, where Boussinesq’s theory stands, exhibits fully elastic behavior. “Area B” is alternatively called “Boussinesq’s Area”. Additionally, it should be mentioned that soils with c- φ pair of values that belong to “area Ø” have the ultimate bearing capacity less than 250kPa.

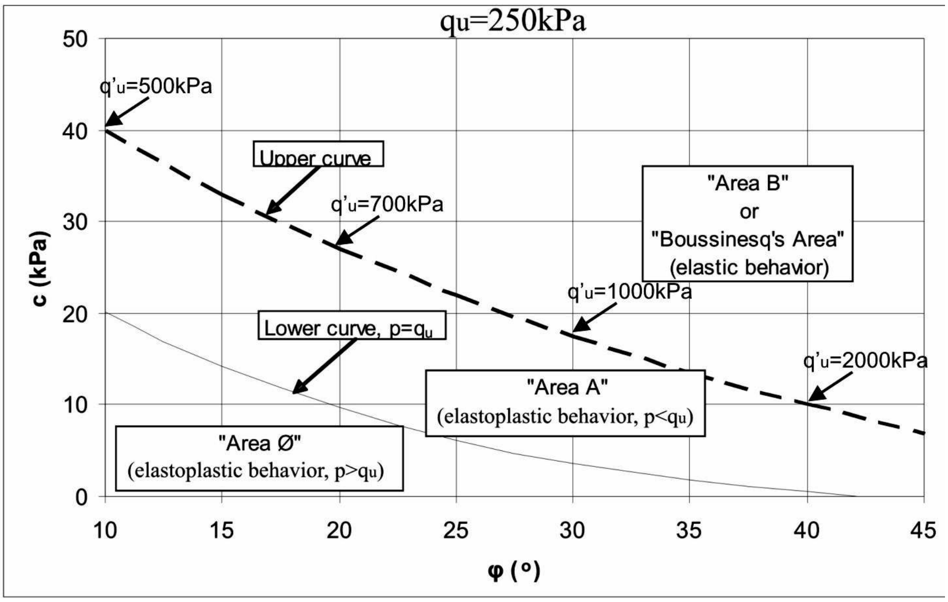

Fig. 4. Classification of soils in three categories (Ø, A and B), for p=250kPa and for rigid circular loading plate with diameter of 2α=300mm. Soils with pair of c- φ values on the lower curve have the ultimate bearing capacity q_u=250kPa.

The “lower curve” of the diagram of Figure 4 corresponds to soils that have the ultimate bearing capacity equal to 250kPa (q_u=250kPa). Although the present analysis was based on numerical methods (finite element software), the same curve can also be reproduced using Terzaghi’s bearing capacity formula completed with De Beer’s [4] empirical shape factors for rigid circular footings (Equation 5.1). The fact that both methodologies led to the same outcome (the plot of the same lower curve), leads to a conclusion that the overall procedure was properly made. The problem was solved for D_f=0.

$$q_u = c \cdot N_c \cdot (1+\frac{N_q}{N_c}) + \gamma \cdot D_f \cdot N_q \cdot (1+tg\phi) + \frac{1}{2} \cdot \gamma \cdot \mathrm{B} \cdot N_\gamma \cdot 0.6\,, \qquad (5.1)$$

where, c=cohesion, φ=the angle of internal friction, B = diameter of circular rigid plate, D_f = the foundation depth, γ = unit weight of soil, N_γ, N_c, N_q=bearing capacity factors.

The bearing capacity factors, N_γ, N_c and N_q, are calculated according to the following mathematical expressions proposed by Terzaghi

$$N_q = \tan^2(45+\phi/2)\cdot e^{\pi\cdot\tan\phi}, \tag{5.2}$$

$$N_c = (N_q - 1)\cdot\cot\phi, \tag{5.3}$$

$$N_\gamma = 2\cdot(\mathrm{N}_q + 1)\cdot\tan\phi. \tag{5.4}$$

Soils with c- φ pair of values that belong to the "upper curve" exhibit marginally elastic behavior for p=250kPa (e.g. c=30kPa and φ=17.5^o). Accordingly, for p>250kPa the behavior exhibited under the same loading conditions is elastoplastic. For the same loading of 250kPa the soil with the pair of values (c- φ) = (11kPa, 20^o) will have high elastoplastic behavior and for a little load increment the shear failure will occur.

An Organic Clay, with shear strength values c=4kPa and φ=22^o, according to Terzaghi's equation, has the ultimate bearing capacity q_u=127.8kPa using the plate with 300mm diameter. As it is clear from the Figure 4, this soil belongs to "Area Ø".

6. CORRELATION BETWEEN THE E_{Young} AND THE E_{Def}. DETERMINING OF THE STRENGHT PARAMETERS OF THE SOIL THROUGH THE PLATE BEARING TEST

As mentioned, the Modulus of Deformation is strongly dependent on the magnitude of the applied load, whilst the Modulus of Elasticity is a unique characteristic of every soil (under specific moisture conditions). A theoretical study based on numerical methods (finite element software) led to a correlation between the Modulus of Elasticity and the Modulus of Deformation. This correlation (Eq.6.1) is given introducing the I_L coefficient, which is dependent on the shear strength parameters (c, φ) of the soil examined, the radius α of the bearing plate and the magnitude of the load p. The correlation is given as

$$E_{Young} = E_{Def}\cdot I_L, \tag{6.1}$$

where I_L=I_L(c, φ, p, α) and $I_L \geq 1$.

DIN 18134 uses Boussineq's mathematical formula (law of elasticity) in order for the modulus of deformation to be defined. This definition is also used in this paper (Eq.6.2)

$$E_{Def} = \frac{\pi \cdot p \cdot \alpha}{2 \cdot dz} \cdot (1 - \nu^2)\,. \tag{6.2}$$

The coefficient I_L derived using the bearing plate with the diameter equal to 300mm and for loads from p=200kPa up to 500kPa is given in Table 1. The present methodology suggests the use of loads greater than 200kPa, avoiding the work-hardening phenomenon occurred in soils during the initial stress increments. Work hardening (or strain hardening) is the strengthening of a material by increasing the material's dislocation density.

Equation (6.1) has four unknown parameters, the Modulus of Elasticity E_{Young}, the cohesion c, the angle of internal friction φ and the Poisson ratio ν. Typical Poisson ratio ν values taken from the international literature could be used for the unknown variables to be reduced to three. A non-linear system with three equations and three unknowns can be defined inserting three pairs of values (p, dz) from the load-settlement curve using Equation (6.1). The solution of the system leads to the determination of the unknown parameters. An important note is that, since the angle of internal friction (φ) is unknown, both cases, for 10<φ≤23 and 23<φ≤45, must be examined and the one that stands is to be selected.

Table 1. The I_L coefficient for the bearing plate with diameter 2α=300mm.

p (*kPa*)	10<φ≤23			23<φ≤45		
	$I_L = \frac{k}{(\sin\phi)^l \cdot \left(\frac{c-c_o}{c_o}\right)^m}$			$I_L = \frac{k}{(\sin\phi)^l \cdot \left(\frac{c}{c_o}\right)^m}$		
	k	*l*	*m*	*k*	*l*	*m*
200	0.296	1.954	2.037-4.034·sinφ	0.535	2.242	2.464-3.176·sinφ
250	0.224	2.234	2.406-4.938·sinφ	0.488	2.417	2.723-3.475·sinφ
300	0.195	2.357	2.619-5.616·sinφ	0.488	2.451	2.575-3.236·sinφ
350	0.216	2.257	2.507-5.381·sinφ	0.480	2.496	2.647-3.285·sinφ
400	0.327	1.898	1.979-3.686·sinφ	0.510	2.376	2.319-2.786·sinφ
450	0.186	2.399	2.594-5.502·sinφ	0.505	2.391	2.416-2.888·sinφ
500	0.146	2.622	2.773-6.045·sinφ	0.483	2.477	2.443-2.859·sinφ
$c_o = 0.03 \cdot p$ and c_{min}=1kPa						

Table note: A minimum value of cohesion was chosen (c_{min}=1kPa) for functional purposes, since a value greater than zero (c>0) was a precondition for the finite element software to run.

In Table 2 three application examples for three different type of soils (OL, CL and CH) have been presented. The Modulus of Deformation has also been calculated according to the mathematical formula (Equation 6.2) used in DIN 18134 [5]. The Standard in question (DIN 18134) requires the determination of the Modulus of Deformation when the settlement value of 5mm or the normal stress below the plate of 500kPa is reached (precondition for the use of the bearing plate of 300mm diameter).

Table 2. Application examples

Soil	Data Values			E_{Def} (MPa) DIN 18134	Calculated Values		
	E_{Young} (MPa)	c (kPa)	φ (°)		E_{Young} (MPa)	c (kPa)	φ (°)
OL	37.0	10.0	22.0	11.0	35.0	14.2	18.7
CL	55.0	15.0	25.0	18.7	51.7	24.3	19.9
CH	25.0	20.0	15.0	11.2	23.4	32.5	8.6

According to Table 2, the E_{Def} values are much smaller than the respective E_{Young} values (data value), where the calculated values of E_{Young} are very close to the data values. This is indicative of the reliability of the proposed methodology regarding the determination of the Modulus of Elasticity. The method also gives an estimation for the c, φ values. A verification example of the proposed methodology is given in Figure 5 by collating the curve produced using the finite element software with the respective curve reproduced directly using Equation (6.1).

As known, the performance of the laboratory Triaxial Test using disturbed specimens may lead to erroneous outcomes regarding the Modulus of Elasticity, as well as the shear strength parameters of soils, especially in case of sensitive clays. Therefore, instead of the elaborated procedure mentioned, the proposed methodology could be used, giving satisfactory approaches for the Modulus of Elasticity and the shear strength parameters of soils.

7. CONCLUSIONS

The accurate determination of the Modulus of Elasticity is necessary for a consistent design and quality control of highway earthworks, since the Modulus of Deformation is strongly dependent on the magnitude of the applied load. Thus, different values of Modulus of Deformation derived from different geological standards, may not be comparable. The common worksite practice is the determination of the Modulus of Deformation using the Plate Bearing Test, instead of the Modulus of Elasticity through the more complex and elaborated laboratory Triaxial Test. In the present paper a theoretical study based on numerical analysis using a finite element software for the correlation of the Modulus of Elasticity with the Modulus of Deformation has been given. This correlation has been given introducing the coefficient I_L related to the shear strength parameters of soils (c and φ), the radius α of the rigid bearing plate and the magnitude of applied pressure p ($I_L = I_L(c, \varphi, p, \alpha)$). The whole analysis was based on the concept that the stress-strain curve is characteristic of every soil. According to the proposed correlation, the Young Modulus is given by the product of the Modulus of Deformation times the I_L coefficient $\left(E_{Young} = E_{Def} \cdot I_L\right)$. The Modulus of Elasticity can be determined through the aforementioned equation and the three pair of values (p, dz) taken from the loading curve (Plate Bearing Test). Additionally, a soil classification is given according to their shear strength parameters. The classification in question separates the soils that exhibit elastic behavior from those exhibiting the elastoplastic one.

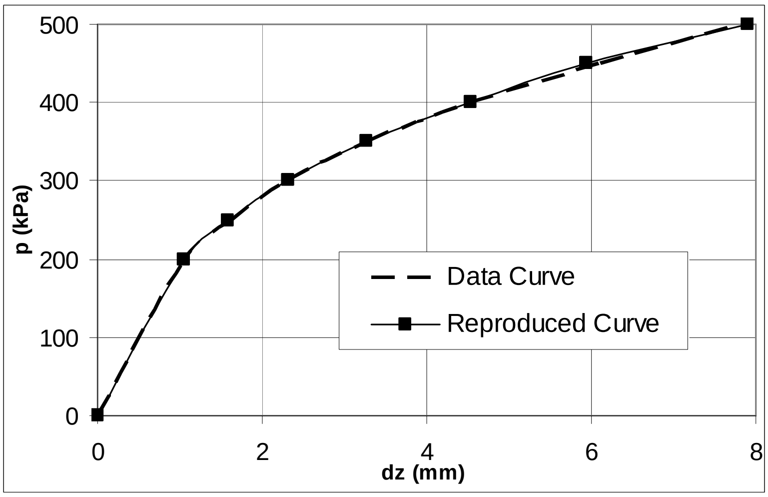

Fig. 5. Verification of the calculated values (soil: CL, Table 2)

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