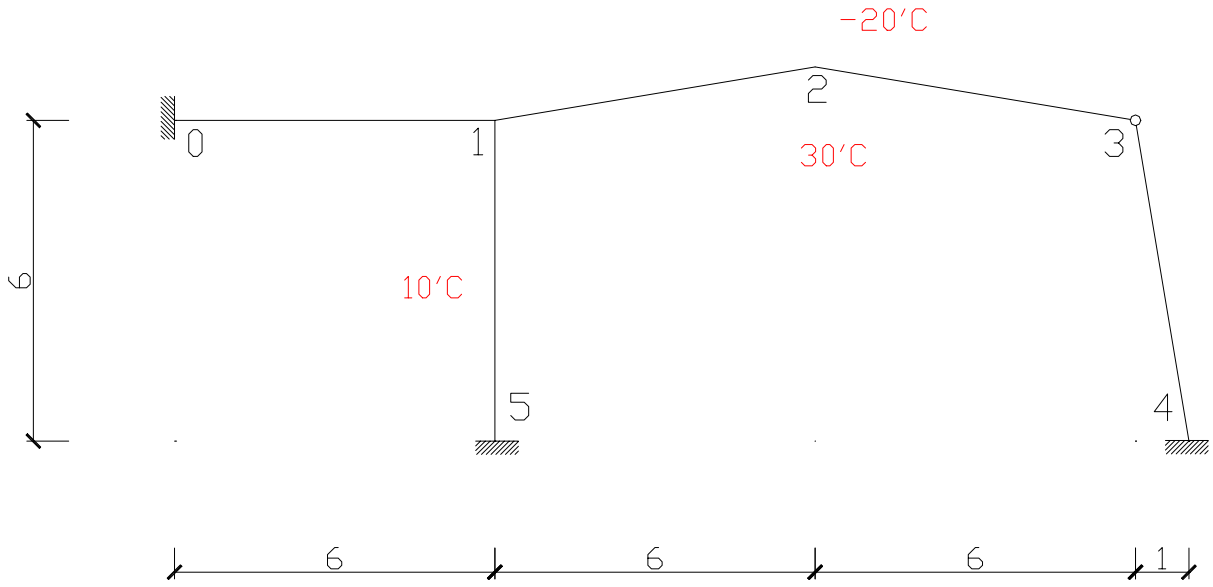
Wpływ temperatury



Temperatura montażu $\mathbf{t_m = 15^0 C}$

Obliczam t_o i Δt dla wszystkich prętów zgodnie z zależnościami:

$$t_0 = \frac{t_d + t_g}{2} - t_m \qquad \Delta t = t_d - t_g$$

$$t_0^{01} = -25^0 C \qquad \Delta t^{01} = 30^0 C$$
$$t_0^{12} = -15^0 C \qquad \Delta t^{12} = 50^0 C$$
$$t_0^{23} = -15^0 C \qquad \Delta t^{23} = 50^0 C$$
$$t_0^{34} = -15^0 C \qquad \Delta t^{34} = 50^0 C$$
$$t_0^{15} = 0^0 C \qquad \Delta t^{15} = 20^0 C$$

Wykorzystując zasadę superpozycji obliczam osobno wpływ od t_o i Δt

Wpływ Δt

Ważne informacje:

$EI_1 = 4243{,}5\,[kNm^2]$

$EI_2 = 8712{,}5\,[kNm^2]$

$\alpha_t = 1{,}2 \cdot 10^{-5}$

$h_1 = 0{,}20[m]$

$h_2 = 0{,}24[m]$

Obliczam momenty w poszczególnych prętach wykorzystując obliczone Δt ze str. 17:

$$M_{01} = M_{10} = \frac{\alpha_t \cdot \Delta t^{01} \cdot EI_2}{h_2} = 13{,}06875\,[kNm]$$

$$M_{15} = M_{15} = \frac{\alpha_t \cdot \Delta t^{15} \cdot EI_1}{h_1} = 5{,}0922\,[kNm]$$

$$M_{12} = M_{21} = \frac{\alpha_t \cdot \Delta t^{12} \cdot EI_2}{h_2} = 21{,}78125\,[kNm]$$

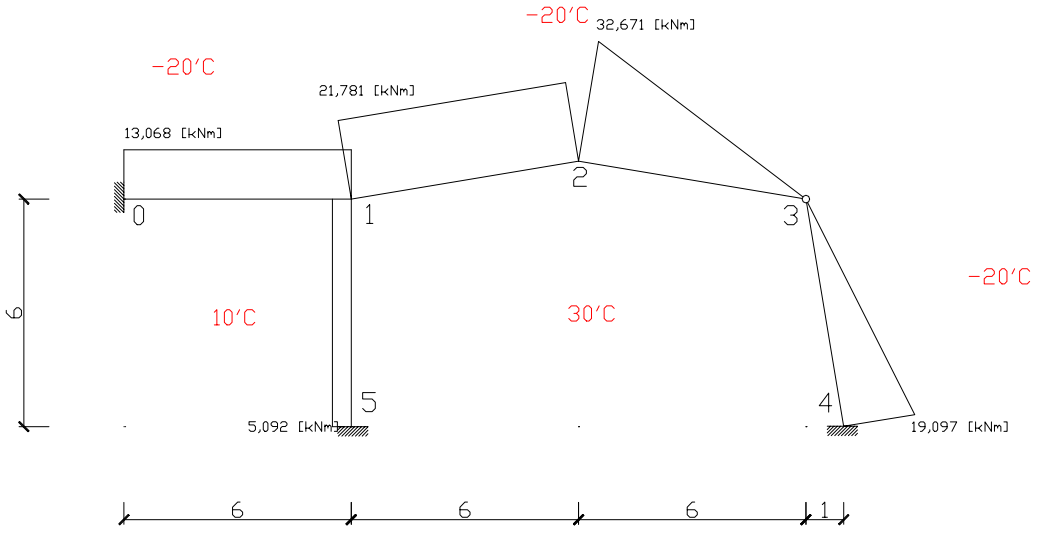
$$M_{23} = \frac{3}{2}\frac{\alpha_t \cdot \Delta t^{23} \cdot EI_2}{h_2} = 32{,}671875\,[kNm]$$

$M_{32} = 0$

$M_{34} = 0$

$$M_{43} = \frac{3}{2}\frac{\alpha_t \cdot \Delta t^{34} \cdot EI_1}{h_1} = 19{,}09575\,[kNm]$$

M(Δt) [kNm]



Wyznaczenie reakcji $R_1(\Delta t)$ i $R_2(\Delta t)$ z równowagi węzłów:

$R_1(\Delta t) = -3{,}6203$ [kNm]

$R_2(\Delta t) = -10{,}890625$ [kNm]

Reakcje $R_3(\Delta t)$ obliczę korzystając z równania pracy wirtualnej (obroty ψ ze strony 2):

$$\psi_{10} = 0 \qquad \psi_{15} = 0 \qquad \psi_{12} = \frac{35}{72}\Delta \qquad \psi_{23} = -\frac{37}{72}\Delta \quad \psi_{34} = \frac{\Delta}{6}$$

$$R_3(\Delta t)\cdot \bar{1} - 32{,}671875\cdot\overline{\psi_{23}} + 19{,}09575\cdot\overline{\psi_{34}} = 0$$

$R_3(\Delta t) = -19{,}97233$ [kNm]

Wpływ t_0

Zapisuję łańcuch kinematyczny z uwzględnieniem wydłużenia pręta

015 $\rightarrow$

$$\psi_{01}\cdot 0 + 6\cdot\alpha_t\cdot(-25) - \psi_{15}\cdot 6 = 0 \qquad \psi_{15} = -0{,}0003$$

510 $\downarrow$

$$\psi_{15}\cdot 0 - 6\cdot\alpha_t\cdot(0) - \psi_{01}\cdot 6 = 0 \qquad \psi_{10} = 0$$

43 $\rightarrow$

$$\psi_{43}\cdot 6 - \alpha_t\cdot(-15) = 0 \qquad \psi_{43} = -0{,}00018$$

0123 $\rightarrow$

$$6\cdot\alpha_t\cdot(-25) + \psi_{12}\cdot 1 + 6\cdot\alpha_t\cdot(-15) - \psi_{23}\cdot 1 + 6\cdot\alpha_t\cdot(-15) = 0 \qquad \psi_{12} = \psi_{23} + 0{,}00396$$

51234 $\downarrow$

$$\psi_{51}\cdot 0 - 6\cdot\alpha_t\cdot(0) + \psi_{12}\cdot 6 - 1\cdot\alpha_t\cdot(-15) + \psi_{23}\cdot 6 + 1\cdot\alpha_t\cdot(-15) + \psi_{34}\cdot 1 + 6\cdot\alpha_t\cdot(-15) = 0$$

$$\psi_{23} = -0{,}001905$$

$$\psi_{12} = 0{,}002055$$

Momenty od t_o z uwzględnieniem obrotów:

$$\varphi_0 = 0 \qquad \varphi_1 = 0 \qquad \psi_{01} = 0 \qquad EI_0 = 8712{,}5\ [kNm^2]$$

$$M_{01} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_0 + \varphi_1 - 3\cdot\psi_{01}) = \frac{2EI_0}{6}\cdot(2\cdot 0 + 0 - 3\cdot 0) = 0\,[kNm]$$

$$M_{10} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_1 + \varphi_0 - 3\cdot\psi_{01}) = \frac{2EI_0}{6}\cdot(2\cdot 0 + 0 - 3\cdot 0) = 0\,[kNm]$$

$$\varphi_1 = 0 \qquad \varphi_5 = 0 \qquad \psi_{15} = -0{,}0003 \qquad EI_0 = 8712{,}5\ [kNm^2]$$

$$M_{15} = \frac{2EI_1}{l}\cdot(2\cdot\varphi_1 + \varphi_5 - 3\cdot\psi_{15}) = \frac{2EI_0\cdot 0{,}487}{6}\cdot(2\cdot 0 + 0 - 3\cdot(-0{,}0003)) = 1{,}2728\,[\mathrm{kNm}]$$

$$M_{51} = \frac{2EI_1}{l}\cdot(2\cdot\varphi_5 + \varphi_1 - 3\cdot\psi_{15}) = \frac{2EI_0\cdot 0{,}487}{6}\cdot(2\cdot 0 + 0 - 3\cdot(-0{,}0003)) = 1{,}2728\,[\mathrm{kNm}]$$

$$\varphi_1 = 0 \qquad \varphi_2 = 0 \qquad \psi_{12} = 0{,}002055 \qquad EI_0 = 8712{,}5\ [kNm^2]$$

$$M_{12} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_1 + \varphi_2 - 3\cdot\psi_{12}) = \frac{2EI_0}{\sqrt{37}}\cdot(2\cdot 0 + 0 - 3\cdot 0{,}002055) = -17{,}661\,[kNm]$$

$$M_{21}=\frac{2EI_2}{l}\cdot(2\cdot\varphi_2+\varphi_1-3\cdot\psi_{12})=\frac{2EI_0}{\sqrt{37}}\cdot(2\cdot 0+0-3\cdot 0{,}002055)=-17{,}661[kNm]$$

$$\varphi_2=0 \qquad \varphi_3=0 \qquad \psi_{23}=-0{,}001905 \qquad EI_0=8712{,}5\ [kNm^2]$$

$$M_{23}=\frac{3EI_2}{l}\cdot(\varphi_2-\psi_{23})=\frac{3EI_0}{\sqrt{37}}\cdot(0+0{,}001905)=8{,}186[kNm]$$

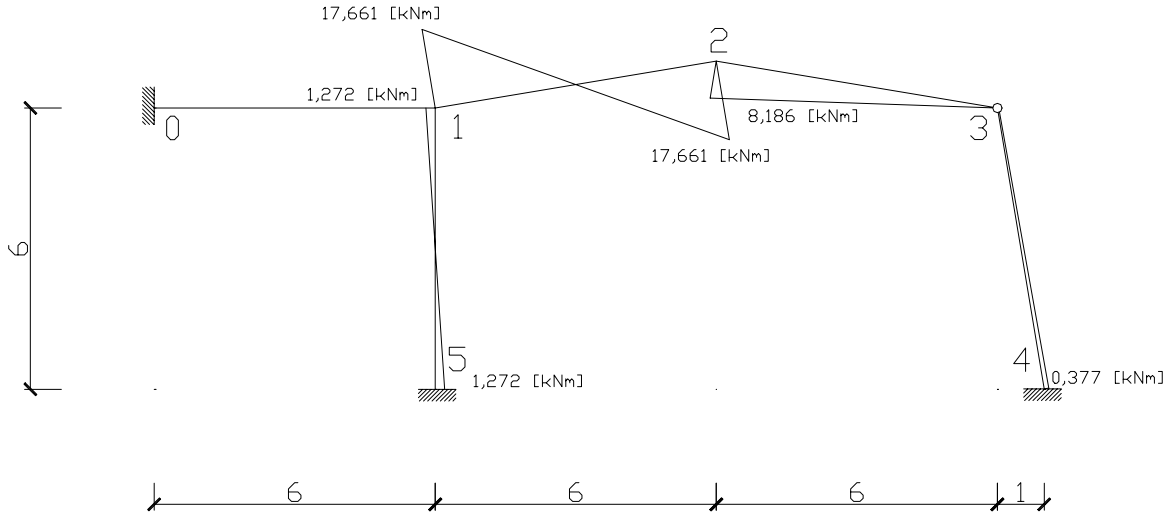
$$M_{32}=0$$

$$\varphi_3=0 \qquad \varphi_4=-0{,}008 \quad \psi_{34}=-0{,}00018 \qquad EI_0=8712{,}5\ [kNm^2]$$

$$M_{34}=0$$

$$M_{43}=\frac{3EI_2}{l}\cdot(\varphi_4-\psi_{43})=\frac{3EI_0\cdot 0{,}487}{\sqrt{37}}\cdot(0+0{,}00018)=0{,}377[kNm]$$

M(t$_0$) [kNm]



Wyznaczenie reakcji $R_1(t_o)$ i $R_2(t_o)$ z równowagi węzłów:

$R_1(t_o)$ = -16,3883 [kNm]

$R_2(t_o)$ = -9,475 [kNm]

Reakcje $R_{3\Delta}$ obliczę korzystając z równania pracy wirtualnej (obroty ψ ze strony 2):

$$\psi_{10}=0 \qquad \psi_{15}=0 \qquad \psi_{12}=\frac{35}{72}\Delta \qquad \psi_{23}=-\frac{37}{72}\Delta \quad \psi_{34}=\frac{\Delta}{6}$$

$$R_{33}\cdot\bar{1}-(2\cdot 17{,}661\cdot 2)\cdot\overline{\psi_{12}}+8{,}186\cdot\overline{\psi_{23}}+0{,}377\cdot\overline{\psi_{34}}=0$$

$R_3(t_o)$ = 21,3142 [kNm]

Korzystając z superpozycji obliczam współczynniki R_{it}, które podstawiam do równania (współczynniki r_{ik} ze strony 8:

$R_1(t_o)$ + $R_1(\Delta t)$ = -20,0085 [kNm]

$R_2(t_o)$ + $R_2(\Delta t)$ = -20,37125 [kNm]

$R_3(t_o)$ + $R_3(\Delta t)$ = 1,34187 [kNm]

$$\begin{cases}1{,}64896\cdot EI_0\cdot\varphi_1+0{,}32879\cdot EI_0\cdot\varphi_2-0{,}47949\cdot EI_0\cdot\Delta_3-20{,}0085=0\\0{,}32879\cdot EI_0\cdot\varphi_1+1{,}15078\cdot EI_0\cdot\varphi_2-0{,}22604\cdot EI_0\cdot\Delta_3-20{,}37125=0\\-0{,}47949\cdot EI_0\cdot\varphi_1-0{,}22604\cdot EI_0\cdot\varphi_2+0{,}60308\cdot EI_0\cdot\Delta_3+1{,}34187=0\end{cases}$$

Rozwiązaniem układu równań:

$$\varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \varphi_2 = 16.82448 \cdot \frac{1}{EI_0} \qquad \Delta_3 = 14.38755 \cdot \frac{1}{EI_0}$$

Wyliczone przemieszczenia podstawiamy do wzorów transformacyjnych na momenty (obroty ψ ze strony 2) i dodajemy do nich momenty od stanu $M(t) = M(t_o) + M(\Delta t)$:

$$\varphi_0 = 0 \qquad \varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \psi_{01} = 0 \qquad M_{10}{}^{t} = -13{,}06875\,[kNm]$$

$$M_{01} = M_{01}{}^{t} + \frac{2EI_2}{l} \cdot (2 \cdot \varphi_0 + \varphi_1 - 3 \cdot \psi_{01}) = \text{-}8{,}7477\,[\text{kNm}]$$

$$\varphi_0 = 0 \qquad \varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \psi_{01} = 0 \qquad M_{10}{}^{t} = 13{,}06875\,[kNm]$$

$$M_{10} = M_{10}{}^{t} + \frac{2EI_2}{l} \cdot (2 \cdot \varphi_1 + \varphi_0 - 3 \cdot \psi_{01}) = 21{,}71075\,[kNm]$$

$$\varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \varphi_5 = 0 \qquad \psi_{15} = 0 \qquad M_{15}{}^{t} = 6{,}21948\ [kNm]$$

$$M_{15} = M_{15}{}^{t} + \frac{2EI_1}{l} \cdot (2 \cdot \varphi_1 + \varphi_5 - 3 \cdot \psi_{15}) = 10{,}5736\,[\text{kNm}]$$

$$\varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \varphi_5 = 0 \qquad \psi_{15} = 0 \qquad M_{51}{}^{t} = 3{,}8194\ [kNm]$$

$$M_{51} = M_{51}{}^{t} + \frac{2EI_1}{l} \cdot (2 \cdot \varphi_5 + \varphi_1 - 3 \cdot \psi_{15}) = -1{,}71607\,[kNm]$$

$$\varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \varphi_2 = 16.82448 \cdot \frac{1}{EI_0} \qquad \psi_{12} = \frac{35}{72} \cdot \Delta \qquad M_{12}{}^{t} = -39{,}44225\ [kNm]$$

$$M_{12} = M_{12}{}^{t} + \frac{2EI_2}{l} \cdot (2 \cdot \varphi_1 + \varphi_2 - 3 \cdot \psi_{12}) = -32{,}285\,[kNm]$$

$$\varphi_1 = 12.96300 \cdot \frac{1}{EI_0} \qquad \varphi_2 = 16.82448 \cdot \frac{1}{EI_0} \qquad \psi_{12} = \frac{35}{72} \cdot \Delta \qquad M_{21}{}^{t} = 4{,}12025\ [kNm]$$

$$M_{21} = M_{21}{}^{t} + \frac{2EI_2}{l} \cdot (2 \cdot \varphi_2 + \varphi_1 - 3 \cdot \psi_{12}) = 12{,}547\,[kNm]$$

$$\varphi_2 = 16.82448 \cdot \frac{1}{EI_0} \qquad \varphi_3 = 0 \qquad \psi_{23} = -\frac{37}{72} \cdot \Delta \qquad M_{23}{}^{t} = -24{,}48587\ [kNm]$$

$$M_{23} = M_{23}{}^{t} + \frac{3EI_2}{l} \cdot (\varphi_2 - \psi_{23}) = -12{,}547\,[kNm]$$

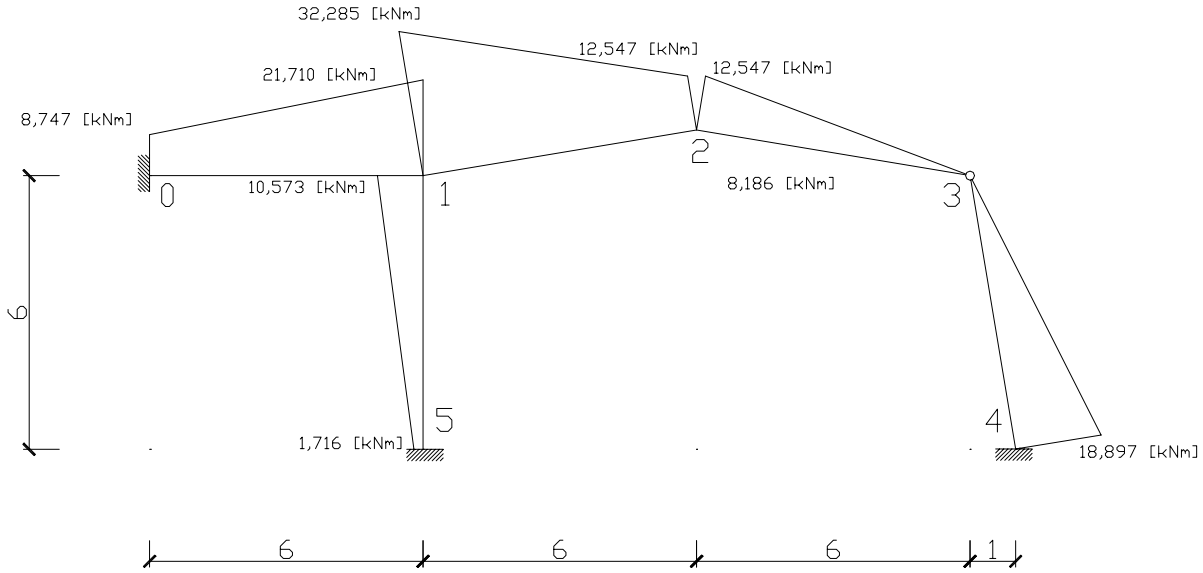
$$M_{32} = 0$$

$$\varphi_3 = 0 \qquad \varphi_4 = 0 \qquad \psi_{34} = \frac{1}{6} \cdot \Delta \qquad M_{43} = 19{,}47275\ [kNm^2]$$

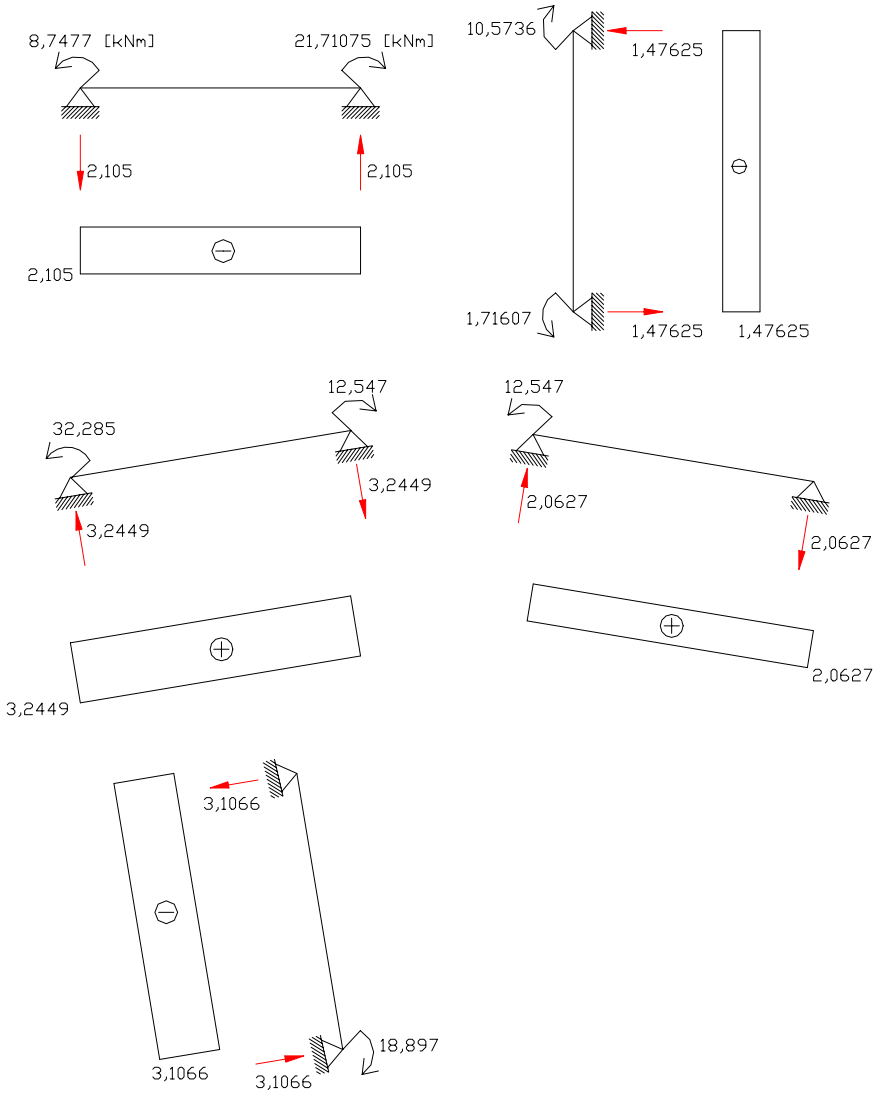
$$M_{34} = 0$$

$$M_{43} = M_{43}{}^{\Delta} + \frac{3EI_2}{l} \cdot (\varphi_4 - \psi_{43}) = 18{,}897\,[kNm]$$

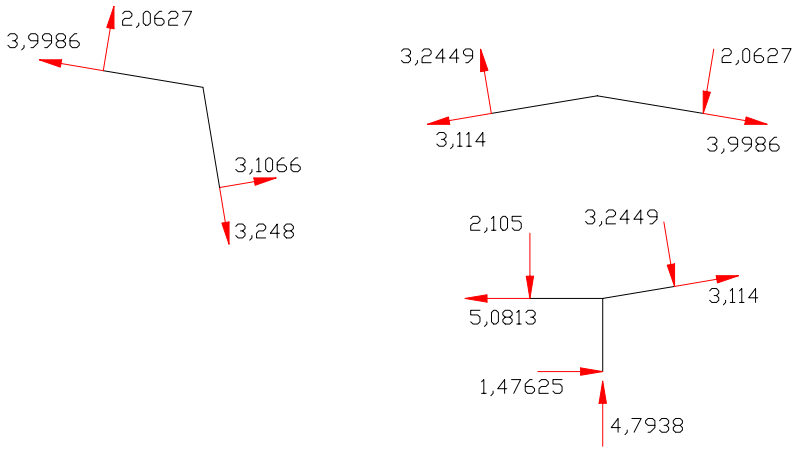
M^n [kNm]



Obliczenie wartości sił tnących:

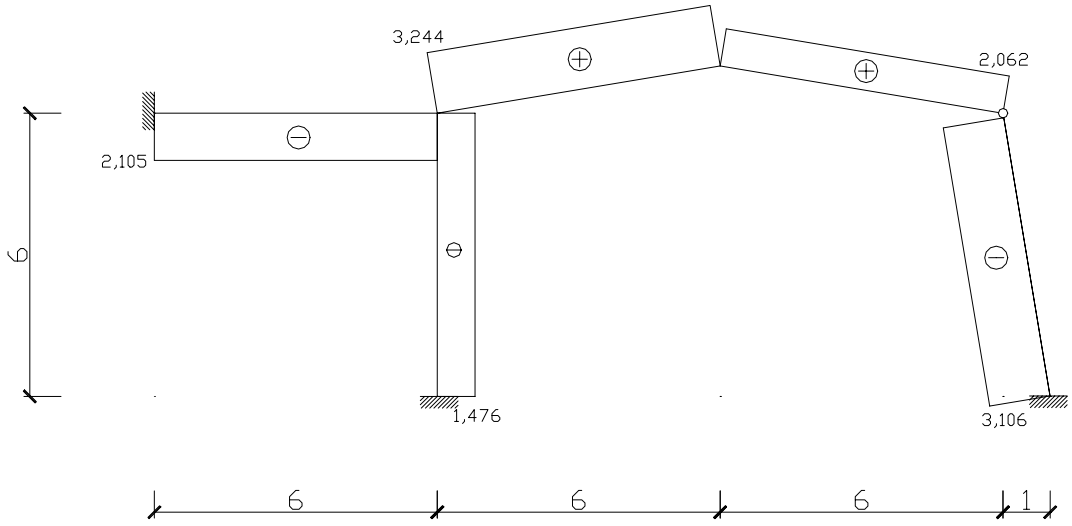


Obliczenie siły normalnej z równowagi węzłów:



Sprawdzenie węzła 2 po osi $\sum y = 0{,}0031$

T^n [kN]



N^n [kN]