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A SIMPLE CRITICAL STATE MODEL WITH SMALL STRAIN NONLINEARITY FOR OVERCONSOLIDATED SOILS

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The paper presents a reasonably advanced constitutive law for soil – a hybrid of the Modified Cam Clay and the Fahey – Carter models. The Modified Cam Clay model is an isotropic hardening elasto–plastic one originated by Burland (1967) within the critical state soil mechanics. This model is widely applied in today's geotechnical analysis. It realistically describes mechanical soil behaviour in the normal consolidation states. The other one is designed to ensure more adequate soil responses to reloading paths (within the overconsolidation stress subspace), particularly in the range of small strains.

The model has been implemented in the FEM computer code Z_SOIL.pc 7.32. To test the influence of the small strain nonlinearity on soil – structure interaction as well as to exhibit the ability of the proposed model to simulate this effect realistically, a comparative study based on the FEM solution has been carried out. As a benchmark, a strip footing founded on a thick homogeneous cohesive soil layer in two different overconsolidation states has been used. The soil response to a load has been simulated assuming two models: original MCC and FC+MCC.

Key words: overconsolidation, small strain nonlinearity, Modified Cam Clay, Fahey – Carter model

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1. THE MODEL CONCEPT

Macroscopic soil behaviour under loading is, in general, very complex. Even for the simple case of a monotonic working load, in the range between 40% and 50% of the foundation bearing capacity, linear elastic idealization of soil is totally incorrect. This approach does not allow to realistically evaluate all the mechanical fields within the loaded soil body. Even if the average settlement was determined accurately as a result of a suitable choice of the soil stiffness represented by the mean value of the deformation modulus in the subsoil area, a reliable prediction of the settlement distribution on the structure – soil contact interface, and even more so – of the contact stress distribution, for the same data, would still be impossible. The reasons for those limitations are obvious. If the soil of which the ground is composed is lightly overconsolidated, its plastic deformations, as well as the stress path sensitivity and convex nonlinearity of the shear characteristics associated with those deformations, occur on the level of the working load in large zones of subsoil, and essentially influence the contact quantities and internal forces in the structure. These effects can be simulated with a reasonable accuracy using one of the simple critical state models in numerical interaction analyses. This could be, for example, the Modified Cam Clay, the elasto - plastic model with isotropic hardening law depending on plastic volumetric strain or, equivalently, on irreversible changes of the void ratio. Its yield surface is assumed to be an ellipsoid of revolution in the stress space, collinear with the hydrostatic axis. The critical state describing the frictional failure mechanism completes this characteristic. A representation of this critical state in the stress space is the straight line connecting zenithal points of successive configurations of the yield surface. This so called critical state line passes through the origin of the stress coordinate system. The elastic deformations are defined by the elastic shear and bulk moduli dependent linearly on the effective mean stress.

Critical state isotropic hardening models are able to simulate deformations of lightly overconsolidated soils with reasonable accuracy. In this case, only short segments of stress paths are situated inside the initial yield surface. The application of the original Modified Cam Clay for the case of heavily overconsolidated deposits usually involves adopting a simple, nonlinear elastic approach using the shear and bulk moduli being linear functions of the mean pressure. Such a description does not account for strong physical nonlinearity in the range of small strains. This phenomenon has been observed in many high quality triaxial tests including local strain and wave velocity measurements, carried out for the last twenty five years. The essence of the above phenomenon is an abrupt (more than tenfold) drop in the soil stiffness at increasing deformations in the range from 0.001% to 0.1%. In light of the results of a number of studies (e.g. Burland, 1989, Jardine et al., 1991) accounting for that experimental fact appears to be crucial for a realistic prediction of subsoil responses to working loads transmitted from structures.

The Fahey – Carter model (Fahey and Carter, 1993) is one of the concepts which have been specially developed in order to describe the nonlinearity defined above in a realistic way and as simply as possible,. This is a hypoelastic model and at the same time a certain improvement on the known Duncan's proposals (Duncan and Chang, 1970, Duncan, 1980). The Fahey – Carter model does not explicitly take into account the plastic deformations and the limit state. Using this model for a correct simulation of the soil behaviour within the whole range of loads up to the bearing capacity is therefore impossible or at least questionable.

This justifies the idea of combining both constitutive laws. The Fahey – Carter model is assumed to be valid within the overconsolidation area, i.e. in the stress subspace confined by the current configuration of MCC yield locus. For normal consolidation states, occurring in the remaining, physically admissible stress subspace, the mechanical soil behaviour is described by the Modified Cam Clay. A continuous transition from one model to another is achieved through treating the bulk and shear moduli as the same parameters, though differently defined within each of the above mentioned stress subspaces.

The second chapter of the paper specifies the proposed model. In the third chapter FEM formulation of the boundary value problem for a strip footing – thick homogeneous soil layer used as a benchmark for the study estimating the effectiveness of the model has been presented. The results of the interaction analysis performed using the FEM computer code Z_SOIL.pc 7.32. (expanded by the second author for the implementation of the combined constitutive model) have been presented in the fourth chapter. The concluding remarks are included in the fifth chapter.

2. MATHEMATICAL DESCRIPTION OF THE MODEL

The hybrid constitutive model for soil developed in this paper differs from the Modified Cam Clay only in "stress – strain" relations inside the state boundary surface in the stress space which encloses a subspace of real physical states of material. Since the Fahey – Carter model has been conceived as a hypoelastic one, the above difference does extend to the nature of the deformation. For both cases strains are assumed to be entirely reversible. In fact, the above divergence only concerns the forms of tangent shear, bulk and Young's elastic moduli as stress path dependent. While for Modified Cam Clay these are simple linear functions of the effective mean stress, for Fahey – Carter concept and thereby for the combined model inside the state boundary surface, the elastic moduli are to adequately describe the above mentioned phenomenon of strong small strain nonlinearity. For this purpose some complex hyperbolic functions of the stress intensity are required beside a simple dependence on the effective mean stress.

The key characteristic of the model proposed is the equation of the state boundary surface

$$F = q^2 + M^2 p' (p' - p'_c) = 0, \tag{2.1}$$

which at the same time is the yield locus for MCC.

In the Eq. 2.1 p’ is the effective mean stress, q – the stress intensity, p_c’ – the preconsolidation pressure, and M – the slope of the critical state line (CSL) in the p’– q space.

Identification of the state boundary (MCC yield) surface enables illustration of the division of the physically admissible subspace into the subregions of validity of the component models (Fig.1). It is necessary to remember that external subregion in which the MCC is valid evolves together with the irreversible changes of the void ratio. This state boundary locus expands when the soil undergoes densification, and contracts at soil loosening.

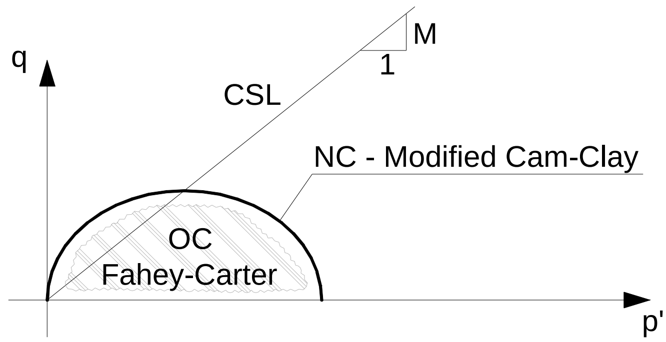

Fig. 1. Graphic representation of the combined FC+MCC model in the p’– q space

The above state boundary surface evolutions are governed by the law of isotropic hardening dependent on the irreversible changes of the void ratio (or volumetric strain)

$$p_c' = p_{c0}' \exp\left(\frac{\Delta e^p}{\lambda - \kappa}\right), \tag{2.2}$$

where p_{c0}' is an initial value of the preconsolidation pressure, Δe^p – the total irreversible change of the void ratio, λ and κ - slopes of the normal consolidation and swelling lines in semi – logarithmic scales.

The tangent elastic bulk modulus is given by

$$K_t = \frac{(1+e_0)p'}{\kappa}, \tag{2.3}$$

whereas the tangent elastic shear modulus by

$$G_t = \frac{3(1-2\nu)}{2(1+\nu)} K_t, \tag{2.4}$$

where ν is the Poisson's ratio assumed to be constant. Formulas (2.3) and (2.4) are valid on the state boundary surface, where they describe the elastic part of the deformation. The combined soil model turns there into Modified Cam Clay.

Inside SBS the hybrid model becomes the Fahey – Carter hypoelastic one. Its original version is defined by the tangent elastic shear modulus expressed as follows

$$G_t = G^* p_a \left(\frac{p'}{p_a}\right)^n \frac{\left(1 - f\left(\frac{q-q_0}{q_f - q_0}\right)^g\right)^2}{1 - f(1-g)\left(\frac{q-q_0}{q_f - q_0}\right)^g}, \tag{2.5}$$

where q_0 - initial (in situ) stress,
q_f - critical shear resistance,
G*, n, f, g - material constants.

The tangent elastic bulk modulus is given by equation

$$K_t = \frac{2(1+\nu)}{3(1-2\nu)} G_t. \tag{2.6}$$

Continuous transition from one model to another is secured when equating the tangent shear modulus given by formulas (2.4) and (2.3) to that defined by (2.5). Hence

$$\kappa = \frac{3(1-2\nu)(1+e_0)\left(1 - f(1-g)\left(\frac{q-q_0}{q_f - q_0}\right)^g\right)}{2(1+\nu)G^* p_a \left(\frac{p'}{p_a}\right)^{n-1}\left(1 - f\left(\frac{q-q_0}{q_f - q_0}\right)^g\right)^2}. \tag{2.7}$$

In Fig. 2, two sets of curves are quoted after Fahey and Carter (1993). The first presents diagrams of the relative secant shear modulus G_s/G_0 as the function of the deviatoric strain γ for various values of parameters f and g. The second shows analogical relations for the relative tangent modulus G_t/G_0. As can be seen, the Fahey – Carter model adequately describes the abruptly decreasing soil stiffness together with a growth of the deformation in the range of small strains.

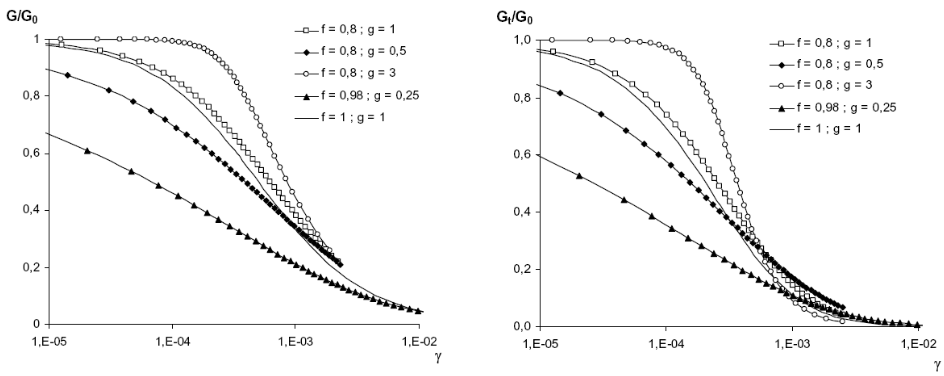

Fig. 2. The relative secant and tangent moduli as functions of deviatoric strain γ in Fahey – Carter model for various values of parameters *f* and *g* (Fahey and Carter, 1993)

3. APPLICATION OF THE MODEL IN AN ANALYSIS OF STRESSES AND DEFORMATION OF FOUNDATION SUBSOIL

The combined FC+MCC soil model has been applied to the static analysis of the subsoil of a strip footing 1 m wide, founded on the depth of 0.5 m. The subsoil is assumed to be a thick homogeneous cohesive soil layer. The relevant boundary value problem has been formulated respecting the plane strain conditions. An appropriate finite element mesh composed of isoparametric, four-noded quadrilaterals has been generated. The selected mesh is shown in Fig. 3.

The model has been implemented by the second author to the FE computer code Z_SOIL. pc 7.32.

The strip footing has been assumed to be the linear elastic, isotropic structure specified by the Young modulus E = 30 GPa and Poissons coefficient $\nu = 0.2$. The parameters for both compared subsoil models have been inserted into Table 1.

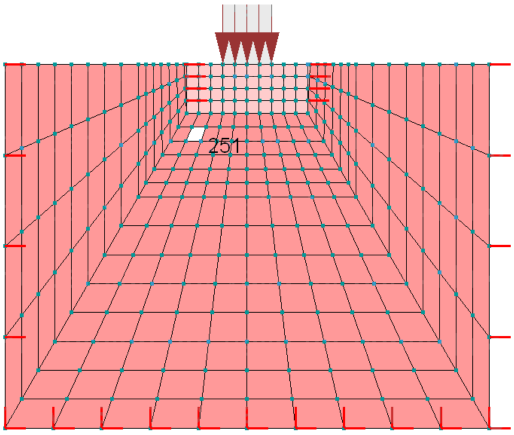

Fig. 3. The FEM mesh applied in the static strip footing – subsoil analysis

Table 1. Parameters of the subsoil model

Parameter	Modified Cam-Clay	FC+MCC
M	0,9	0,9
λ	0,05	0,05
κ	0,012	0,012 (κ_{max})
e_0	0,8	0,8
p_{c0}	50 kPa	50 kPa
OCR	5 ; 21	5 ; 21
ν	0,3	0,3
G^*	-	6000
n	-	0,7 ; 0,9
f	-	0,94
g	-	0,636

The parameters of MCC are based on the results of high quality triaxial tests carried out by Jastrzębska (2002) on the reconstituted samples of kaolin, subject to simple loading programs. The slope of the critical state line M has been evaluated from the shear characteristics in the critical state (for advanced distortional deformations), slopes λ and κ of the isotropic normal consolidation and the swelling lines in the semi logarithmic scales from compressibility tests, and modulus G from the unloading shear tests for a deviatoric stress path. FC parameters G*, n, f and g are the results of the optimal fitting of the model's "shear strain – shear modulus" relationship to the experimental characteristic given by Viggiani and Atkinson (1995).

The data inserted in Table 1 refer to two subsoil loading histories: a light overconsolidation identified by OCR = 5 on the foundation depth and a heavy one (OCR = 21). The selected values correspond with the so called erosion loads amounting to 40 kPa and 200 kPa.

4. RESULTS OF ANALYSIS

Fig. 4 shows the theoretical "load – settlement" characteristics for the subsoil model.

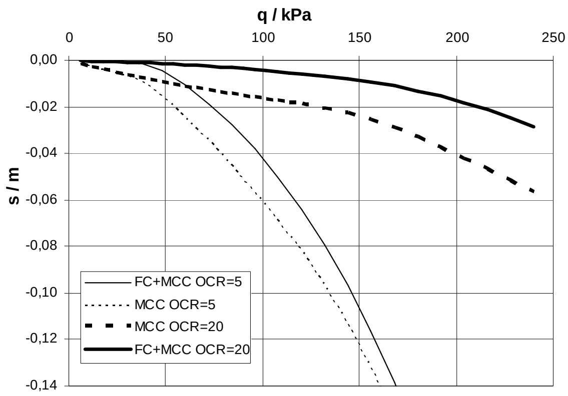

Fig. 4. Load – settlement characteristics for a lightly and a heavily overconsolidated subsoil described by the MCC and FC+MCC models

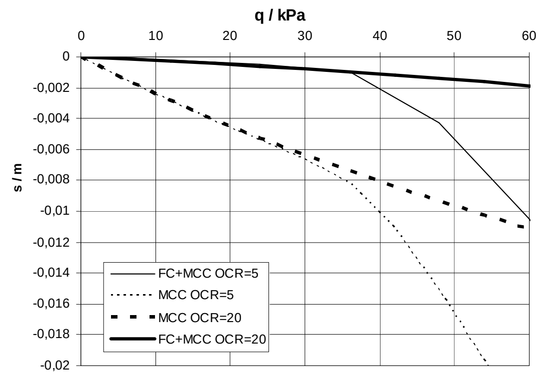

Fig. 5. Comparison of the theoretical load – settlement characteristics (presented in Fig. 3) for a range of small loads of the subsoil

The thin curves represent the case of light overconsolidation, the thicker ones - heavily overconsolidated soil. The continuous lines refer to the model analysed, and the dotted ones to MCC, used as the comparative base.

The results of the analysis confirm the crucial role of soil overconsolidation in stiffening and strengthening of the soil, already known from observations. For the purposes of the study, the difference in the settlement predictions determined by means of the proposed model and the original Modified Cam Clay is more important.

Particularly substantial divergences occur in the range of small loads transmitted from the footing to heavily overconsolidated subsoils (Fig. 5). Accounting for small strain nonlinearity by using the FC+MCC model enables more realistic settlement predictions for these cases. They appear to be more than four times smaller than the predictions obtained for the classic MCC.

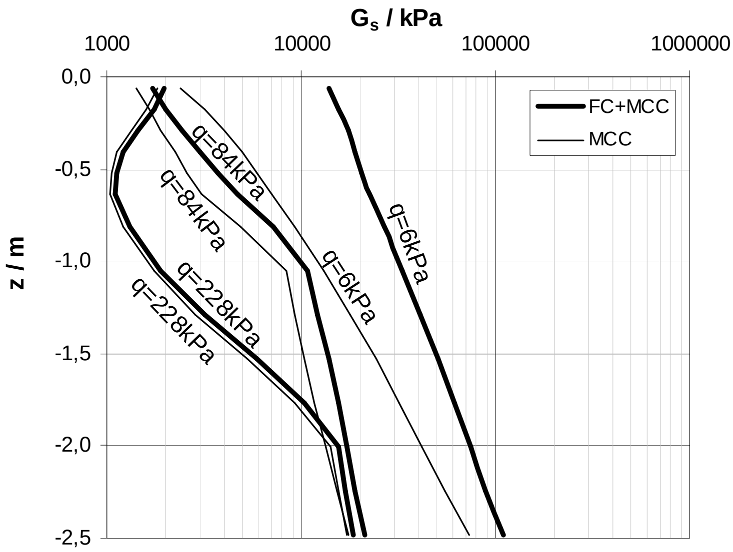

Fig. 6. Comparison of the distributions on the depth of the secant modulus along the footing axis for FC+MCC and MCC at different levels of load

The reason for such a divergence becomes evident when comparing distributions on the depth of the secant modulus along the footing axis for FC+MCC and MCC at different levels of load (Fig. 6). The secant modulus for FC+MCC on the foundation level at very small load, i.e. in the conditions close to those in situ, appears to be three times higher than that for classic MCC. This divergence decreases distinctly with a growth of the depth and, first of all, with the increasing load, but the global effect of the modulus on the settlement remains significant.

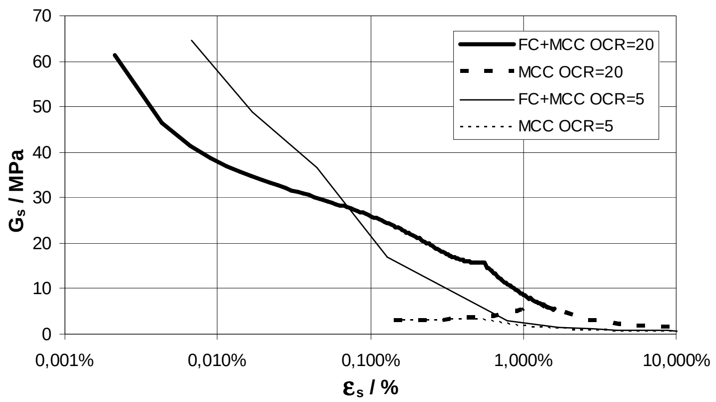

Fig. 7. A comparison of "deviatoric strain ε_s – secant shear modulus G_s" characteristics for
FC+MCC and MCC in the centre of element 251

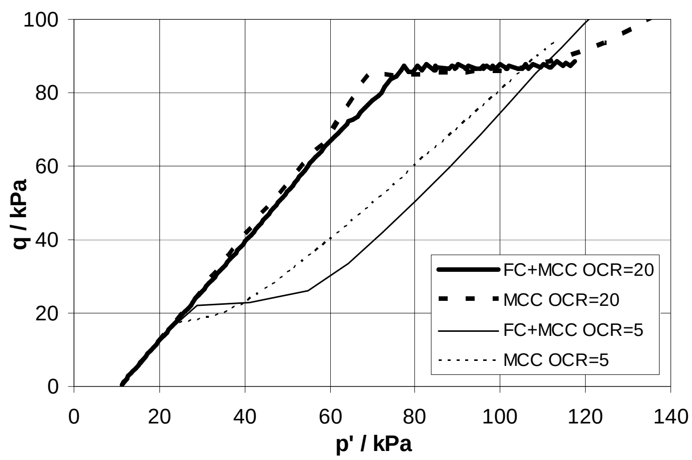

Fig. 8. The stress paths in the p'– q space in the centre of element 251

The comparison of the conventional characteristics "ε_s – G_s" in Fig. 7 provides even more convincing explanation of the differences between the models. The divergence occurs only in the overconsolidation subspace, but is quite fundamental.

On the other hand, the stress paths in the space of stress invariants p', q for FC+MCC and MCC at any point of subsoil differ very little from one another in the case of heavy overconsolidation and only somewhat more for lightly overconsolidated soil (both of the above properties are shown in Fig. 8).

5. CONCLUDING REMARKS

This paper presents a new simple critical state model which is a combination of the Modified Cam Clay describing the mechanical behaviour of soil in normal consolidation states and the Fahey – Carter model developed for improved simulation of soil response to loading paths in the overconsolidation states. A special emphasis has been put on the qualitatively correct mathematical description of a strong physical nonlinearity phenomenon in the range of small strains and on its implementation to the FEM analysis of the 3D geotechnical problem. The suitable algorithm has been included in the computer code Z_SOIL.pc 7.32. The numerical study of stress, strain and displacement fields in a subsoil beneath a strip footing has been focused on their dependences on the small strain nonlinearity and on the proposed model ability in this aspect.
Less attention has been devoted to precise model calibration. The MCC parameters have been adapted from the results of triaxial tests carried out by Jastrzębska (2002) on the samples of kaolin from Tułowice. On the other hand, the Fahey – Carter model parameters are based on fitting theoretical characteristics $G_s – \varepsilon_s$ in experimental data given by Viggiani and Atkinson (1995) for a silty clay. To fully verify the model, it is necessary to perform triaxial tests on samples of the same cohesive soil while applying local strain gauges and bender elements. Such tests are to be carried out next year in the laboratory of Geotechnical Department at Silesian University of Technology. Additionally, the inclinometer measurements in the subsoil of a tank are also planned for the purposes of verification.

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