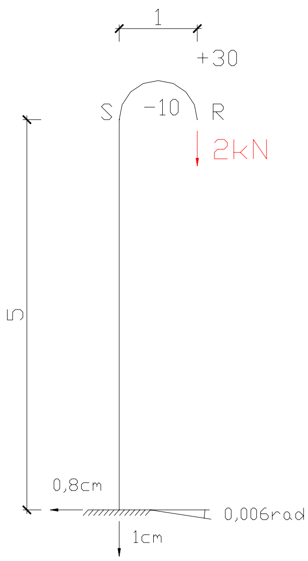
OBLICZANIE PRZEMIESZCZEŃ UKŁADÓW STATYCZNIE WYZNACZALNYCH Z ZASTOSOWANIEM RÓWNANIA PRACY WIRTUALNEJ.

1. Projektuję przekrój pręta:



$$M_{\max} = 2kN = 200kNcm$$

$$1{,}2 \cdot \frac{200}{W} \le \sigma_{dop}$$

$$\sigma_{dop} = 200MPa = 20\frac{kN}{cm^2}$$

$$W \ge 12$$

Dobieram odpowiedni pręt rurowy:

$$d = 7{,}5cm = 0{,}075m$$

$$g = 0{,}35cm = 0{,}0035$$

$$W = 13{,}43$$

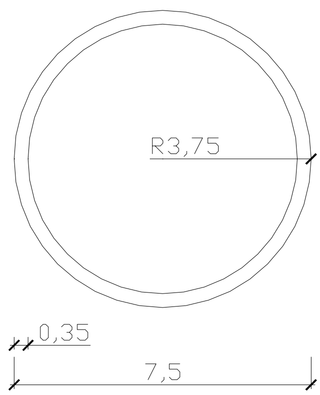
$$E = 205GPa = 205 \cdot 10^6 \frac{kN}{m^2}$$

$$I = 50{,}36cm^4 = 50{,}36 \cdot 10^{-8} m$$

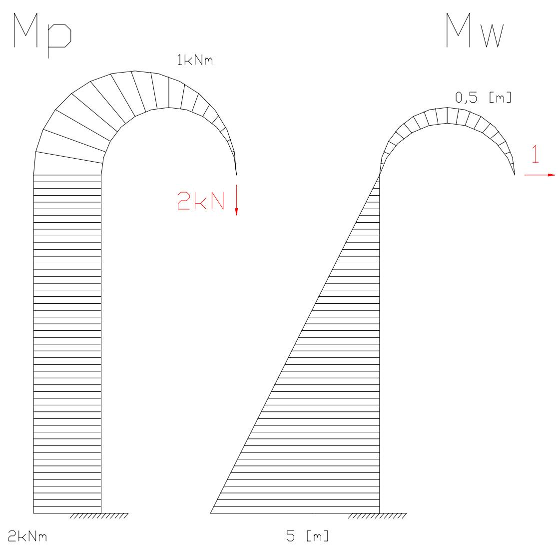
$$EI = 103{,}238$$

$$A = 7{,}862 \cdot 10^{-4} m^2$$

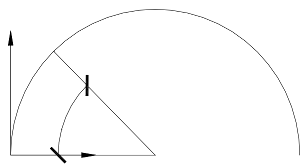
$$G = 78{,}84 \cdot 10^6$$



2. Obliczenie przemieszczenia punktu K (składowa pozioma) od obciążenia zewnętrznego (bez wpływu N i T).



$$x = R(1-\cos\varphi)$$
$$y = R\sin\varphi$$
$$\varphi \in \langle 0;\pi \rangle$$
$$R = 0{,}5$$



$$\bar{1}\cdot\Delta = \int_0^{\pi} \frac{M\cdot\overline{M}}{EI}\cdot ds \Rightarrow \left[\frac{kNm\cdot m}{kNm^2}m\right] = [m]$$

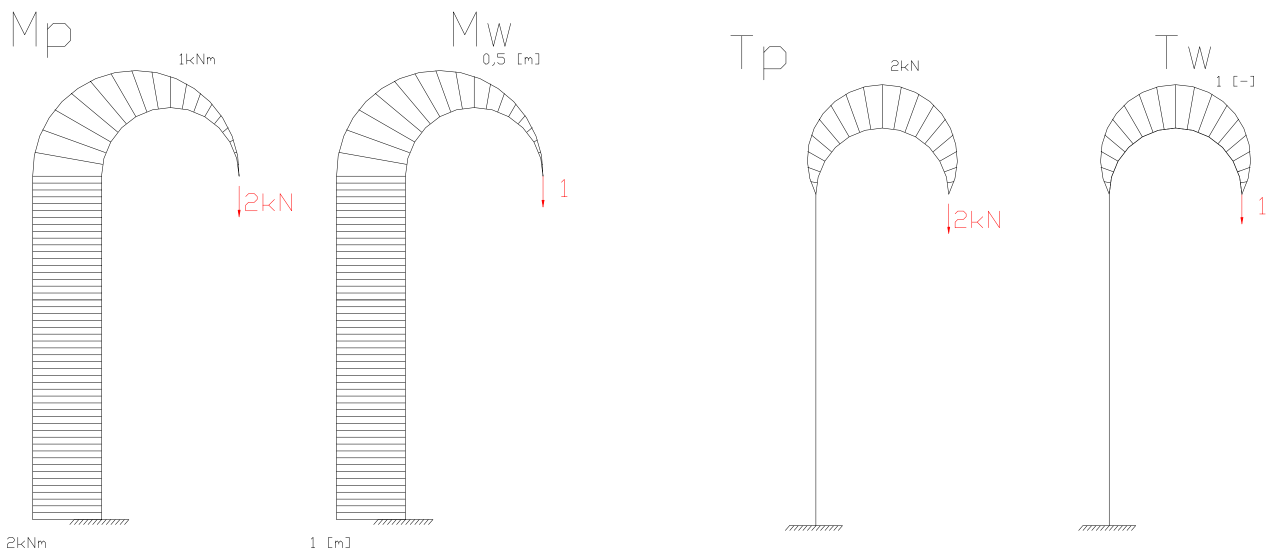
$$\Delta = \frac{1}{EI}\left[25 + \int_0^{\pi} R\sin\varphi 2R(1-\cos\varphi)Rd\varphi\right]$$

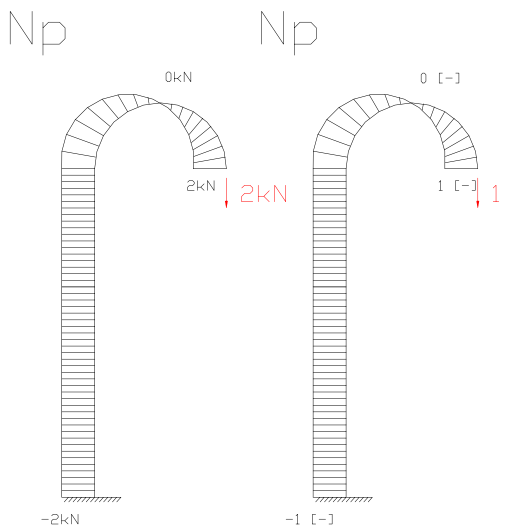
$$\Delta = \frac{1}{EI}\left[25 + 0{,}25\int_0^{\pi} \sin\varphi(1-\cos\varphi)d\varphi\right]$$

$$\int_0^{\pi} \sin\varphi(1-\cos\varphi)d\varphi = \begin{vmatrix}\cos\varphi = t \\ \sin\varphi d\varphi = dt\end{vmatrix} = \int_0^{\pi} \sin\varphi(1-t)\frac{dt}{\sin\varphi} = \left[t - 0{,}5t^2\right]_0^{\pi} = \left[\cos\varphi - 0{,}5\cos^2\varphi\right]_0^{\pi} = -3$$

$$\Delta = 0{,}23m$$

3. Obliczenie przemieszczenia punktu K (składowa pionowaa) od obciążenia zewnętrznego łącznie z wpływem N i T.





$$M_p = 2\cdot R(1-\cos\varphi)$$
$$\overline{M} = \bar{1}\cdot R(1-\cos\varphi)$$
$$T_p = 2\cdot\sin\varphi$$
$$\overline{T} = \bar{1}\cdot\sin\varphi$$
$$N_p = 2\cdot(\sin\varphi-1)$$
$$\overline{N} = \bar{1}\cdot(\sin\varphi-1)$$

$$\bar{1}\cdot\Delta=\int_0^\pi \frac{\overline{M}\cdot M}{EI}ds+\int_0^\pi\frac{\overline{T}\cdot T}{GA}\kappa ds+\int_0^\pi\frac{\overline{N}\cdot N}{EA}ds \Rightarrow \left[\frac{kNm\cdot m}{kNm^2}m\right]+\left[\frac{kN}{kN}m\right]+\left[\frac{kN}{kN}m\right]=[m]$$

$$\bar{1}\cdot\Delta=\frac{1}{EI}\left[1\cdot 2\cdot 5+\int_0^\pi 2\cdot R^3(1-\cos\varphi)^2 d\varphi\right]+\frac{1}{GA}\left[0+\int_0^\pi 2R\sin^2\varphi\kappa d\varphi\right]+\frac{1}{EA}\left[2\cdot 5\cdot 1+\int_0^\pi 2R(1-\sin\varphi)^2 d\varphi\right]$$

$$\Delta=\frac{1}{EI}\left[10+0{,}25\int_0^\pi(1-\cos\varphi)^2 d\varphi\right]+\frac{1}{GA}\left[0+1\cdot 1\int_0^\pi\sin^2\varphi d\varphi\right]+\frac{1}{EA}\left[10+1\int_0^\pi(1-\sin\varphi)^2 d\varphi\right]$$

$$\int_0^\pi(1-\cos\varphi)^2 d\varphi=\int_0^\pi 1d\varphi-\int_0^\pi 2\cos\varphi d\varphi+\int_0^\pi\cos^2\varphi d\varphi=\varphi\Big|_0^\pi-2\sin\varphi\Big|_0^\pi+\int_0^\pi(1-\sin^2\varphi)d\varphi=$$

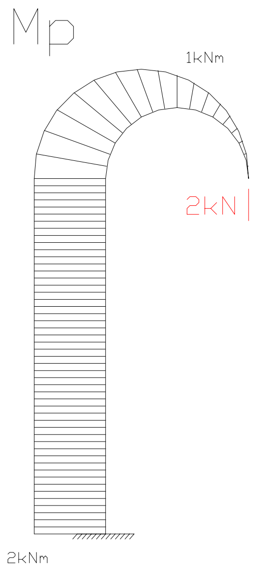
$$\pi+\varphi\Big|_0^\pi-\left|-\frac{1}{2}\sin\varphi\cos\varphi+\frac{1}{2}\varphi\right|_0^\pi=2\pi-\frac{1}{2}\pi=1{,}5\pi$$

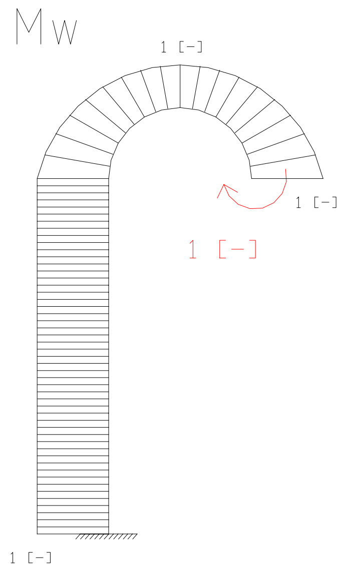
$$\int_0^\pi\sin^2\varphi d\varphi=\left|-\frac{1}{2}\sin\varphi\cos\varphi+\frac{1}{2}\varphi\right|_0^\pi=0{,}5\pi$$

$$\int_0^\pi(1-\sin\varphi)^2 ds=\varphi\Big|_0^\pi-\left|-\frac{1}{2}\sin\varphi\cos\varphi+\frac{1}{2}\varphi\right|_0^\pi=\pi-0{,}5\pi=0{,}5\pi$$

$$\Delta=\frac{1}{EI}[10+0{,}25\cdot 1{,}5\pi]+\frac{1}{GA}[0{,}5\pi]+\frac{1}{EA}[10+0{,}5\pi]=0{,}108+0{,}000025+0{,}0000097=0{,}108$$

4. Obliczenie obrotu przekroju od obciążenia zewnętrznego (bez wpływu N i T).





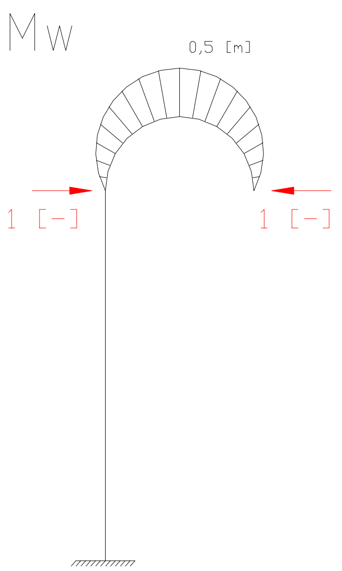
$$\bar{1} \cdot \varphi = \int_0^{\pi} \frac{M \cdot \overline{M}}{EI} \cdot ds \Rightarrow \left[\frac{kNm \cdot}{kNm^2} m \right] = [-] = [rad]$$

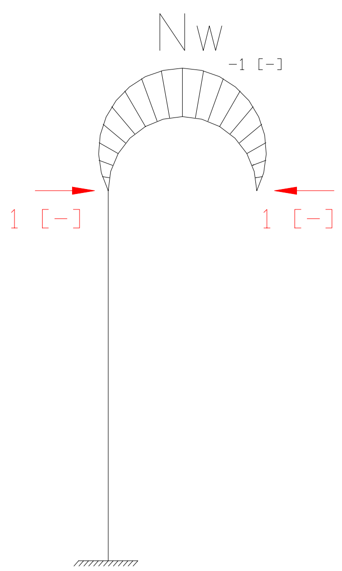
$$\varphi = \frac{1}{EI}\left[1 \cdot 2 \cdot 5 + \int_0^{\pi} 2R^2 (1 - \cos\varphi) d\varphi \right]$$

$$\varphi = \frac{1}{EI}\left[10 + 0{,}5\left(\int_0^{\pi} 1 d\varphi - \int_0^{\pi} \cos\varphi d\varphi \right) \right]$$

$$\varphi = \frac{1}{EI}\left[10 + 0{,}5\left(\pi - \sin\varphi\big|_0^{\pi} \right) \right] = \frac{1}{EI}(10 + 0{,}5\pi) = 0{,}112 rad$$

5. Obliczenie wzajemnego przemieszczenia punktów R i S od zmiany temperatury.





$$t_d = -10^o$$

$$t_g = 30^o$$

$$t_m = 5^o$$

$$h = 7{,}5 cm = 0{,}075 m$$

$$\alpha_t = 1{,}2 \cdot 10^{-5}$$

$$t_o = \frac{t_g + t_d}{2} - t_m = 5$$

$$\Delta t = t_g - t_d = 40^o$$

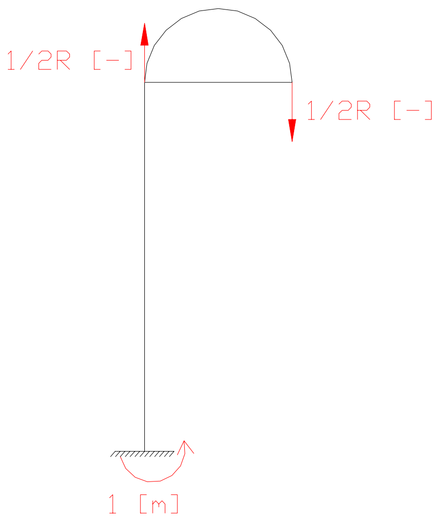
$$\bar{1}\cdot\Delta RS=\int_0^{\pi}\overline{M}\frac{\alpha_t\Delta t}{h}ds+\int_0^{\pi}\overline{N}\alpha_t t_0 ds\Rightarrow\left[m\frac{1\cdot{}^0C}{{}^0C\cdot m}\cdot m\right]=[m]$$

$$\Delta RS=\int_0^{\pi}R^2\sin\varphi 0{,}0064d\varphi+\int_0^{\pi}R\sin\varphi 0{,}00006d\varphi$$

$$\Delta RS=0{,}0016\int_0^{\pi}\sin\varphi d\varphi+0{,}00003\int_0^{\pi}\sin\varphi d\varphi$$

$$\Delta RS=0{,}0016\cdot 2+0{,}00003\cdot 2=0{,}00314m$$

6. Obliczenie obrotu cięciwy RS od osiadania podpór.



$$\bar{1}\cdot\varphi RS+\sum\overline{R}\Delta=0\Rightarrow[rad]$$

$$\bar{1}\cdot\varphi RS-\bar{1}\cdot(0{,}006)=0$$

$$\varphi RS=0{,}006rad$$