Analysis of cross-wind response of steel chimneys with spoilers

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Abstract

A method for prediction of vortex-induced response of steel chimneys with spoilers is presented. A 3-start helical strake system with strakes of pitch 5D was analysed. The dependence of the displacement of the top of a chimney on the parameter of excitation \( C_v \) was proved.

1. Basic mathematical model describing vortex excitation of steel chimneys without spoilers

A starting point for calculations in this model is a horizontal stationary circular cylinder and the value of the total lift coefficient for smooth flow, \( \tilde{c}_{L0} \). This is then changed into the local lift coefficient, \( \tilde{c}_{\gamma} \), allowing for correlation length. Then the increase of local lift coefficient, \( \tilde{c}_{\gamma} \), for the oscillatory cylinder as a function of the vibration amplitude is analysed. Taking into consideration the correlation length for turbulent flow, on the value of total lift coefficient, \( \tilde{c}_{\gamma} \), for turbulent flow is obtained. Assuming a simplified distribution of lift coefficient along the cantilever, equivalent static load acting on the chimney is achieved. Deflection of the top of the chimney calculated from the above allows the next calculational stage according to the scheme in Fig. 1a.

RMS values of the total lift coefficient, \( \tilde{c}_{L0} \), as the basis for vortex excitation calculations, were adopted according to Schewe investigations [1] and presented in simplified form in Fig. 2a. When the geometric dimensions of the circular cylinder used for these investigations are known, the local RMS lift coefficient can be easily determined. The method presented by Hovell and Novak [2] was used for this purpose. In agreement with it

\[
\tilde{c}_{L0}/\tilde{c}_{\gamma} = \left(0.2(L_0/D)[1 - 0.1(L_0/D)(1 - e^{-10(L_0/D)})]\right)^{1/2},
\]

(1)

where \( L_0/D \) is the correlation length for a stationary cylinder.

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As proposed in Ref. [3], the relation of RMS values of the lift coefficients in the synchronization range is as follows:

$$
\tilde{c}_l/\tilde{c}_{l0} = (C_D/C_{D0})(f_v/f_s)(l/l_0)\Psi,
$$

where $C_D$ is the drag coefficient, $f_v$ is the vortex-shedding frequency (for an oscillating cylinder), $f_s \equiv f_{s0}$ is the Strouhal frequency (for stationary cylinder), $l$ is the longitudinal spacing of vortices, $\Psi$ is the coefficient resulting from the difference of the vortex shedding from a stationary and oscillating cylinder. Quantities with subscript "0" relate to the case of absence of variations.
Fig. 2. RMS of the total lift coefficient $\tilde{c}_{Lo}$ and Strouhal number $Sr$ adopted on the calculations.

Coefficient $\Psi$ was assumed as

$$\Psi = Sr/0.2,$$

where the values of the Strouhal number are adopted from Fig. 2b based on the research carried out by Schewe [1].

Relation $C_D/C_{Do}$ was determined on the basis of investigations of Diana and Falco [4], Stansby [5], Griffin and Ramberg [6,7] as

$$C_D/C_{Do} = 1 + 0.879(F - 0.868)^{0.387},$$

where $F$ is a function of two variables – dimensionless amplitude of transverse cylinder displacement $Y/D$ and $f_v/f_s$:

$$F = (Y/D) - 190.25(f_v/f_s)^4 + 705.93(f_v/f_s)^3 - 978.67(f_v/f_s)^2$$
$$+ 600.48(f_v/f_s) - 136.62.$$
The change in the longitudinal spacing of the vortices in a vortex street was adopted on the basis of the observations of Koopmann [8] and the investigations of Griffin and Ramberg [6] as depending only on changes in frequency,

\[ l/l_0 = 1.93 - 0.93(f_c/f_e). \]  \hspace{1cm} (6)

The above relation was obtained from investigations carried out at small Reynolds numbers. With regard to lack of any other information, it was adopted for the whole Re range.

Regarding the coefficient \( \Psi \) as increasing dynamic pressure of wind \( q \), the quantity \( \sqrt{\Psi} \) can be assumed as an increase of critical velocity \( U_c \). Determined in that way, wind velocity, which probably equals real critical velocity, may be expressed as

\[ U_{cr} = U_c \sqrt{\Psi}. \]  \hspace{1cm} (7)

From Eq. (7) and on the basis of calculations carried out in accordance to the proposed model, real wind velocity while maximal amplitude of transverse displacement occurs, may be described as

\[ U_{mr} = (1.3-1.4)U_c \sqrt{\Psi}. \]  \hspace{1cm} (8)

On this basis, the nondimensional frequency corresponding to maximal cross-wind displacement of the top of a stack, is

\[ S = \sqrt{0.2Sr \over (1.3-1.4)}. \]  \hspace{1cm} (9)

Assuming that within the supercritical range \( Sr \approx 0.45 \), the nondimensional frequency \( S \approx 0.22 \) is established. This value is in agreement with the results of the research of Wootton [9], who has stated “in supercritical flow the moving stacks showed an amplitude peak corresponding to a Strouhal number of 0.2”.

Relation (9) has been also verified for transcritical range. Assuming in this case \( Sr \approx 0.28 \), maximal value of displacement amplitude is obtained for \( S \approx 0.18 \). Analysing vortex excitation of television towers, Schneider and Wittmann [10] established the Strouhal number for which the maximal response of the structure has appeared, equal to about 0.17–0.18.

The authors of this paper have also analysed a steel chimney with the following specifications: height over foundation 40 m, height of foundation above the ground level 5 m, diameter up to height of 38 m 1.22 m, diameter at the top 0.78 m, equivalent mass per unit length of chimney 241.5 kg/m, and the natural frequency of chimney 0.90 Hz. The chimney under went cross-wind vibrations of very large amplitudes which could result in catastrophe. Vibrations were observed on 29th December 1992 and 22nd November 1993. The meteorological station located at the distance of about 3 km from the chimney registered in both days the following: wind speed 4–5 m/s (10 minute-mean speed at the height of 16 m above ground level) for constant duration and with no gusts, air temperature of about – 5°C.
Using for the forest/suburban terrain the following relationship according to [11]

\[ \alpha = 0.480 U_{10}^{-0.495} e^{0.0142 U_{10}} \]  

(10)

and

\[ \frac{U_z}{U_{10}} = (z/10)^\alpha, \]  

(11)

in which \( \alpha \) is the power-law exponent and \( U_{10} \) is the mean wind speed at level of 10 m, wind speed average over the top third of the chimney \( U = 5.1 - 6.2 \) m/s was determined. As the result of calculations carried out the following data were also received: \( \text{Sr} = 0.32, \ U_c = 3.5 \text{ m/s}, \ U_{cr} = 4.4 \text{ m/s}, \ U_{mr} = 5.9 \text{ m/s} \), and finally, nondimensional frequency, \( S = 0.19 \), corresponding to maximal response.

For estimation of equivalent load of a chimney, \( p_x(z) \), the equivalent lift coefficient has been determined

\[ \tilde{c}_x = \tilde{c}_x (1 - a/\lambda) \left\{ \frac{2}{(\lambda - a)(L/D)} \left[ 1 - \frac{1}{(\lambda - a)(L/D)} (1 - e^{a - \lambda(L/D)}) \right] \right\}^{1/2}, \]  

(12)

assuming, however, that the spectrum of local lift is constant along the height. In this equation, \( \lambda \) is the aspect ratio (= \( H/D \)), \( a \) is the nondimensional end-effect, \( L/D \) is the correlation length. Eq. (12) was developed from a modification of Howell and Novak [2] proposition which was justified by them for a horizontal cylinder.

For the determination of the correlation length in Eq. (12), Novak and Tanaka’s [12] investigation results, obtained for turbulent flow, were used. The adopted relationship \( L/L_0 \) is described by the following equation:

\[ \frac{L/D}{L_0/D} = 1 + 54.58 (2Y/D)^{1.81}. \]  

(13)

An equivalent load of the chimney was adopted with the assumption that during frequency synchronization (in lock-in range) vibrations are quasi-harmonic. This load is described by the following relation:

\[ p_x(z) = \frac{\pi}{8} \tilde{c}_x D g q \frac{z}{H}, \]  

(14)

where \( \delta_s \) is the logarithmic structural damping decrement, \( q \) is the dynamic pressure of wind, \( g \) is the peak factor.

Taking into account the method of calculation in successive steps, factor \( q \) was interpreted as a peak factor of original load (without vibrations). It was assumed that in sub- and transcritical-ranges vortex shedding is periodic, but in the supercritical range, it is a narrow band Gaussian process.

1.1. Simplified method

On the basis of calculation results carried out according to the model described above, it was proposed to adopt a parameter describing the structural response to
vortex excitation. The parameter was expressed by

\[ C_v = \frac{Sc}{\tilde{c}_{r0}} (0.1 + s), \]  

(15)

where Sc is the Scruton number, \( \tilde{c}_{r0} \) is the local RMS lift coefficient and \( s \) is a coefficient dependent on the aspect ratio

\[ s = 0.01H/D \quad \text{for} \quad H/D \leq 40, \]

\[ s = 0.04 \quad \text{for} \quad H/D > 40. \]

Values of the local RMS lift coefficient were calculated on the basis of examination of a stationary circular cylinder in a wind-tunnel. According to Ref. [13] the calculated values of the \( \tilde{c}_{r0} \) were described by

\[ \tilde{c}_{r0} = -0.274 \log \text{Re} + 1.8425, \quad \text{for} \quad 5.0 \times 10^4 \leq \text{Re} < 8.0 \times 10^5, \]

\[ \tilde{c}_{r0} = -0.877 \log \text{Re} + 5.4020, \quad \text{for} \quad 8.0 \times 10^5 \leq \text{Re} < 1.0 \times 10^6, \]

\[ \tilde{c}_{r0} = +0.049 \log \text{Re} - 0.1540, \quad \text{for} \quad 1.0 \times 10^6 \leq \text{Re} < 1.6 \times 10^6, \]

\[ \tilde{c}_{r0} = -0.179 \log \text{Re} + 1.2606, \quad \text{for} \quad 1.6 \times 10^6 \leq \text{Re} < 4.2 \times 10^6, \]

\[ \tilde{c}_{r0} = +0.133 \log \text{Re} - 0.8059, \quad \text{for} \quad 4.2 \times 10^6 \leq \text{Re} < 1.0 \times 10^7. \]  

(16)

Variability ranges were adopted for the constant value of the Strouhal number, \( Sr = 0.2 \) in all ranges of the Reynolds number.

According to Ref. [14], depending on the quantity \( C_v \), the maximal displacement of the chimney top can be determined

\[ Y_{\max}/D = (21.83 - 2C_v)/23, \quad \text{for} \quad C_v < 8.5, \]

\[ Y_{\max}/D = (22.15 - C_v)/65, \quad \text{for} \quad 8.5 \leq C_v < 18.9, \]

\[ Y_{\max}/D = (5.19 - 0.1C_v)/66, \quad \text{for} \quad 18.9 \leq C_v < 35.4, \]

\[ Y_{\max}/D = 0.025, \quad \text{for} \quad 35.4 \leq C_v. \]  

(17)

The formulae were adapted to the value of the structural damping advised in the code CICIND [15]. According to this code \( \delta = 0.02 \) should be adopted as the minimal value.

2. Vortex excitation model of steel chimneys with spoilers

The basic mathematical model describing vortex excitation of steel chimneys without spoilers was extended to chimneys with spoilers. This was made by introducing an effective length of aerodynamic stabilizer \( L_{s\text{ef}} \) into the calculation procedure. The influence of effective length of spoiler was defined as the length on which a zero value of the lift coefficient can be adopted in calculations. \( L_{s\text{ef}} \) is determined from
formula:

\[ L_{s\, ef} = L_s + nD, \]

where \( L_s \) is the length of the chimney at which the spoiler was installed.

The scheme of the analytic procedure of deflection calculations of a chimney is shown in Fig. 1b.

In the proposed model, the main problem is the assumption of a relevant value for the effective length of the aerodynamic stabilizer. The published results obtained by Hirsch et al. [16] from tunnel tests permitted to determine the value of \( L_{s\, ef} \) in the case,

<table>
<thead>
<tr>
<th>( L_s/H )</th>
<th>( r = c_{y}/c_{y, max} )</th>
<th>( r = Y/Y_{max} )</th>
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<tbody>
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<td>Ref. [16]</td>
<td>( n = f(L_s/H) )</td>
<td>( n = f(L_s/H) )</td>
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<tr>
<td>0.15</td>
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<td>0.03</td>
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<tr>
<td>0.6</td>
<td>0.01</td>
<td>0.003</td>
</tr>
</tbody>
</table>

\[ Y/Y_{max} \]

\[ H/D = 20^\circ \]

\[ L_s/H = 0.15 \]

\[ 0.20 \]

\[ 0.30 \]

\[ 0.40 \]

\[ 0.50 \]

\[ 0.60 \]

\[ 0.70 \]

\[ 0.80 \]

\[ 0.90 \]

\[ C_v \]

Fig. 3. Calculated values \( Y/Y_{max} \) for \( H/D = 20 \) and \( L_{s\, ef} = L_s - D \).
when the strake length ratio $L_{\text{st}}/H$ is 0.322. Then $L_{\text{st}} = L_{\text{st}} + 1.5D$ was obtained. Adopting the top effect as equal to $1D$, a linear relation $n = f(L_{\text{st}}/D)$ has been found. On this basis, the proposed model has been verified. The obtained results of maximal displacement $Y$ of the top of a chimney with spoiler, in relation to the respective maximal displacement $Y_{\text{max}}$ of a chimney without a spoiler were compared with Hirsch et al. results as shown in Table 1. It is easy to notice that a good agreement has been obtained.

![Graph](image1.png)

**Fig. 4.** Calculated values $Y/Y_{\text{max}}$ for $H/D = 36$ and $L_{\text{st}} = L_{\text{st}} - D$.

![Graph](image2.png)

**Fig. 5.** Effect of increase of amplitude with decreasing value $C_v$ for $H/D = 20$. 

![Graph](image3.png)
Results given by Scruton and Flint [17] however, prove that in natural scale the values $L_{sef}$ must be adopted more carefully. Finally, in further calculations it is assumed that $L_{sef}$ is less than the strake length by one diameter of the chimney, i.e. $n = -1$.

Examples of calculation results of a chimney with aspect ratio of 20 and 36 are presented in Figs. 3 and 4. Fig. 5 also shows changes of nondimensional amplitude, $Y/D$, of the top of the stack, depending on parameter $C_v$. In this figure, one can observe an effect of increase of amplitude with decreasing value of $C_v$ (the same with decreasing Scruton number). This effect has been observed in wind-tunnel tests of Ruscheweyh [18]. It was noticed that for large vibration amplitudes (it means for small Scruton numbers) a distinct vortex street was formed behind the straked stack, while at small amplitudes the wake is irregular. However, from experimental research, it can be concluded that the increase of amplitude of vibrations with decreasing damping and mass is larger than that obtained from calculations shown above.

In order to allow for the effect of vortex street stabilization and lost of spoiler effectiveness during significant vibrations amplitudes, the calculation procedure has been modified. It was completed applying to the part of the stack with spoilers additional forces. It was assumed that these forces are localised every third of pitch of strake. Additional exciting forces appear in these places after nondimensional amplitude of $y/D = 0.07$ is exceeded. With increase of vibrations new forces are applied to lower parts of the chimney stack with spoiler. Original calculation results are shown in Fig. 6.

![Fig. 6. Calculation results with allowance for additional forces caused by vortex shedding on the part of the stack with spoilers for $H/D = 20$.](image-url)
References