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THE DETERMINATION OF SHEAR MODULUS IN OVERCONSOLIDATED COHESIVE SOILS

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The mechanical behaviour of soils is characterized through constitutive models which require a correct definition of the soil parameters. Prediction of subsoil deformation of geotechnical structures depends on the stiffness-strain curve of the soil and the stiffness at very small strains. Recently, the shear wave velocity measurement using bender elements in laboratory conditions has become a technique which permits to determine the initial shear modulus at strains of the order of 0.0001%. In this paper the criteria for the determination of the shear wave velocity in triaxial tests with piezoelements in cohesive soils have been suggested. The importance of initial signal frequency and optimal range of frequency to measure the shear wave velocity in clays have been discussed. Moreover, the paper presents some results of triaxial tests performed in order to obtain a shear modulus in overconsolidated Pliocene Warsaw clays.

Key words: shear wave, bender elements, shear modulus, stiffness

1. INTRODUCTION

The shear modulus G_0 at a very small strain (initial shear modulus) is a fundamental soil property important in practical geotechnical solutions, especially in earthquake engineering and in the prediction of soil structure interaction ([1-4]). Hardin & Black [5] identified major factors which influence the

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actual value of the shear modulus including vertical effective stress, void ratio, OCR, soil fabric, temperature and degree of saturation [5].

The shear wave velocity is related to the shear modulus of the soil. Consequently, the measurement of shear wave velocity provides a convenient method for determining the soil stiffness. It is possible to obtain the initial shear modulus of soils at induced strain levels less than 0.0001% according to the following equation

$$G_0 = \rho \cdot V_s^2 , \tag{1.1}$$

where: G_0 – shear modulus [MPa],
ρ – mass density [Mg/m^3],
V_s – velocity of shear wave for linear, elastic and isotropic medium [m/s].

Shear wave velocity can be measured with the bender elements – small electro-mechanical transducers which either bend as an applied voltage is changed, or generate the voltage as they bend [6]. Bender element system can be set up in most laboratory equipment. This technique is still being developed, and it still awaits development of rules regarding its applicability and uniqueness of results.

2. LABORATORY TESTS

The laboratory tests carried out on undisturbed clay samples included: general index tests for the classification and characterization of the clay – density, grain size distribution and measurement of shear wave velocity at a very small strain. Triaxial tests were performed on 10 undisturbed clay specimens in three stages: saturation (back pressure method), consolidation and shearing (strain controlled mode with strain rate 0.005 mm/min).

The triaxial tests were performed in a cell which had internal linking bars enabling an easy access to a specimen at each stage of its preparation, and in addition, was equipped with bender elements located in the top and bottom platens. This type of cell also results in more reliable measurements of the deformation characteristics obtained during the consolidation and shearing stages.

In this method the bender elements were located at the top and bottom of the soil specimen. A change of voltage applied to the transmitter causes bending and transmission of a shear wave through the specimen. The arrival of the shear wave at the other end of the specimen is recorded as a change in voltage by the receiver ([7-9], Fig. 2).

Table 1. Index properties of tested soil samples

sample	depth	w	LL	PL	PI	LI	p'	ρ	Δe
	[m]	[%]	[%]	[%]	[-]	[-]	[kPa]	[t/m^3]	[-]
P1	22.5-23.0	21.5	45.7	19.1	26.6	0.09	400	2.00	0.644-0.613
P2	21.5-22.2	17.4	42.8	17.7	25.1	-0.01	450	2.05	0.620-0.530
P3	21.5-22.2	17.4	42.8	17.7	25.1	-0.01	350	2.04	0.630-0.550
P4	17.0-17.5	22.1	36.9	18.6	18.3	0.20	500	2.00	0.697-0.439
P5	14.0-14.5	22.5	36.9	18.6	18.3	0.21	330	2.03	0.650-0.610
P6	3.0-3.7	12.1	26.5	9.9	16.6	0.13	350	2.24	0.370-0.340
P7	3.0-3.7	10.7	26.5	11.7	14.8	-0.07	400	2.25	0.360-0.320
P8	3.0-3.7	12.4	26.5	11.7	14.8	0.05	300	2.23	0.390-0.350
P9	3.0-3.7	10.6	26.5	11.7	14.8	-0.08	250	2.27	0.340-0.320
P10	3.0-3.7	10.7	26.5	11.7	14.8	-0.07	250	2.26	0.350-0.320

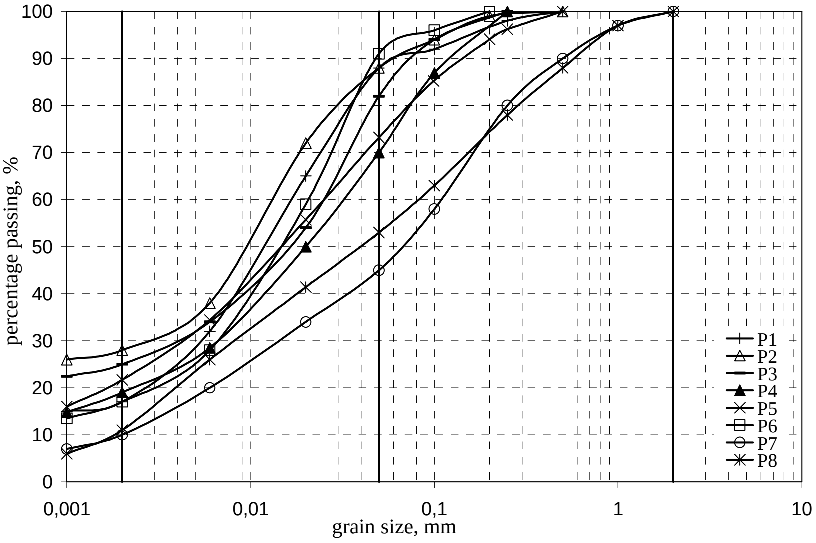

Fig. 1. Grain size distributions for tested soils

The shear wave velocity is calculated according to the relationships presented below

$$V_s = \frac{h}{t}, \tag{2.1}$$

where: h – distance between the transmitter and the receiver [m],
t – travel time [s].

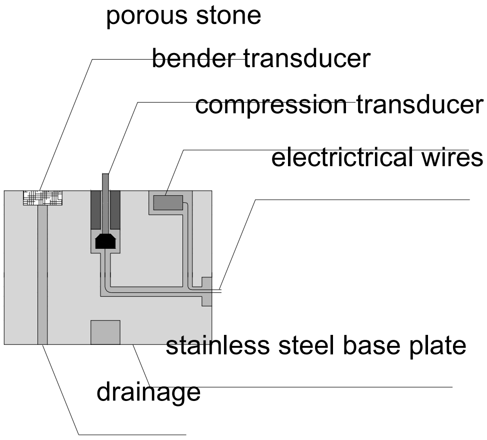

Fig. 2. Bender element [8]

The shear wave velocity measurements were carried out at the end of each saturation and consolidation stage during the triaxial tests. In order to assure repeatability of results, multiple measurements were taken at each stress level and at different frequencies of the input signal [10]. The results presented in this paper are from the tests carried out at the Department of Geotechnical Engineering WULS-SGGW (Fig.3).

The main problem in the assessment of the soil stiffness is to determine the shear wave propagation time through the soil sample. The tests carried out on different soils show that the distance between the transmitter and the receiver bender element should be the effective wave travel path through the sample. To find the effective wave travel path the height of the sample should have been reduced by the height of the bender elements ("tip-to-tip", [6, 8]). The shear wave propagation time is determined from the output signal on the oscilloscope. Figure 4 illustrates an example of the output oscilloscope signal. The arrival of the shear wave at the receiver is not always clearly defined. The first deflection of the signal occurs at point A, and it is a common practice to accept this point as the first arrival of the shear wave; however, this deflection may not correspond to the shear wave alone. It may also contain the compression wave velocity and this component is called the near-field effect. In this case the best results are obtained when the arrival of the shear wave is identified to be between points B and C. It is necessary to take into account the same polarisations of the transmitted and the received signals.

The evidence for the existence of near-field component in the bender element tests was found by Brignoli & Gotti [8]. The near-field effect may mask the arrival of the shear wave when the distance between the source and the receiver is in the range 0.25-4 wavelengths ([6]). The increase of the distance between the bender elements results in a more distinct arrival signal. The shape of the received signal can be controlled by means of the R_d ratio calculated from ([11])

$$R_d = \frac{h}{\lambda} = \frac{h \cdot f_{in}}{V_s}, \tag{3.1}$$

where: h – distance between the transmitter and receiver bender element [m],
λ – wavelength [m],
f_{in} – frequency of the input signal [kHz],
V_s – velocity of shear wave [m/s].

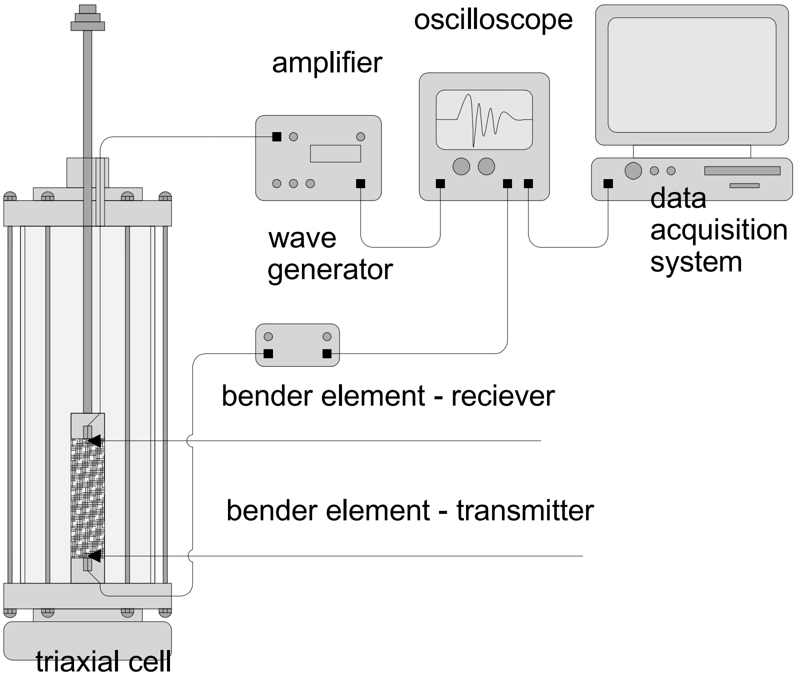

Fig. 3. Schematics of a bender element system
(Department of Geotechnical Engineering WULS-SGGW)

For higher values of R_d ratio the near-field effect is negligible and the received signals are much better for proper interpretation. The test results confirmed that the increase in frequency of the input signals increases the R_d ratio and it diminishes the near-field effect. The best results for the tested samples were obtained at frequencies of input signal from 4 kHz to 10 kHz [10].

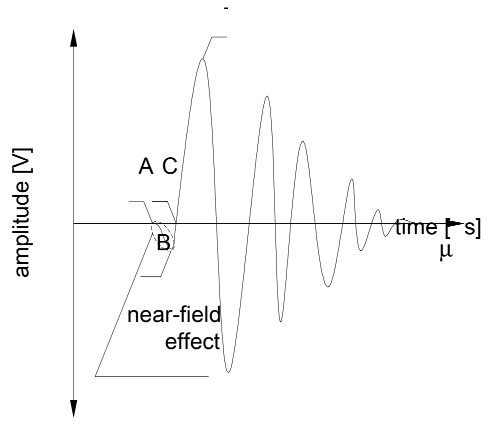

Fig. 4. The near-field effect [6]

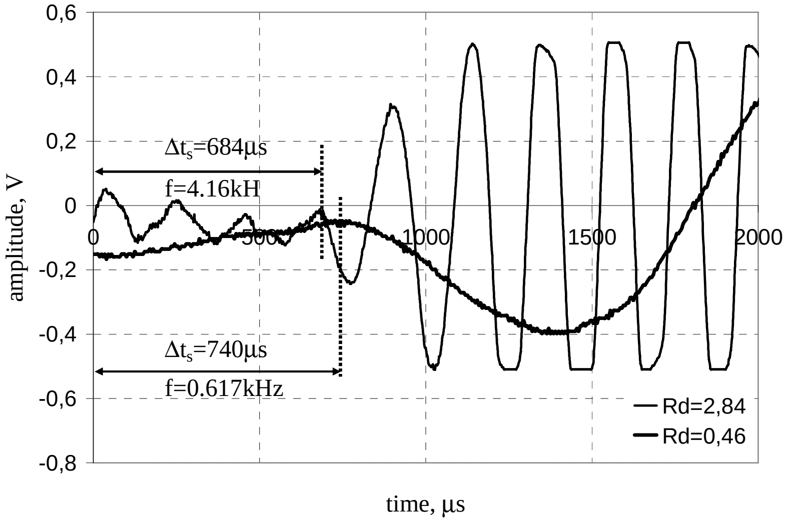

Fig. 5. The oscilloscope signals obtained in different frequencies of input signals at the same effective stress (Tymiński & Markowska-Lech, 2005)

Figure 5 shows typical oscilloscope signals from bender element tests. The input signals at different frequencies were transmitted in the same stress conditions. The determination of the arrival time at higher frequencies was more precise (4.16 kHz, [9]). Changes of the input signal frequencies did not produce the changes of polarization of the shear waves. For all ranges of frequencies, both the input and output signals, they exhibited the same polarization. Moreover, in many cases at low mean effective stress and high frequencies, the measurement of the shear wave propagation time in the tested soils was not possible.

3. THE RESULTS

The test results obtained in the laboratory using the bender elements are presented in the figures below. Figure 6 shows relationships between the shear wave velocity and the mean effective stress. Values of the shear wave velocity vary from 100 to 450 m/s at the applied mean effective stress in the range between 15 and 500 kPa. The increase in the mean effective stress during the subsequent stage of consolidation causes the decrease in the void ratio, and it results in the increase of the shear wave velocity. The largest increase in the shear wave velocity was observed for sample P9 and the smallest one for sample P3. The smaller gradient of the lines in Fig. 6 indicates lower influence of the stress change on the shear wave velocity.

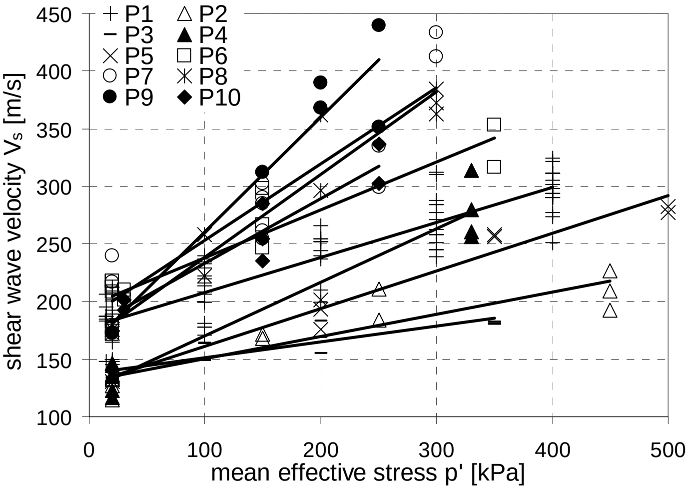

Figure 6. Shear wave velocity vs. mean effective stress in the tested clays

The shear modulus at very small strains calculated from equation (1.1) for Pliocene clays does not exceed 400MPa. The test results are presented in Fig. 7.

These tests indicate a clear dependence of the shear modulus in cohesive soils on the mean effective stress and the void ratio. This relationship would be helpful to estimate the shear modulus at very small strain without shear wave velocity measurement according to the following equation

$$V_s = 60.87 \cdot p'^{0.21} \cdot e^{-0.46} . \tag{3.2}$$

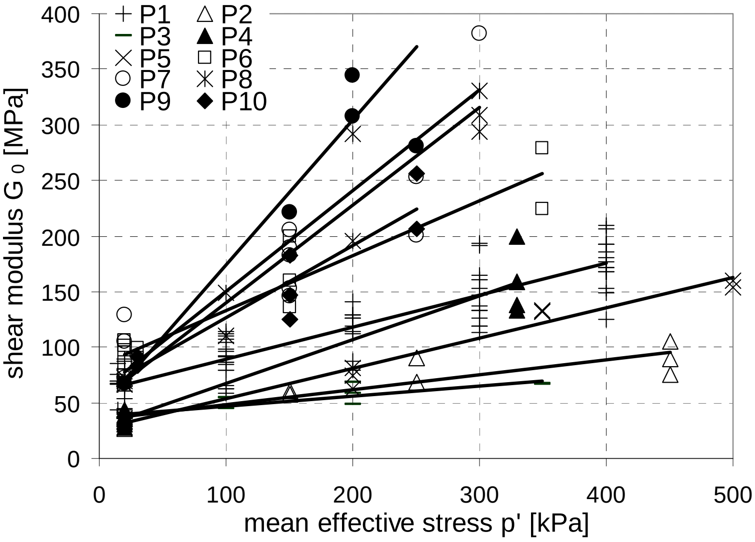

Figure 7. Shear modulus vs. mean effective stress in the tested clays

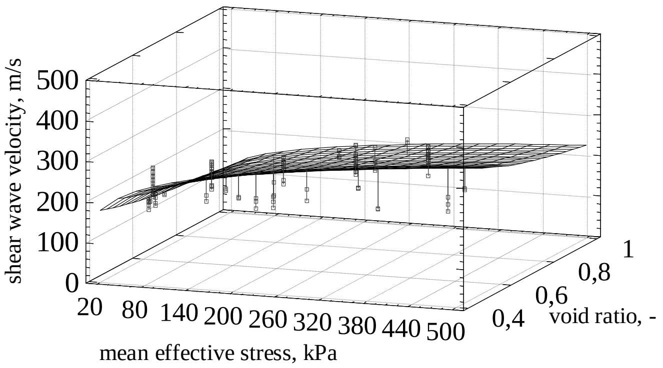

Figure 8. The correlation of experimental and estimated values of shear wave velocity with mean effective stress and void ratio in the tested soils

The correlation between the values calculated from relationship (3.2) and the results of laboratory tests is about 83%, mean relative error is 15%. Similarly, the relationship between the shear modulus, the mean effective stress and the void ratio can be expressed in the following form

$$G_0 = 2.16 \cdot p'^{0.4} \cdot e^{-1.26}\,. \tag{3.3}$$

In this case, the correlation is as good as in the equation (3.3) - 82%, but the mean relative error is twice as large (32%, [12]). Therefore, for the tested soils it seems a better solution to calculate the shear wave velocity from equation (3.2) and then calculate the shear modulus from equation (1.1), rather than estimate it directly from equation (3.3). The relationships in equations (3.2) and (3.3) are shown in Figs. 8 and 9.

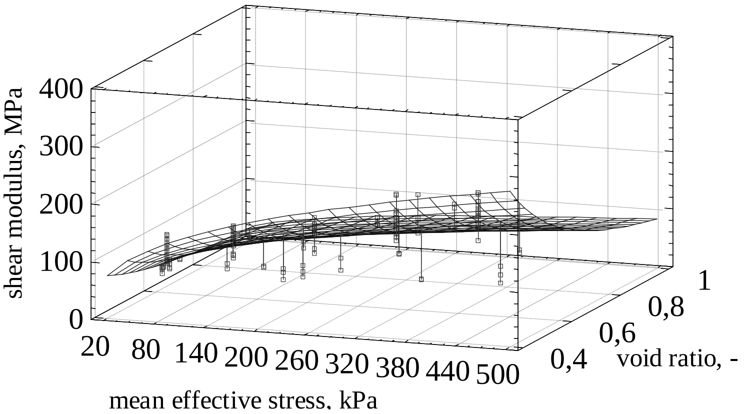

Figure 9. The correlation of experimental and estimated values of shear modulus with mean effective stress and void ratio in the tested soils

6. CONCLUSIONS

The experimental results have shown that:

- the mean effective stress and the void ratio have a visible influence on the shear wave velocity in the tested soils, which is consistent with the results dating back to the 1960's,
- there are linear relationships between the mean effective stress and the shear wave velocity at a very small strain for the tested soils,
- it is difficult to obtain the oscilloscope signal of a good quality which allows to determine unequivocally the travel time at low effective stress (15-100 kPa) and high frequency of the input signal,
- the increase in the frequency of the input signal results in the decrease of the "near-field effect" and a risk of making incorrect measurements,
- the best measurements of travel time for the tested soils were obtained at frequencies of the input signal from 4 kHz to 10 kHz,

- good correlations obtained for equations (3.2) and (3.3) allow estimating the initial shear modulus according to the proposed empirical formulas.

Although a good correlation among the shear wave velocity, the mean effective stress, and the void ratio were obtained for the tested soils, the analysis of the shear wave propagation in cohesive soils requires testing in triaxial cells fitted with two pairs of bender elements to compare the impulses and the travel times. It is necessary to continue the investigation to determine the initial stiffness for strongly overconsolidated cohesive soils.

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