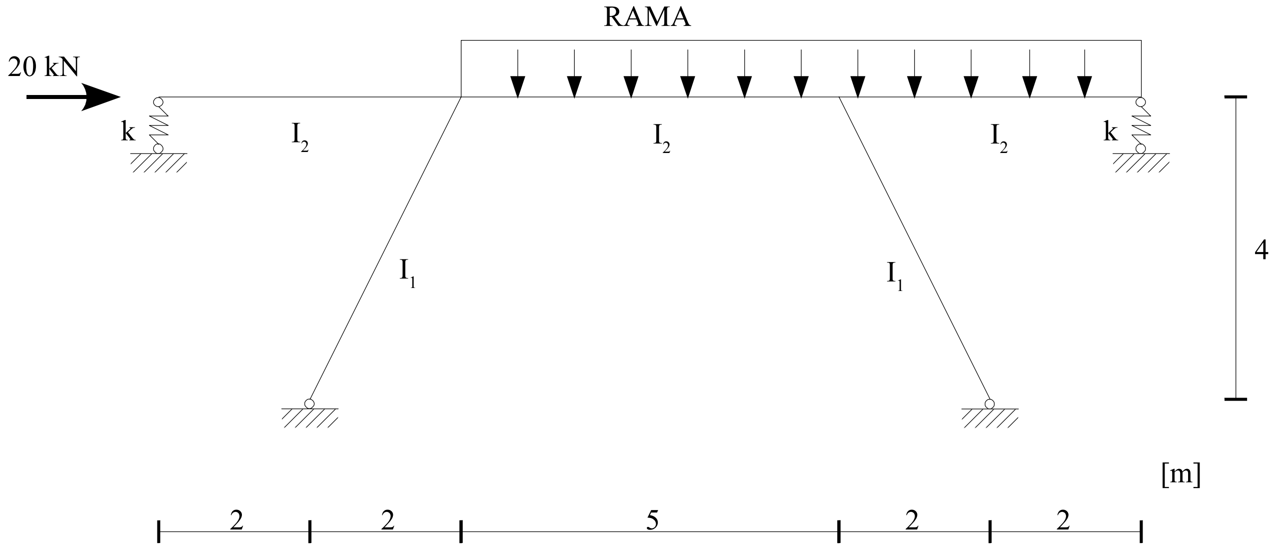
POLITECHNIKA POZNAŃSKA
INSTYTUT KONSTRUKCJI BUDOWLANYCH
ZAKŁAD MECHANIKI BUDOWLI

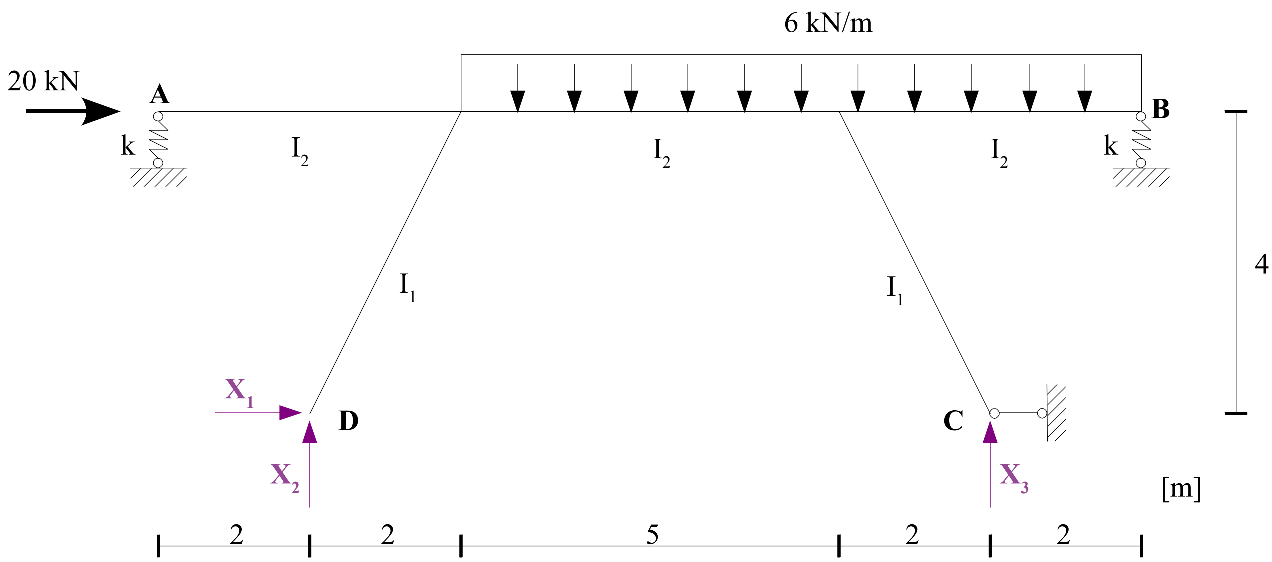
ĆWICZENIE NR 3

# OBLICZANIE UKŁADÓW STATYCZNIE NIEWYZNACZALNYCH METODĄ SIŁ OD OBCIĄŻENIA ZEWNĘTRZNEGO




SSN = 3

Przyjmujemy układ podstawowy




Zapisujemy warunki kinematycznej zgodności przyjętego układu podstawowego z układem wyjściowym:

$$v_c = 0$$
$$v_d = 0$$
$$u_d = 0$$

$$u_d = \delta_{1p} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + X_3 \cdot \delta_{13} = 0$$
$$v_d = \delta_{2p} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} + X_3 \cdot \delta_{23} = 0$$
$$v_c = \delta_{3p} + X_1 \cdot \delta_{31} + X_2 \cdot \delta_{32} + X_3 \cdot \delta_{33} = 0$$

Zapisujemy wzór na pojedyncze przemieszczenie $\delta_{ij}$:

$$\delta_{ij} = \sum \int \frac{M_i \cdot M_j}{E \cdot I} dx + \sum_m \frac{R_m^{(i)} \cdot R_m^{(j)}}{k}$$

Z twierdzenia Maxwella:

$$\delta_{ij} = \delta_{ji}$$

czyli:

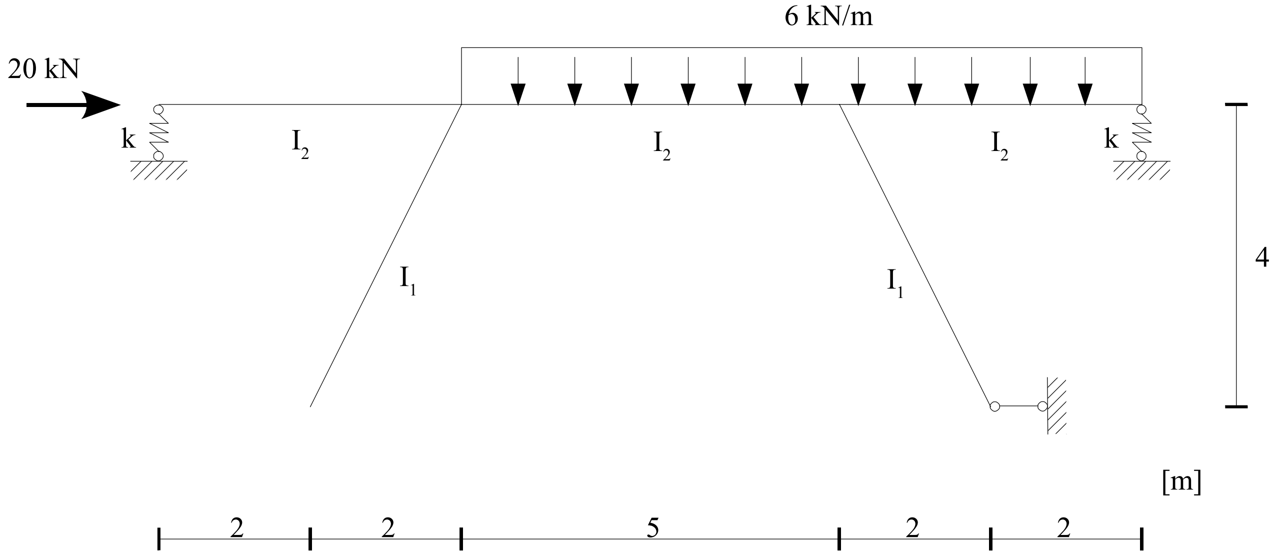
$$\delta_{12} = \delta_{21}$$

$$\delta_{13} = \delta_{31}$$

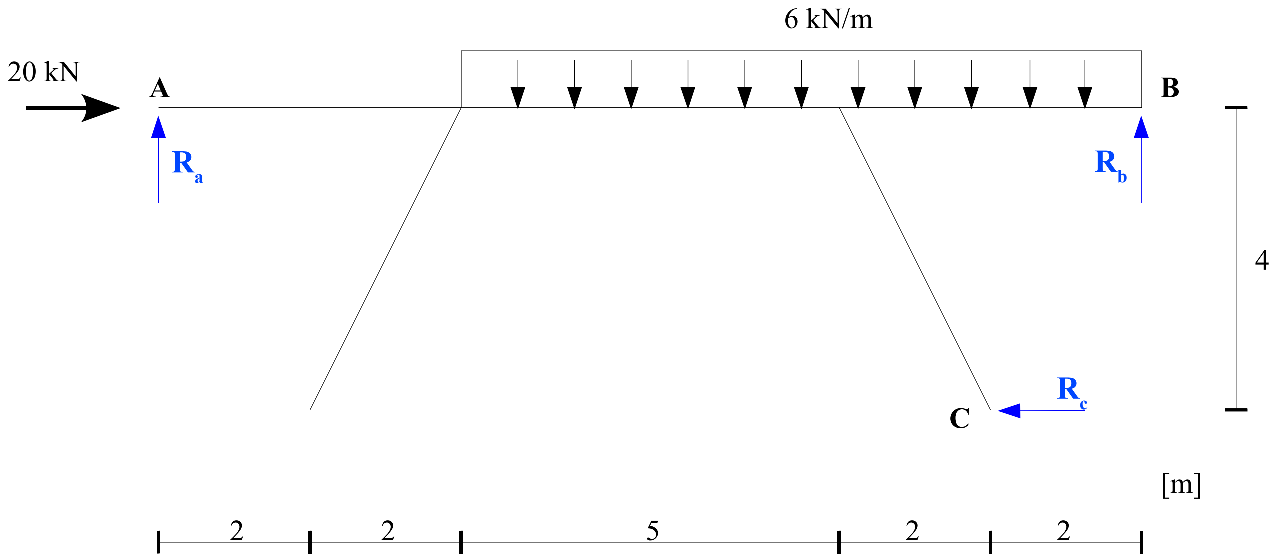
$$\delta_{23} = \delta_{32}$$

Dzielimy na stany od poszczególnych obciążeń, obliczamy reakcje podpór i rysujemy wykresy momentów zginających.

- stan od obciążenia rzeczywistego (P)

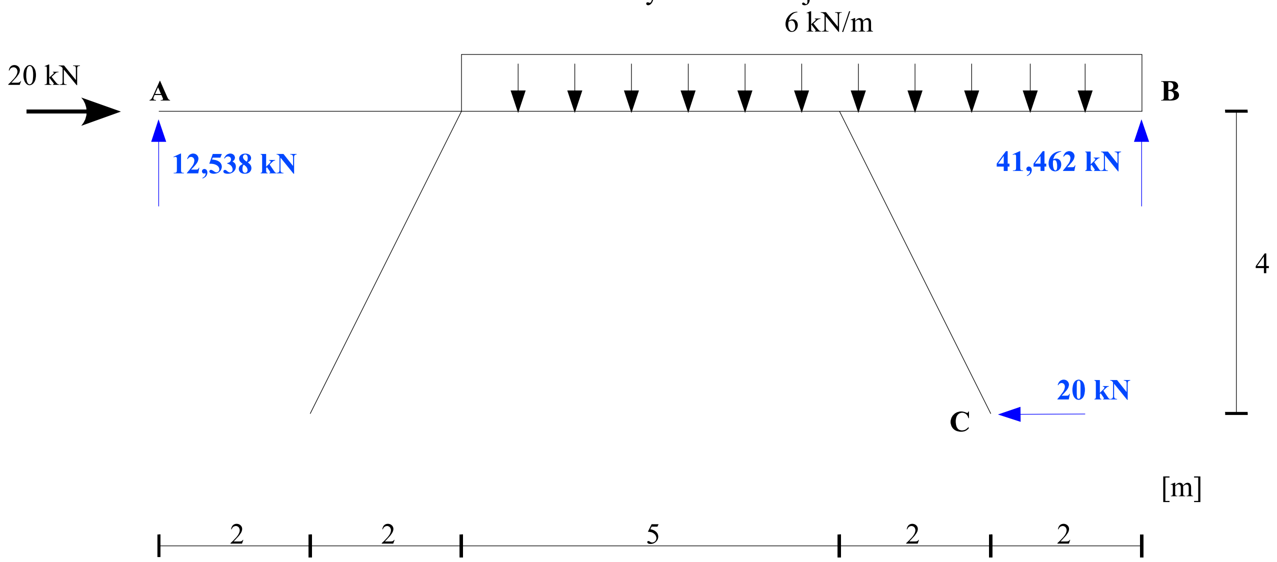



Obliczenie reakcji

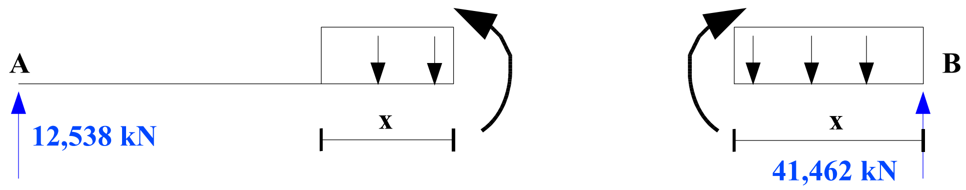


$$\sum X: \quad 20-R_c=0 \quad \Rightarrow \quad R_c=20\,[kN]$$
$$\sum M_A: \quad R_c\cdot 4+6\cdot 9\cdot(4+4{,}5)-R_b\cdot 13=0 \quad \Rightarrow \quad R_b=41{,}462\,[kN]$$
$$\sum M_B: \quad R_c\cdot 4+R_a\cdot 13-6\cdot 9\cdot 4{,}5=0 \quad \Rightarrow \quad R_a=12{,}538\,[kN]$$
$$\sum Y_{spr}: \quad R_a+R_b-6\cdot 9=41{,}462+12{,}538-54=\underline{0}$$

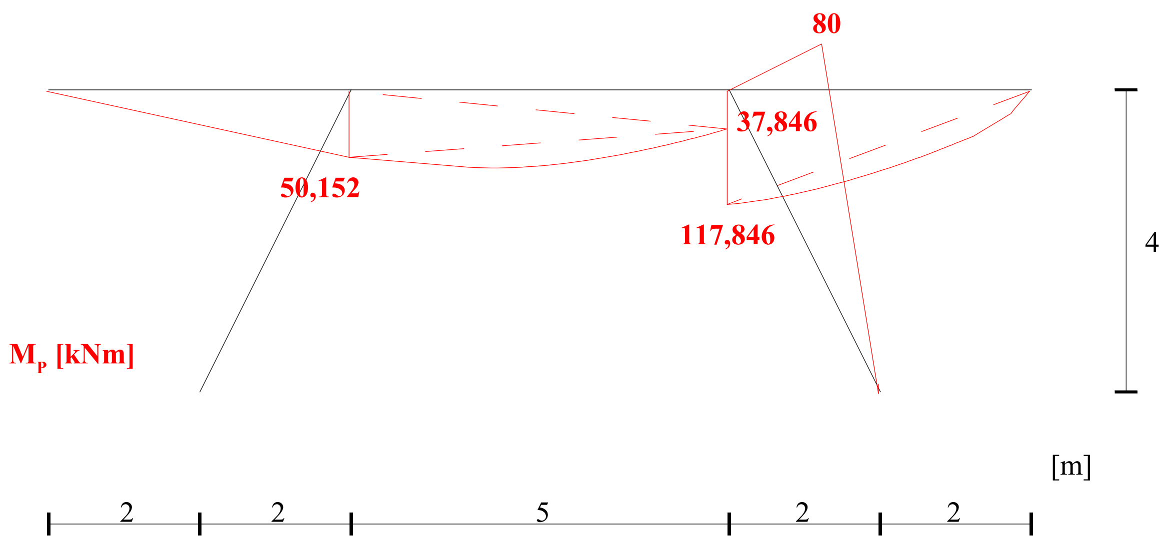
Zestawienie wyników reakcji



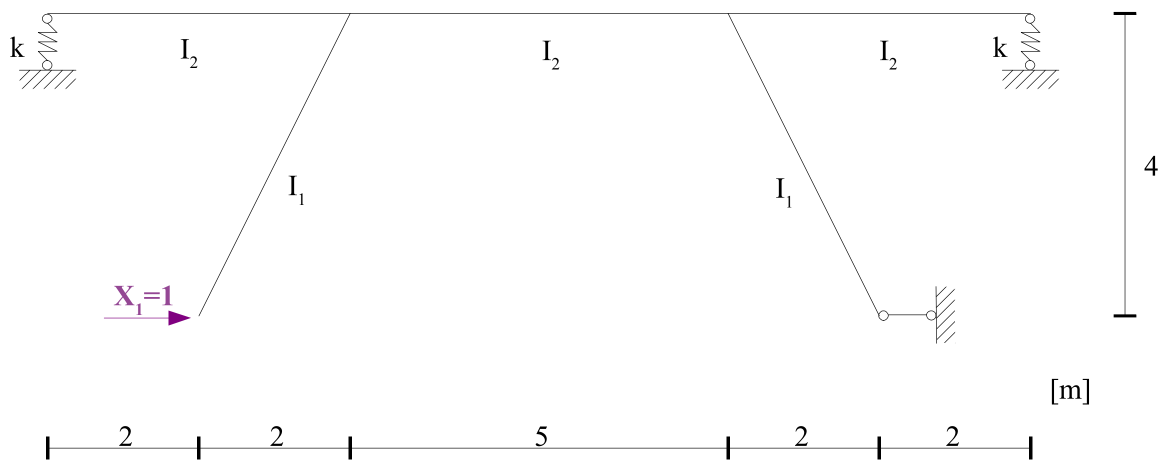

Rysunki pomocnicze do wykonania wykresu



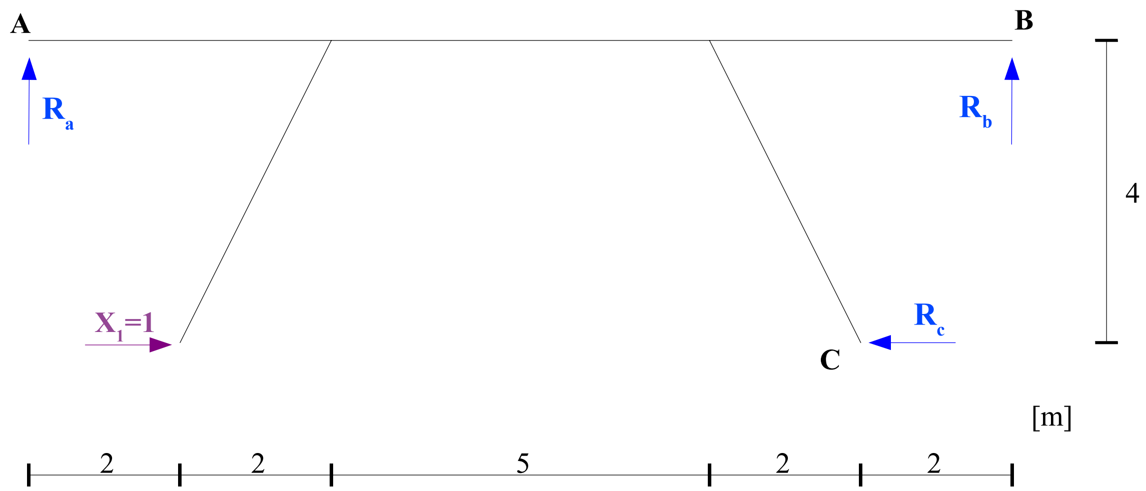

Wykres momentów zginających od obciążenia rzeczywistego P



- Stan od obciążenia $X_1$

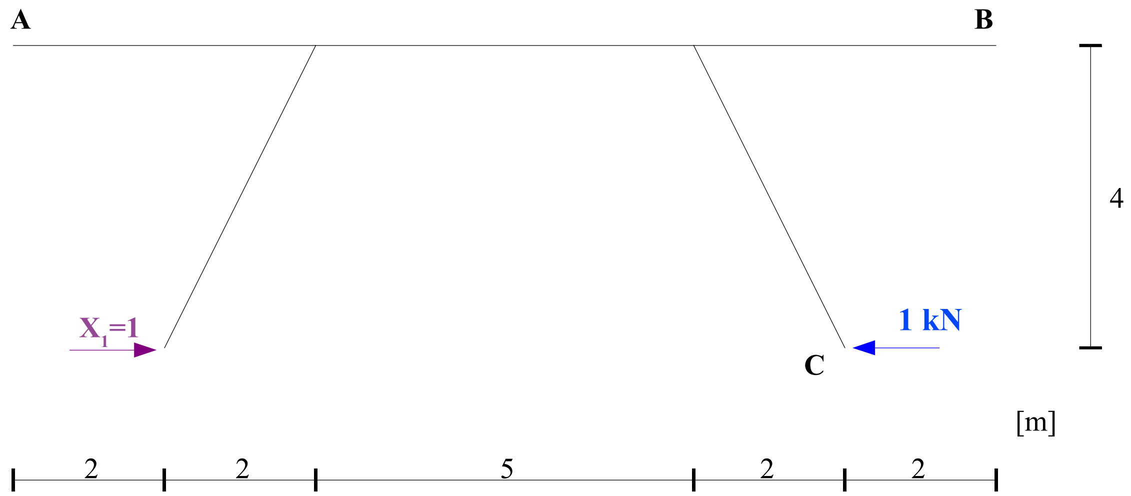



Obliczenie reakcji

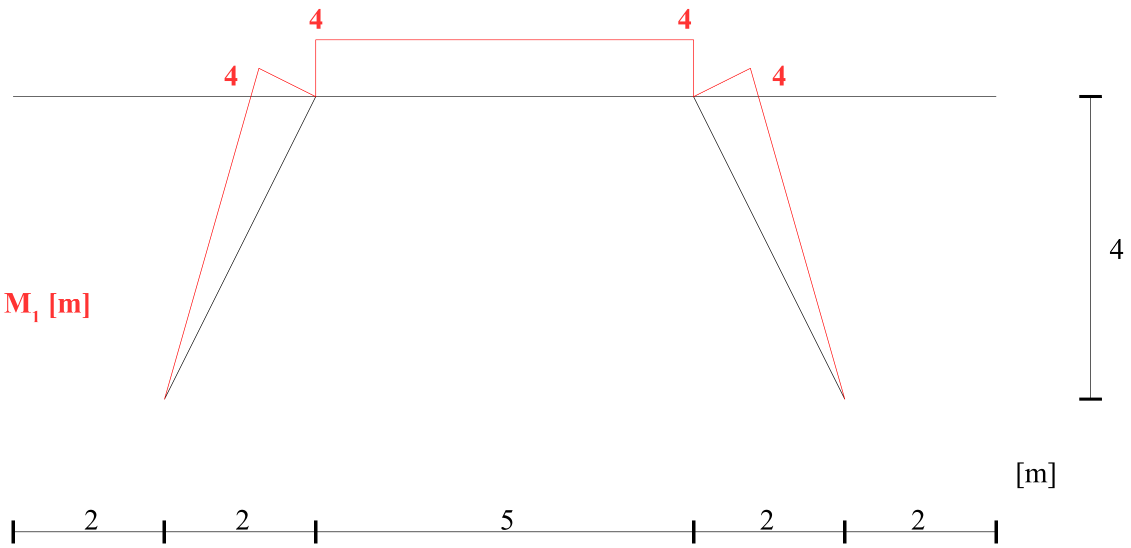



$$\sum X: \quad 1-R_c=0 \quad \Rightarrow \quad R_c=1\,[kN]$$
$$\sum M_A: \quad R_c\cdot 4-1\cdot 4-R_b\cdot 13=0 \quad \Rightarrow \quad R_b=0[kN]$$
$$\sum M_B: \quad R_c\cdot 4-1\cdot 4+R_a\cdot 13=0 \quad \Rightarrow \quad R_a=0[kN]$$
$$\sum Y_{spr}: \quad R_a+R_b=0+0=\underline{0}$$

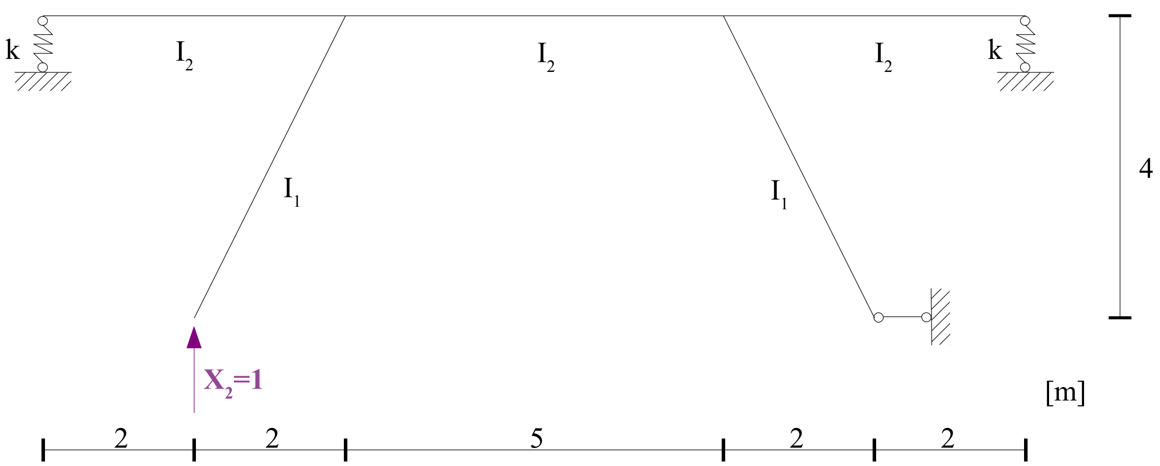
Zestawienie wyników reakcji



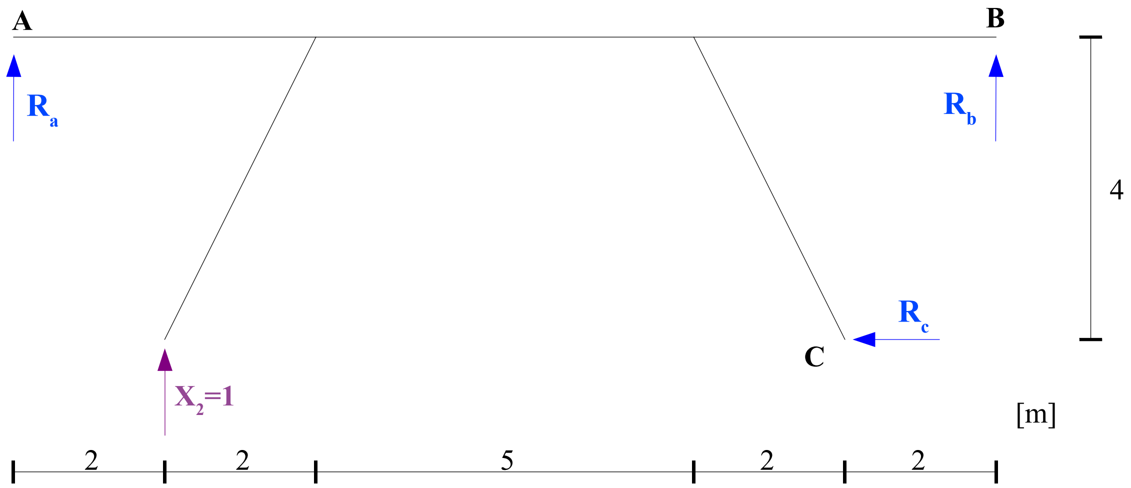

Wykres momentów zginających od $X_1$




- Stan od obciążenia $X_2$

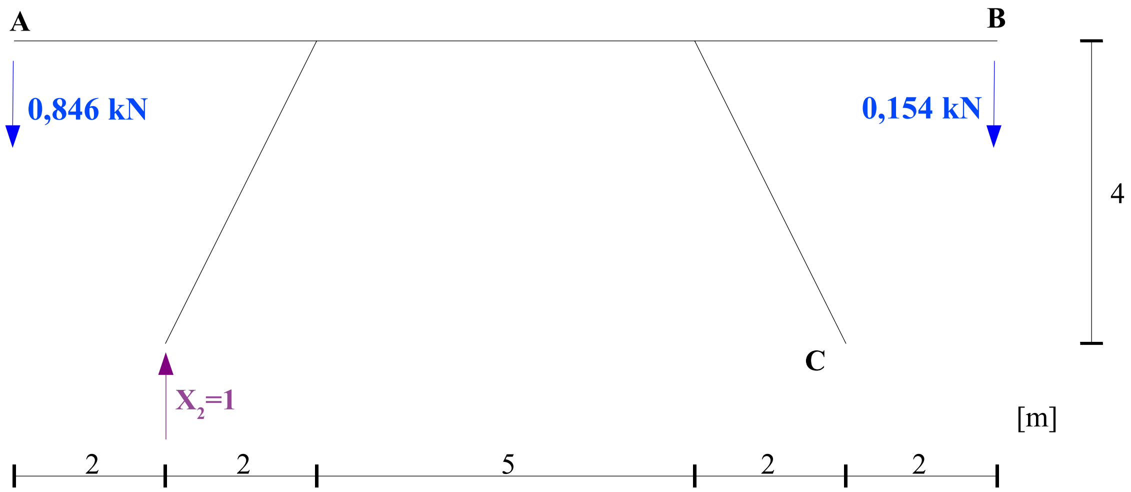


Obliczenie reakcji

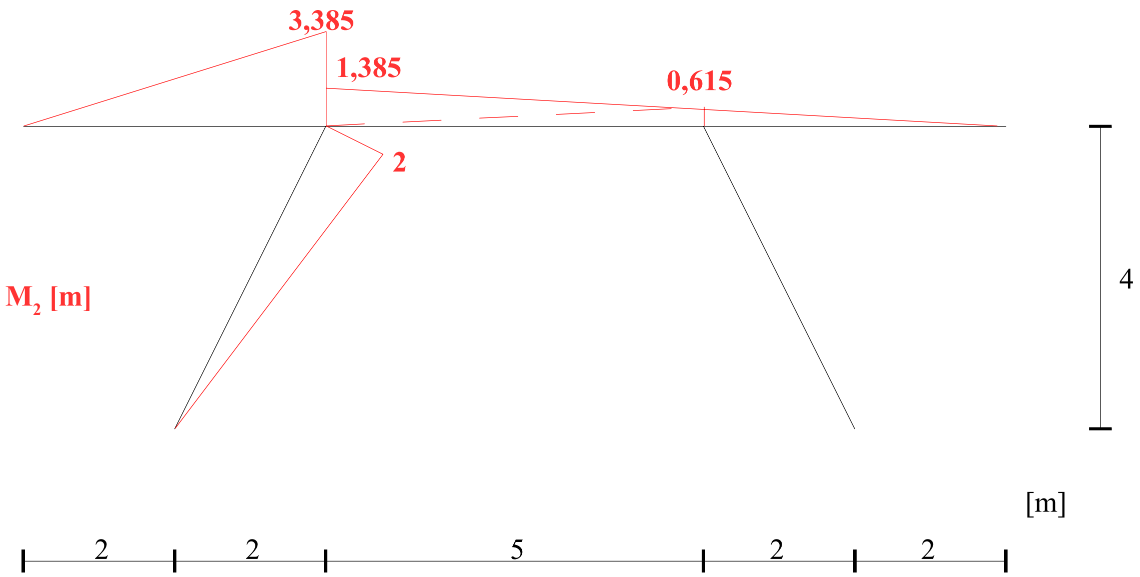



$$\sum X: \quad R_c = 0\ [kN]$$
$$\sum M_A: \quad 1 \cdot 2 + R_b \cdot 13 = 0 \quad \Rightarrow \quad R_b = -0{,}154\,[kN]$$
$$\sum M_B: \quad 1 \cdot 11 + R_a \cdot 13 = 0 \quad \Rightarrow \quad R_a = -0{,}846\,[kN]$$
$$\sum Y_{spr}: \quad R_a + R_b + 1 = -0{,}846 + (-0{,}154) + 1 = \underline{0}$$

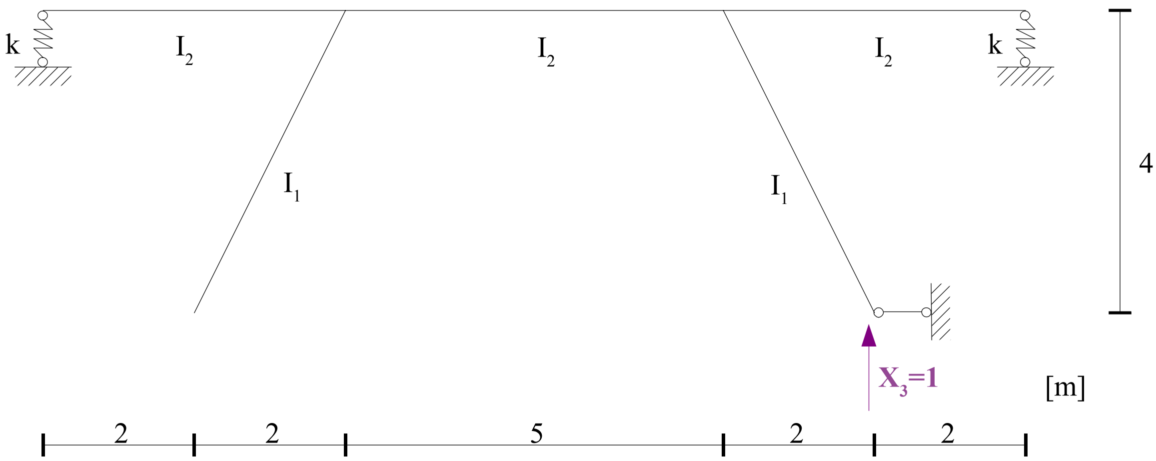
Zestawienie wyników reakcji



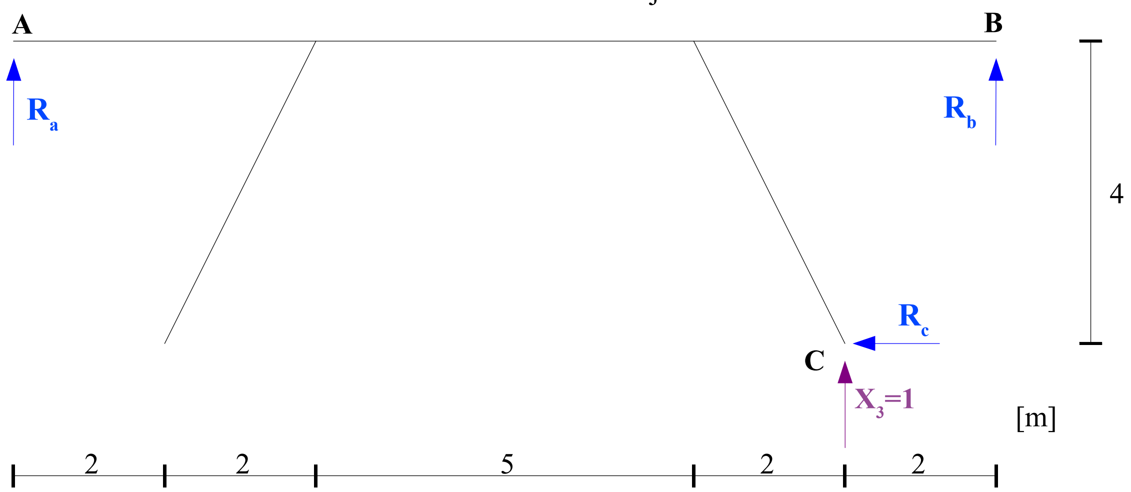
Wykres momentów zginających od $X_2$




- Stan obciążenia od $X_3$

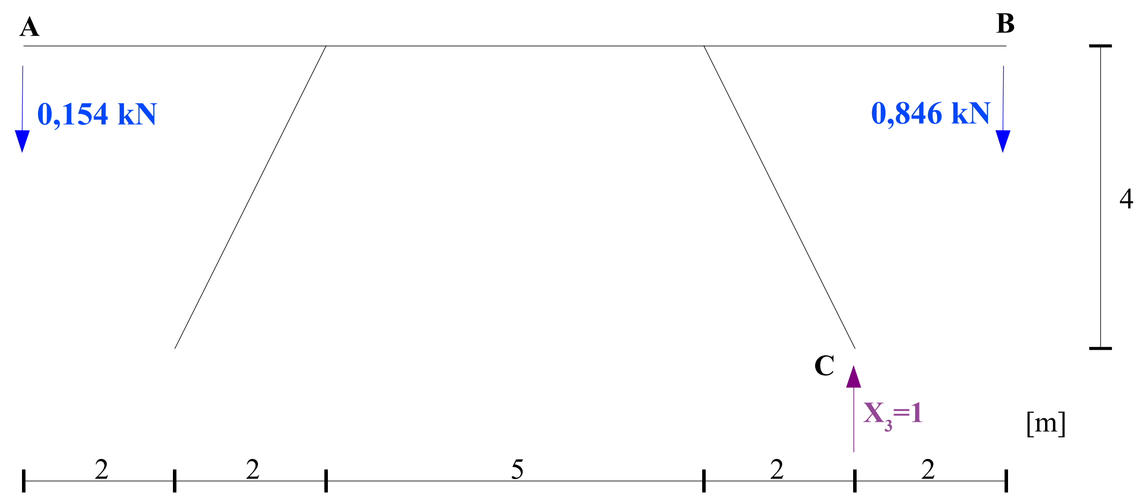



Obliczenie reakcji

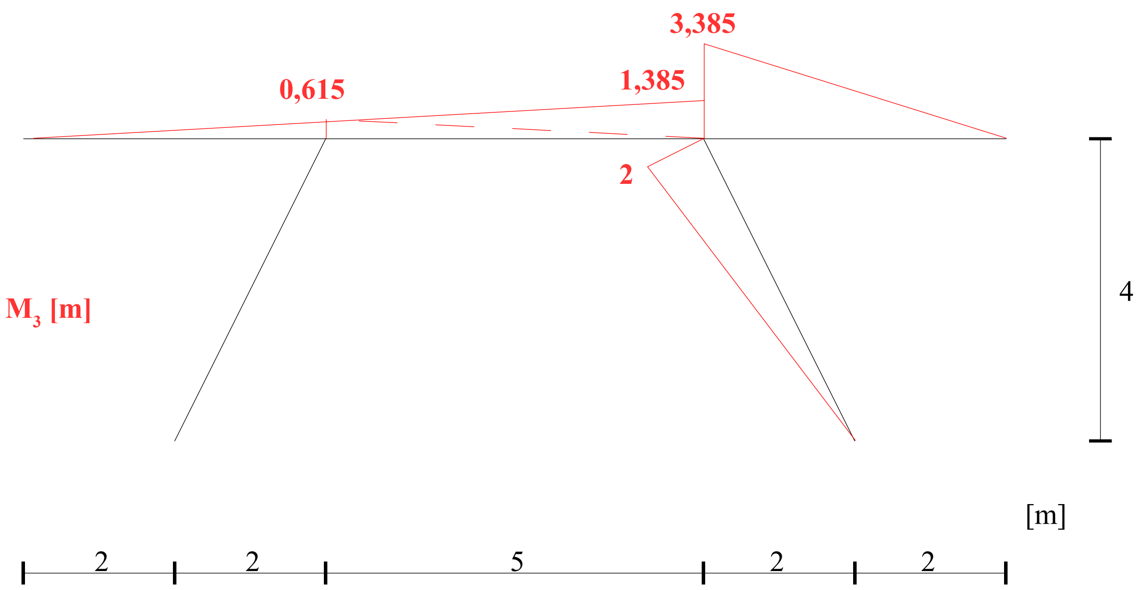


$$\sum X: \quad R_c = 0\,[kN]$$
$$\sum M_A: \quad 1\cdot 11 + R_b\cdot 13 = 0 \quad \Rightarrow \quad R_b = -0{,}846\,[kN]$$
$$\sum M_B: \quad 1\cdot 2 + R_a\cdot 13 = 0 \quad \Rightarrow \quad R_a = -0{,}154\,[kN]$$
$$\sum Y_{spr}: \quad R_a + R_b + 1 = -0{,}154 + (-0{,}846) + 1 = \underline{0}$$

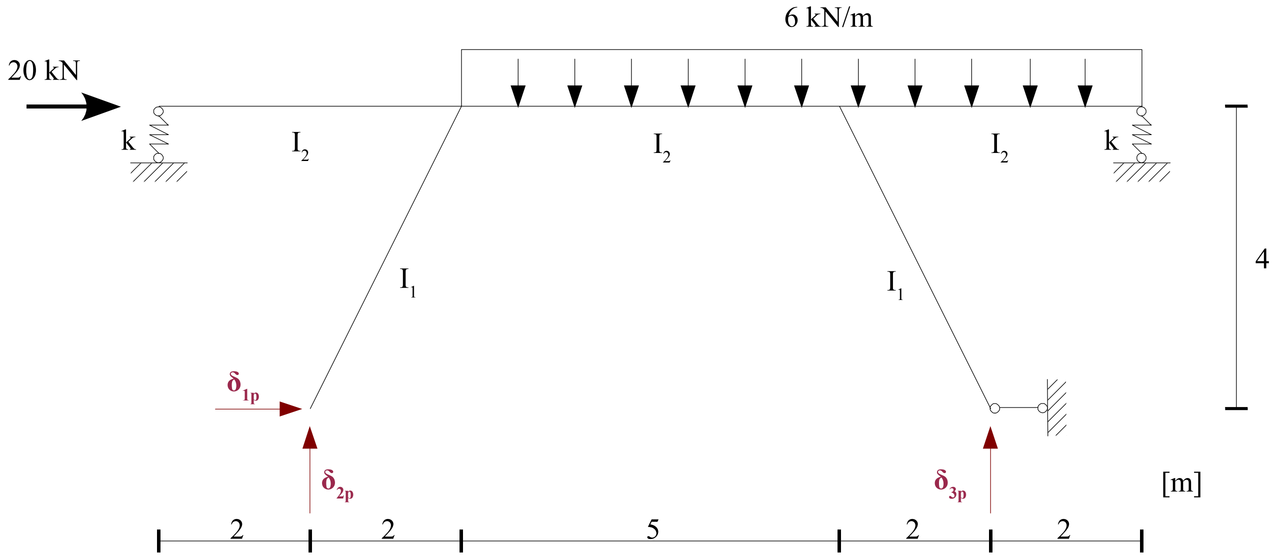
Zestawienie wyników reakcji

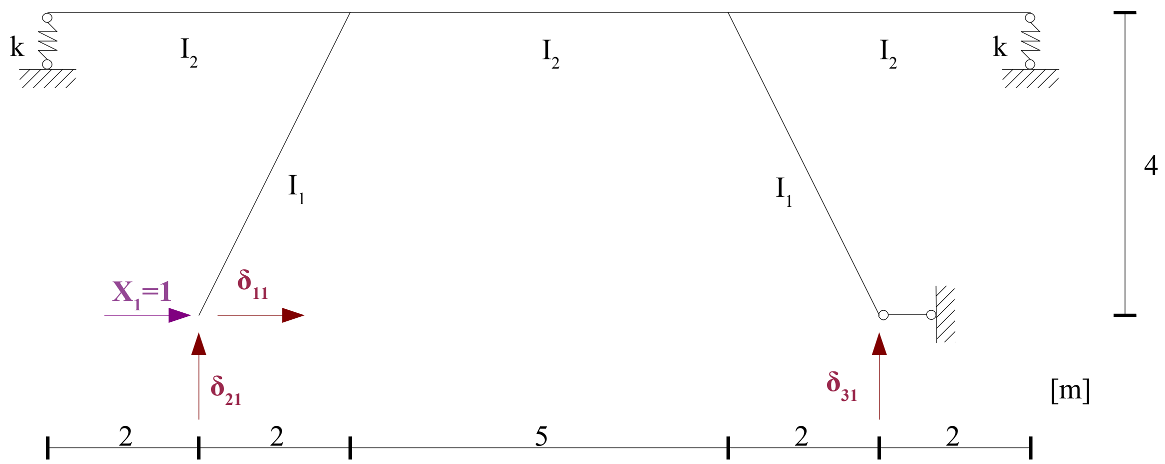


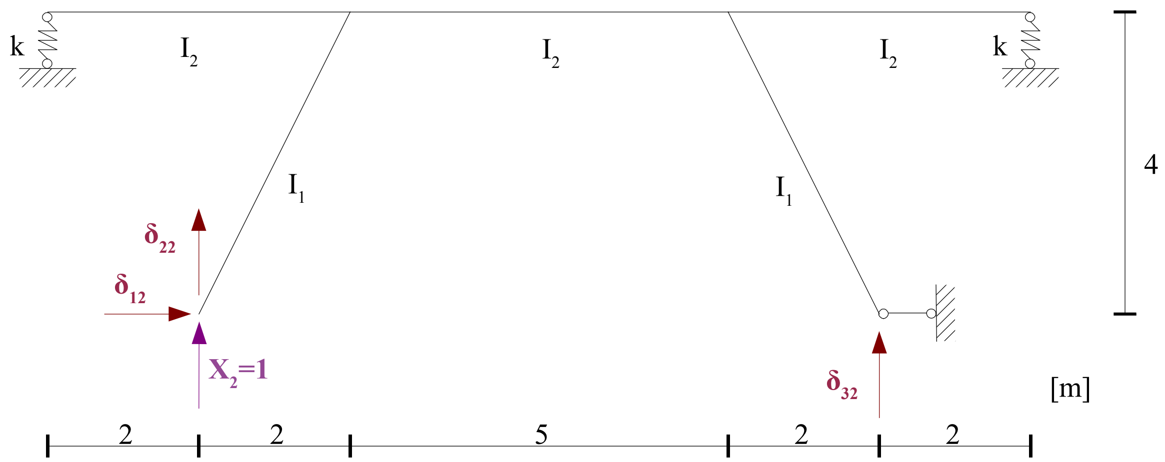

Wykres momentów zginających od $X_3$

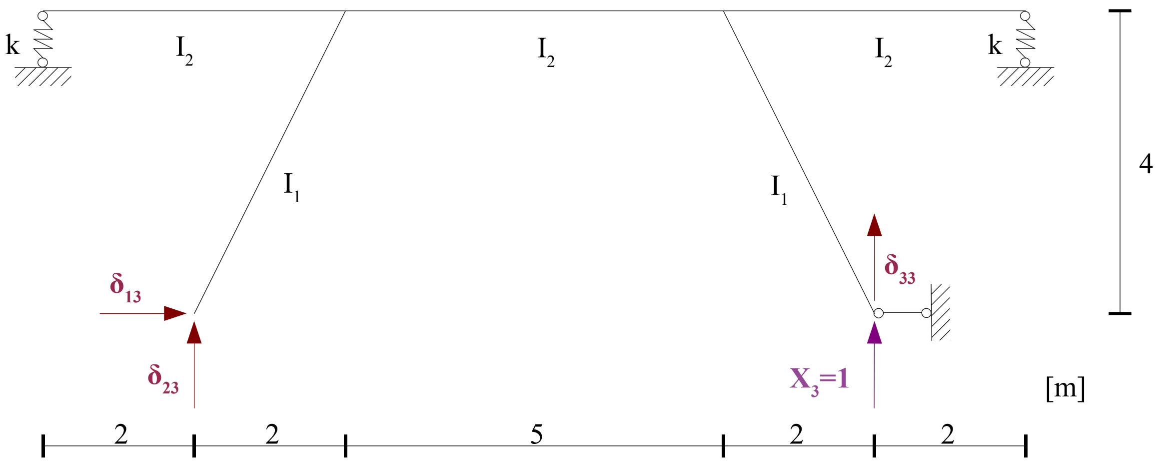


Zaznaczamy potrzebne przemieszczenia na schematach






Dane geometryczne i fizyczne:

I 220 I 240

$I_1 = 3060\ cm^4 = 3060\cdot10^{-8}\ m^4$ $I_2 = 4250\ cm^4 = 4250\cdot10^{-8}\ m^4$

$E = 205\ GPa = 205\cdot10^6\ [kN/m^2]$

$EI_1 = EI$

$EI_2 = nEI$ gdzie $n = I_2/I_1$

$EI_2 = 1{,}389\cdot EI$

$k = 0{,}167\cdot EI$

Obliczenia:

$$\delta_{11}=\sum\int\frac{M_1\cdot M_1}{EI}dx+\sum_m\frac{R_m^{(1)}\cdot R_m^{(1)}}{k}$$

$$\delta_{11}=\frac{1}{EI}\cdot\left[2\cdot\left(\frac{1}{2}\cdot4\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot4\right)\right]+\frac{1}{1{,}389\cdot EI}\cdot(4\cdot5\cdot4)=\frac{105{,}303}{EI}$$

$$\delta_{22}=\sum\int\frac{M_2\cdot M_2}{EI}dx+\sum_m\frac{R_m^{(2)}\cdot R_m^{(2)}}{k}$$

$$\delta_{22}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot2\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot2\right)+\frac{1}{1{,}389\cdot EI}\cdot\left[\left(\frac{1}{2}\cdot3{,}385\cdot4\cdot\frac{2}{3}\cdot3{,}385\right)+\left(\frac{1}{2}\cdot1{,}385\cdot9\cdot\frac{2}{3}\cdot1{,}385\right)\right]$$
$$+\frac{0{,}846\cdot0{,}846+0{,}154\cdot0{,}154}{0{,}167\cdot EI}=\frac{25{,}543}{EI}$$

$$\delta_{33}=\sum\int\frac{M_3\cdot M_3}{EI}dx+\sum_m\frac{R_m^{(3)}\cdot R_m^{(3)}}{k}$$

$$\delta_{33}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot2\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot2\right)+\frac{1}{1{,}389\cdot EI}\cdot\left[\left(\frac{1}{2}\cdot1{,}385\cdot9\cdot\frac{2}{3}\cdot1{,}385\right)+\left(\frac{1}{2}\cdot3{,}385\cdot4\cdot\frac{2}{3}\cdot3{,}385\right)\right]$$
$$+\frac{0{,}154\cdot0{,}154+0{,}846\cdot0{,}846}{0{,}167\cdot EI}=\frac{25{,}543}{EI}$$

$$\delta_{12}=\delta_{21}=\sum\int\frac{M_1\cdot M_2}{EI}dx+\sum_m\frac{R_m^{(1)}\cdot R_m^{(2)}}{k}$$

$$\delta_{12}=\delta_{21}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 4\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-2)\right)+\frac{1}{1,389\cdot EI}\cdot\left[4\cdot 5\cdot(\frac{1}{2}\cdot 1,385+\frac{1}{2}\cdot 0,615)\right]=\frac{2,474}{EI}$$

$$\delta_{13}=\delta_{31}=\sum\int\frac{M_1\cdot M_3}{EI}dx+\sum_m\frac{R_m^{(1)}\cdot R_m^{(3)}}{k}$$

$$\delta_{13}=\delta_{31}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 4\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-2)\right)+\frac{1}{1,389\cdot EI}\cdot\left[4\cdot 5\cdot(\frac{1}{2}\cdot 0,615+\frac{1}{2}\cdot 1,385)\right]=\frac{2,474}{EI}$$

$$\delta_{23}=\delta_{32}=\sum\int\frac{M_2\cdot M_3}{EI}dx+\sum_m\frac{R_m^{(2)}\cdot R_m^{(3)}}{k}$$

$$\delta_{23}=\delta_{32}=\frac{1}{1,389\cdot EI}\cdot\left[(\frac{1}{2}\cdot 3,385\cdot 4\cdot\frac{2}{3}\cdot 0,615)+(\frac{1}{2}\cdot 1,385\cdot 5\cdot(\frac{1}{3}\cdot 1,385+\frac{2}{3}\cdot 0,615))\right.$$
$$\left.+(\frac{1}{2}\cdot 0,615\cdot 5\cdot(\frac{2}{3}\cdot 1,385+\frac{1}{3}\cdot 0,615))+(\frac{1}{2}\cdot 0,615\cdot 4\cdot\frac{2}{3}\cdot 3,385)\right]$$
$$+\frac{0,846\cdot 0,154+0,154\cdot 0,846}{0,167\cdot EI}=\frac{8,986}{EI}$$

$$\delta_{1p}=\sum\int\frac{M_1\cdot M_p}{EI}dx+\sum_m\frac{R_m^{(1)}\cdot R_m^{(p)}}{k}$$

$$\delta_{1p}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 80\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 4\right)+\frac{1}{1,389\cdot EI}\cdot\left[(\frac{1}{2}\cdot 50,152\cdot 5\cdot(-4))+(\frac{1}{2}\cdot 37,846\cdot 5\cdot(-4))\right.$$
$$\left.+(\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\cdot(-4))\right]=\frac{-336,558}{EI}$$

$$\delta_{2p}=\sum\int\frac{M_2\cdot M_p}{EI}dx+\sum_m\frac{R_m^{(2)}\cdot R_m^{(p)}}{k}$$

$$\delta_{2p}=\frac{1}{1,389\cdot EI}\cdot\left[(\frac{1}{2}\cdot 50,152\cdot 4\cdot\frac{2}{3}\cdot(-3,385))+(\frac{1}{2}\cdot 37,846\cdot 5\cdot(-\frac{1}{3}\cdot 1,385-\frac{2}{3}\cdot 0,615))\right.$$
$$+(\frac{1}{2}\cdot 50,152\cdot 5\cdot(-\frac{2}{3}\cdot 1,385-\frac{1}{3}\cdot 0,615))+(\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\cdot(-\frac{1}{2}\cdot 1,385-\frac{1}{2}\cdot 0,615))$$
$$\left.+(\frac{1}{2}\cdot 117,846\cdot 4\cdot\frac{2}{3}\cdot(-0,615))+(\frac{2}{3}\cdot\frac{6\cdot 4^2}{8}\cdot 4\cdot\frac{1}{2}\cdot(-0,615))\right]$$
$$+\frac{12,538\cdot(-0,846)+41,462\cdot(-0,154)}{0,167\cdot EI}=\frac{-547,828}{EI}$$

$$\delta_{3p}=\sum\int\frac{M_3\cdot M_p}{EI}dx+\sum_m\frac{R_m^{(3)}\cdot R_m^{(p)}}{k}$$

$$\delta_{3p}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 80\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-2)\right)+\frac{1}{1{,}389\cdot EI}\cdot\left[(\frac{1}{2}\cdot 50{,}152\cdot 4\cdot\frac{2}{3}\cdot(-0615))\right.$$

$$+(\frac{1}{2}\cdot 37{,}846\cdot 5\cdot(-\frac{2}{3}\cdot 1{,}385-\frac{1}{3}\cdot 0{,}615))+(\frac{1}{2}\cdot 50{,}152\cdot 5\cdot(-\frac{1}{3}\cdot 1{,}385-\frac{2}{3}\cdot 0{,}615))$$

$$\left.+(\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\cdot(-\frac{1}{2}\cdot 0{,}615-\frac{1}{2}\cdot 1{,}385))+(\frac{1}{2}\cdot 117{,}846\cdot 4\cdot\frac{2}{3}\cdot(-3{,}385))+(\frac{2}{3}\cdot\frac{6\cdot 4^2}{8}\cdot 4\cdot\frac{1}{2}\cdot(-3{,}385))\right]$$

$$+\frac{12{,}538\cdot(-0{,}154)+41{,}462\cdot(-0{,}846)}{0{,}167\cdot EI}=\frac{-1112{,}671}{EI}$$

Gdy mamy obliczone już wszystkie potrzebne $\delta_{ij}$ podstawiamy je do wzorów na przemieszczenia i otrzymujemy w ten sposób układ równań, z którego obliczymy niewiadome $X_1$, $X_2$, $X_3$:

$$\begin{cases}\frac{-336{,}558}{EI}+X_1\cdot\frac{105{,}303}{EI}+X_2\cdot\frac{2{,}474}{EI}+X_3\cdot\frac{2{,}474}{EI}=0\\ \frac{-547{,}828}{EI}+X_1\cdot\frac{2{,}474}{EI}+X_2\cdot\frac{25{,}543}{EI}+X_3\cdot\frac{8{,}986}{EI}=0\\ \frac{-1112{,}671}{EI}+X_1\cdot\frac{2{,}474}{EI}+X_2\cdot\frac{8{,}986}{EI}+X_3\cdot\frac{25{,}543}{EI}=0\end{cases}$$

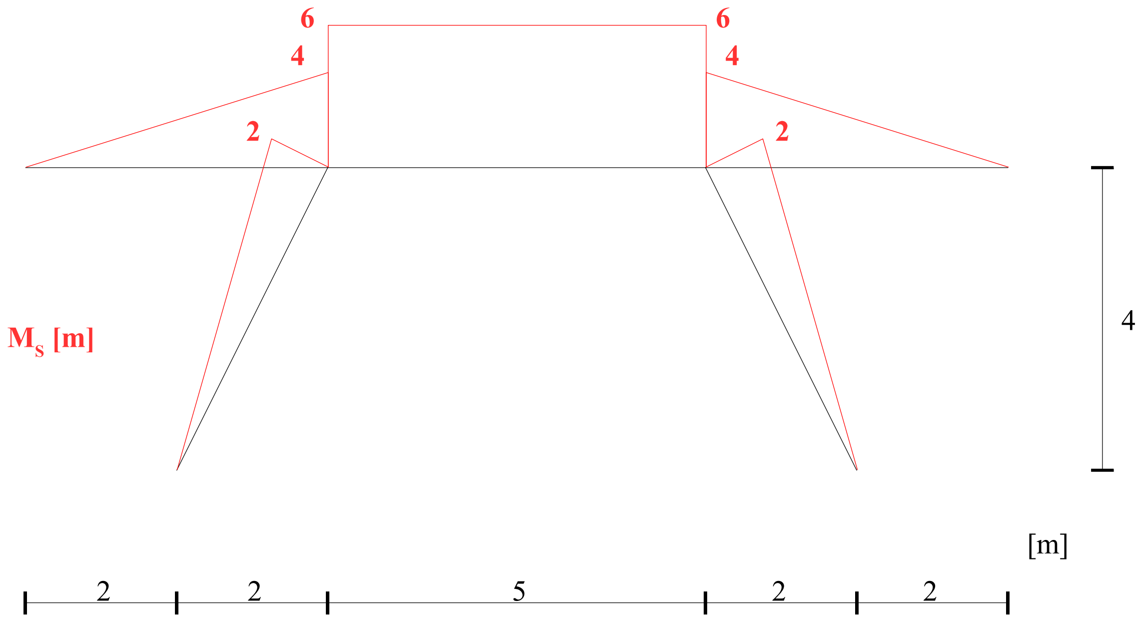
$$\begin{cases}X_1\cdot 105{,}303+X_2\cdot 2{,}474+X_3\cdot 2{,}474=336{,}558\\ X_1\cdot 2{,}474+X_2\cdot 25{,}543+X_3\cdot 8{,}986=547{,}828\\ X_1\cdot 2{,}474+X_2\cdot 8{,}986+X_3\cdot 25{,}543=1112{,}671\end{cases}$$

$$\begin{cases}X_1=2{,}073\ [kN]\\ X_2=6{,}839\ [kN]\\ X_3=40{,}954\ [kN]\end{cases}$$

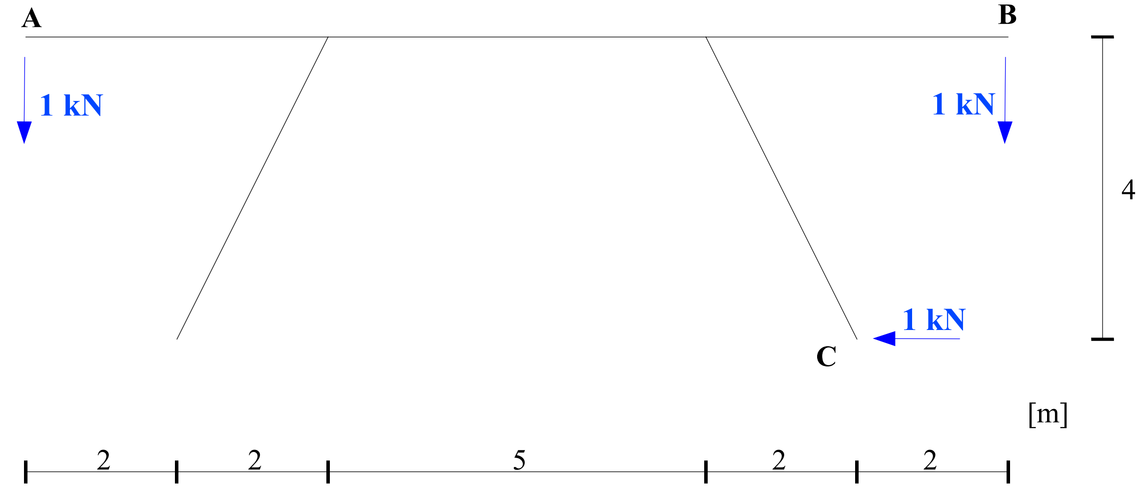
*obliczenia wykonano w programie Derive

<u>sprawdzenie</u>

$M_S = M_1 + M_2 + M_3$




Reakcje




$$\sum\int \frac{M_s\cdot M_p}{EI}dx+\sum_m \frac{R_m^{(s)}\cdot R_m^{(p)}}{k}=\delta_{1p}+\delta_{2p}+\delta_{3p}$$

$$\sum\int \frac{M_s\cdot M_p}{EI}dx+\sum_m \frac{R_m^{(s)}\cdot R_m^{(p)}}{k}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 80\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 2\right)+\frac{1}{1,389\cdot EI}\cdot\left[\frac{1}{2}\cdot 50,152\cdot 4\cdot\frac{2}{3}\cdot(-4)\right.$$

$$+\frac{1}{2}\cdot 37,846\cdot 5\cdot(-6)+\frac{1}{2}\cdot 50,152\cdot 5\cdot(-6)+\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\cdot(-6)+\frac{1}{2}\cdot 117,846\cdot 4\cdot\frac{2}{3}\cdot(-4)$$

$$\left.+\frac{2}{3}\cdot\frac{6\cdot 4^2}{8}\cdot 4\cdot\frac{1}{2}\cdot(-4)\right]+\frac{12,538\cdot(-1)+41,462\cdot(-1)}{0,167\cdot EI}=\frac{-1997,057}{EI}$$

$$\delta_{1p}+\delta_{2p}+\delta_{3p}=\frac{-336,558}{EI}+\frac{-547,828}{EI}+\frac{-1112,671}{EI}=\frac{-1997,057}{EI}$$

$$\sum\int\frac{M_s\cdot M_1}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(1)}}{k}=\delta_{11}+\delta_{12}+\delta_{13}$$

$$\sum\int\frac{M_s\cdot M_1}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(1)}}{k}=\frac{1}{EI}\cdot 2\cdot\left(\frac{1}{2}\cdot 2\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 4\right)+\frac{1}{1{,}389\cdot EI}\cdot(6\cdot 5\cdot 4)=\frac{110{,}251}{EI}$$

$$\delta_{11}+\delta_{12}+\delta_{13}=\frac{105{,}303}{EI}+\frac{2{,}474}{EI}+\frac{2{,}474}{EI}=\frac{110{,}251}{EI}$$

$$\sum\int\frac{M_s\cdot M_2}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(2)}}{k}=\delta_{21}+\delta_{22}+\delta_{23}$$

$$\sum\int\frac{M_s\cdot M_2}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(2)}}{k}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 2\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-2)\right)$$

$$+\frac{1}{1{,}389\cdot EI}\cdot\left[\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 3{,}385+6\cdot 5\cdot\left(\frac{1}{2}\cdot 1{,}385+\frac{1}{2}\cdot 0{,}615\right)+\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 0{,}615\right]$$

$$+\frac{0{,}846\cdot 1+0{,}154\cdot 1}{0{,}167\cdot EI}=\frac{36{,}997}{EI}$$

$$\delta_{21}+\delta_{22}+\delta_{23}=\frac{2{,}474}{EI}+\frac{25{,}543}{EI}+\frac{8{,}986}{EI}=\frac{37{,}003}{EI}$$

$$\sum\int\frac{M_s\cdot M_3}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(3)}}{k}=\delta_{31}+\delta_{32}+\delta_{33}$$

$$\sum\int\frac{M_s\cdot M_3}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(3)}}{k}=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot 2\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-2)\right)$$

$$+\frac{1}{1{,}389\cdot EI}\cdot\left[\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 0{,}615+6\cdot 5\cdot\left(\frac{1}{2}\cdot 0{,}615+\frac{1}{2}\cdot 1{,}385\right)+\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 3{,}385\right]$$

$$+\frac{0{,}154\cdot 1+0{,}846\cdot 1}{0{,}167\cdot EI}=\frac{36{,}997}{EI}$$

$$\delta_{31}+\delta_{32}+\delta_{33}=\frac{2{,}474}{EI}+\frac{8{,}986}{EI}+\frac{25{,}543}{EI}=\frac{37{,}003}{EI}$$

$$\sum\int\frac{M_s\cdot M_s}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(s)}}{k}=\delta_{11}+\delta_{12}+\delta_{13}+\delta_{21}+\delta_{22}+\delta_{23}+\delta_{31}+\delta_{32}+\delta_{33}$$

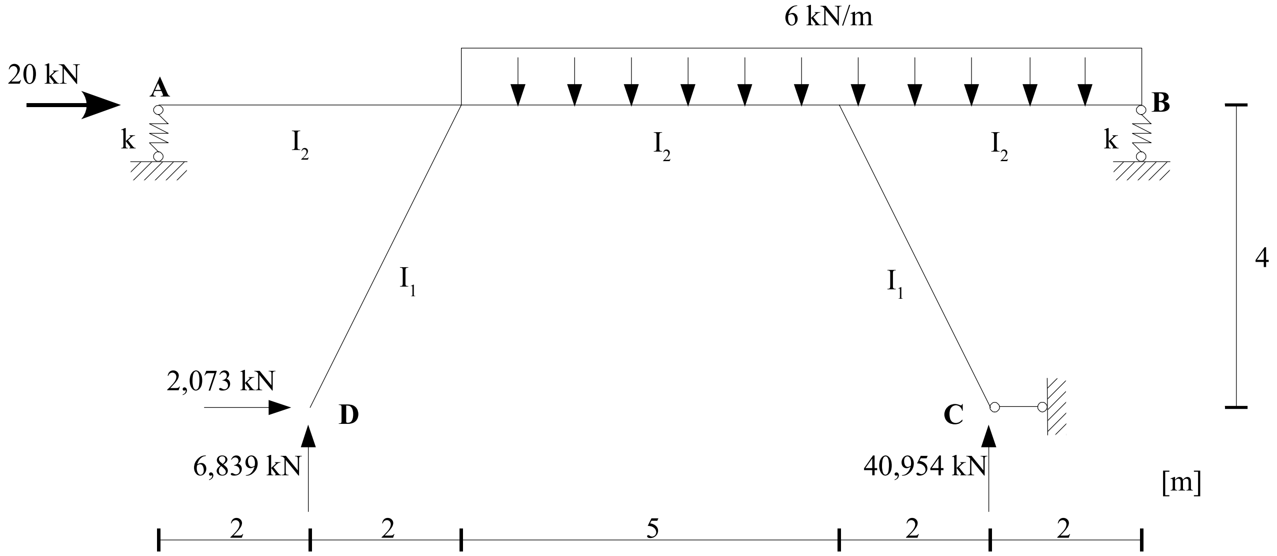
$$\sum\int\frac{M_s\cdot M_s}{EI}dx+\sum_m\frac{R_m^{(s)}\cdot R_m^{(s)}}{k}=\frac{1}{EI}\cdot 2\cdot\left(\frac{1}{2}\cdot 2\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 2\right)$$

$$+\frac{1}{1{,}389\cdot EI}\cdot\left[2\cdot\left(\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4\right)+6\cdot 5\cdot 6\right]+\frac{1\cdot 1+1\cdot 1}{0{,}167\cdot EI}=\frac{184{,}246}{EI}$$

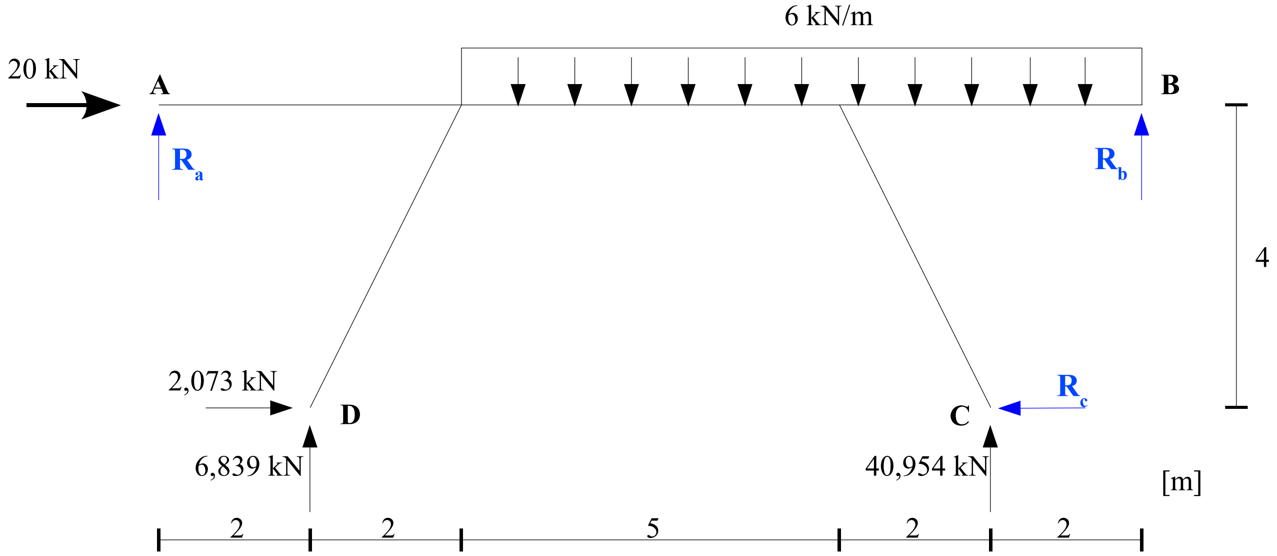
$$\delta_{11}+\delta_{12}+\delta_{13}+\delta_{21}+\delta_{22}+\delta_{23}+\delta_{31}+\delta_{32}+\delta_{33}=\frac{105{,}303}{EI}+\frac{2{,}474}{EI}+\frac{2{,}474}{EI}+\frac{2{,}474}{EI}+\frac{25{,}543}{EI}$$

$$\frac{+8{,}986}{EI}+\frac{2{,}474}{EI}+\frac{8{,}986}{EI}+\frac{25{,}543}{EI}=\frac{184{,}257}{EI}$$

Zestawienie wyników




Obliczenie reakcji




$$\sum X: \quad 20+2{,}073-R_c=0 \quad \Rightarrow \quad R_c=22{,}073\,[kN]$$

$$\sum M_A: \quad 6{,}839\cdot 2+40{,}954\cdot 11+R_b\cdot 13-6\cdot 9\cdot(4+4{,}5)+2{,}073\cdot 4-R_c\cdot 4=0$$

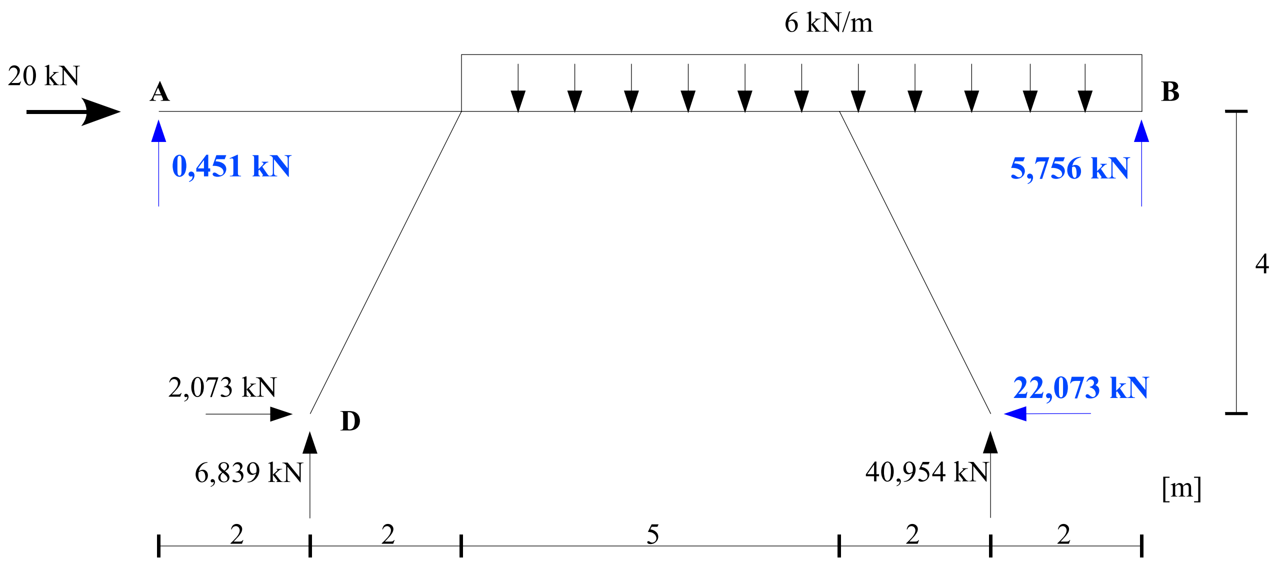
$$\Rightarrow \quad R_b=5{,}756\,[kN]$$

$$\sum M_B: \quad 40{,}954\cdot 2+6{,}839\cdot 11+R_a\cdot 13-6\cdot 9\cdot 4{,}5+R_c\cdot 4-2{,}073\cdot 4=0$$

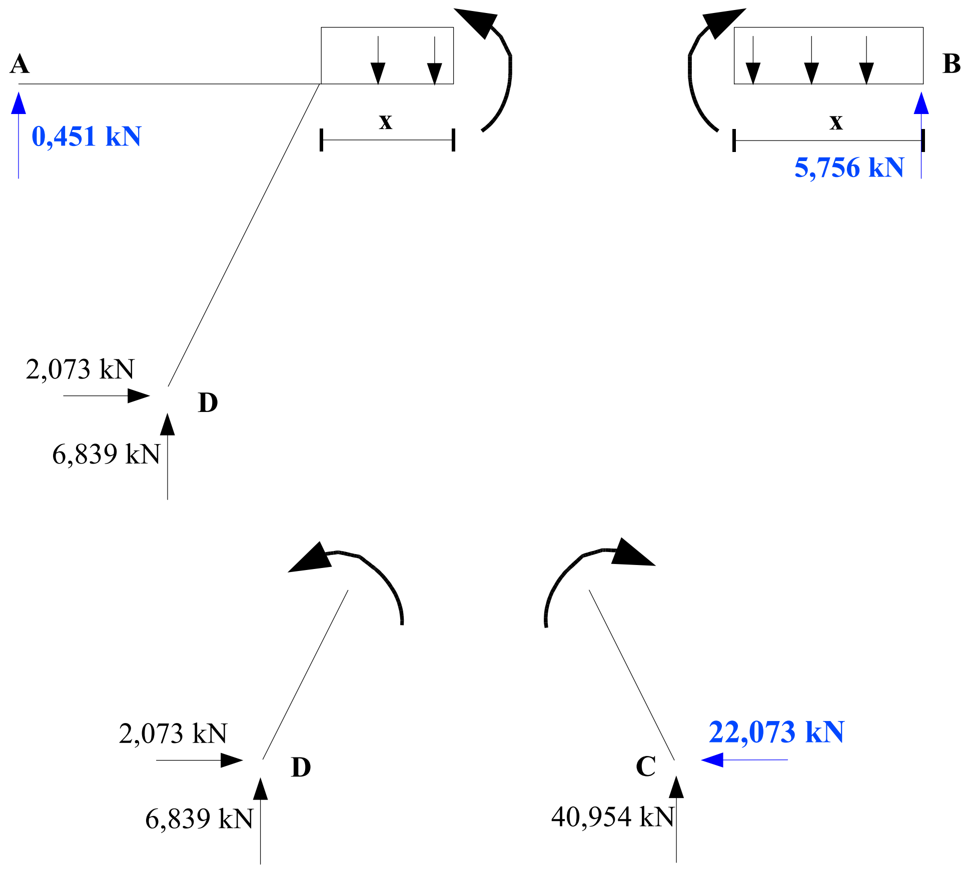
$$\Rightarrow \quad R_a=0{,}451\,[kN]$$

$$\sum Y_{spr}: \quad R_a+R_b+6{,}839+40{,}954-6\cdot 9=0{,}451+5{,}756+6{,}839+40{,}954-54=\underline{0}$$

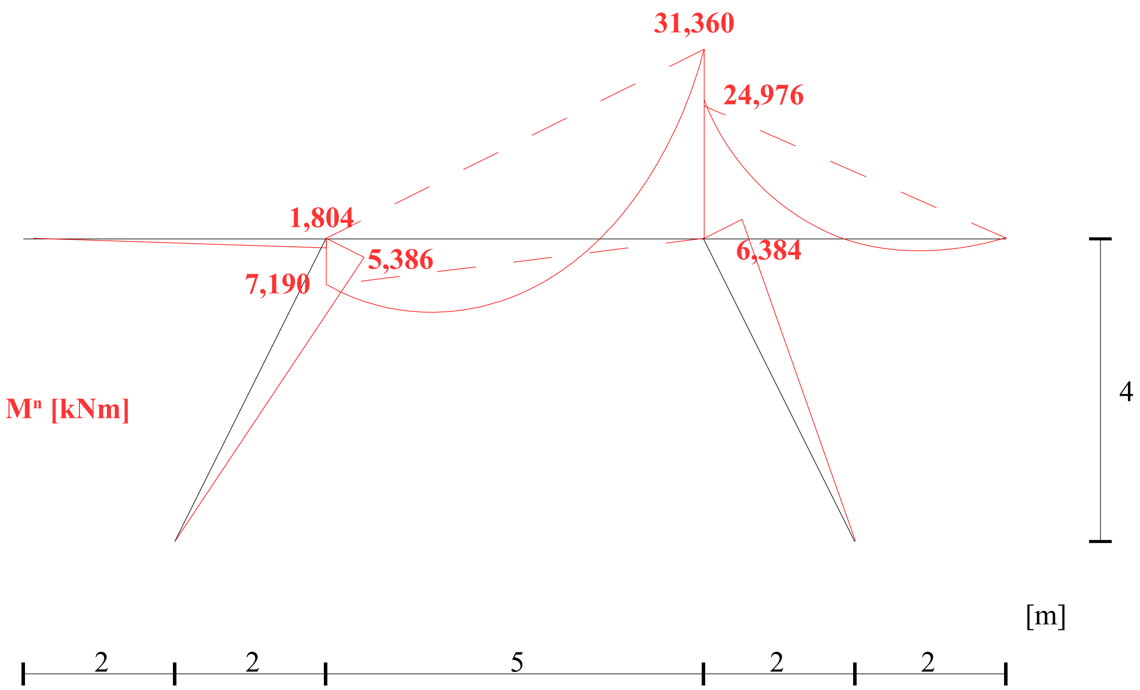
Zestawienie wyników reakcji




Rysunki pomocnicze do wykonania wykresu



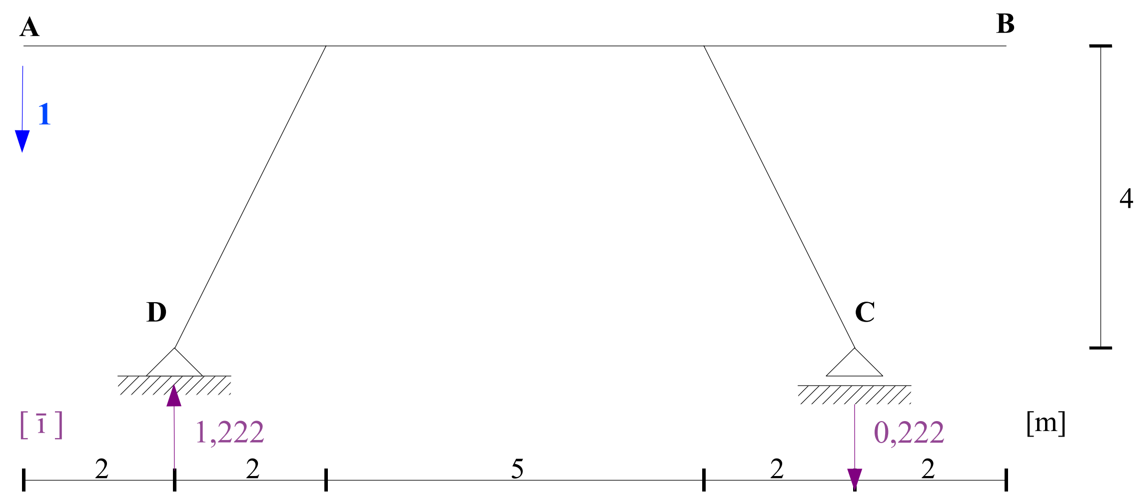
Wykres momentów zginających



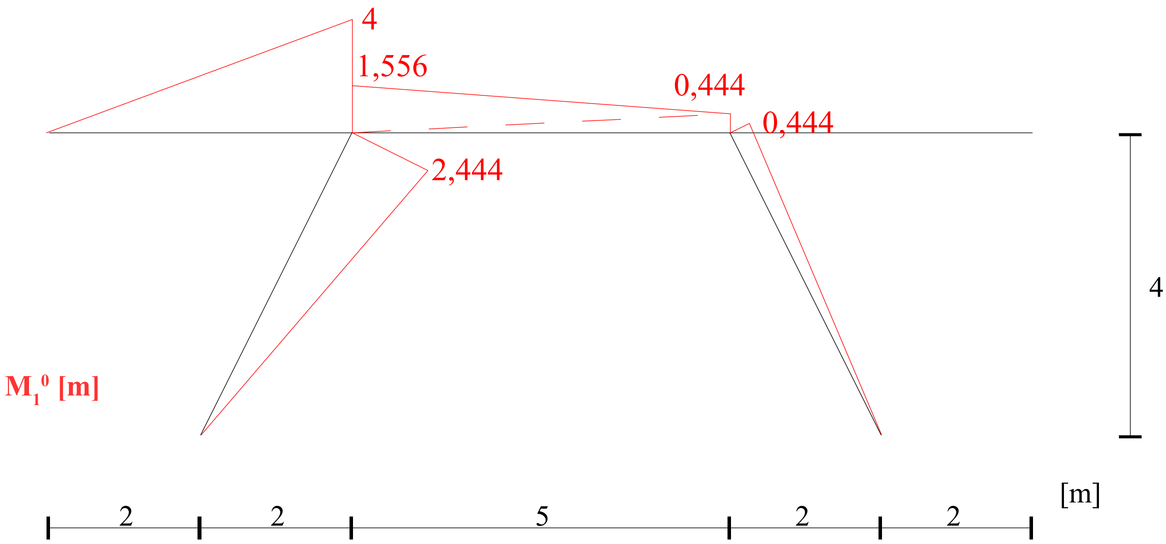

kontrola kinematyczna

$$\bar{1}\cdot\delta=\sum\int\frac{M^{(n)}\cdot\bar{M}^{(0)}}{E\,I}dx+\frac{\sum_m R_m^{(n)}\cdot\bar{R}_m^{(0)}}{k}$$

- obliczenie zerowego przemieszczenia pionowego $v_A$ dla nowego układu podstawowego:

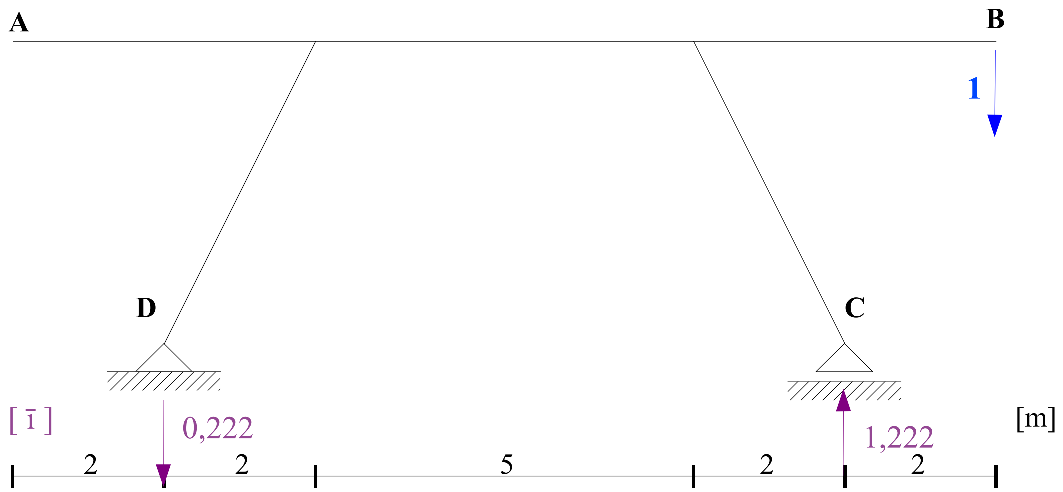


Wykres momentów zginających od jednostkowej siły w punkcie A

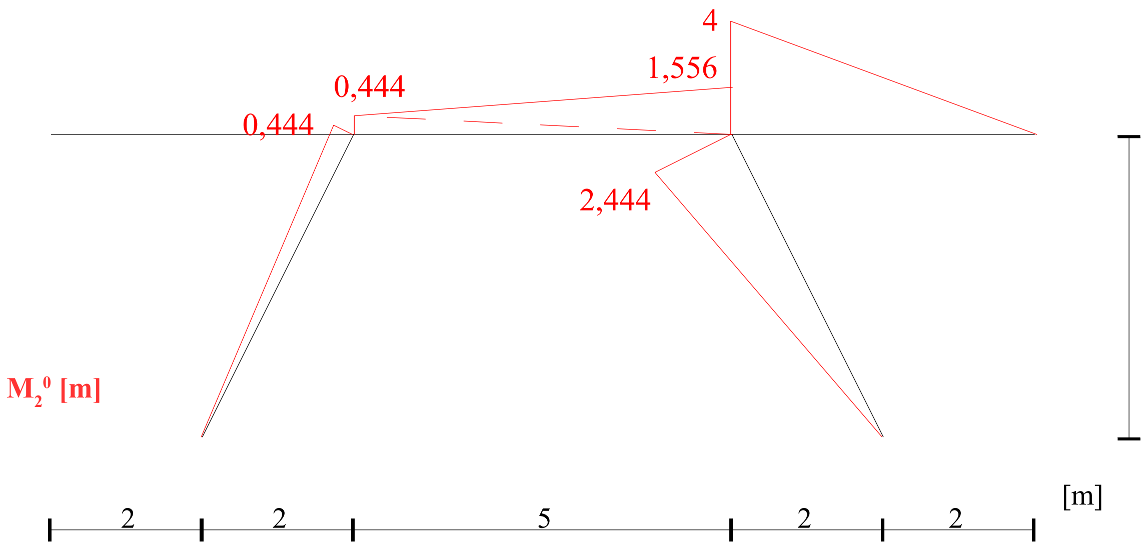



$$v_A=\frac{1}{EI}\cdot\left[\frac{1}{2}\cdot 5{,}386\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 2{,}444+\frac{1}{2}\cdot 6{,}384\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 0{,}444\right]$$
$$+\frac{1}{1{,}389\cdot EI}\cdot\left[\frac{1}{2}\cdot 4\cdot 1{,}804\cdot\frac{2}{3}\cdot(-4)+\frac{1}{2}\cdot 5\cdot 7{,}190\cdot\left(-\frac{2}{3}\cdot 1{,}556-\frac{1}{3}\cdot 0{,}444\right)\right.$$
$$\left.+\frac{1}{2}\cdot 5\cdot 31{,}360\cdot\left(\frac{1}{3}\cdot 1{,}556+\frac{2}{3}\cdot 0{,}444\right)+\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\cdot\left(-\frac{1}{2}\cdot 1{,}556-\frac{1}{2}\cdot 0{,}444\right)\right]$$
$$+\frac{1\cdot(-0{,}451)}{0{,}167\cdot EI}=\frac{-0{,}139}{EI}=-0{,}000022\ [m]$$
$$\left(\frac{0{,}139}{EI}\div\frac{31{,}360}{1{,}389\cdot EI}\right)\cdot 100‰=0{,}62‰$$

- obliczenie zerowego przemieszczenia pionowego $v_B$ dla nowego układu podstawowego:

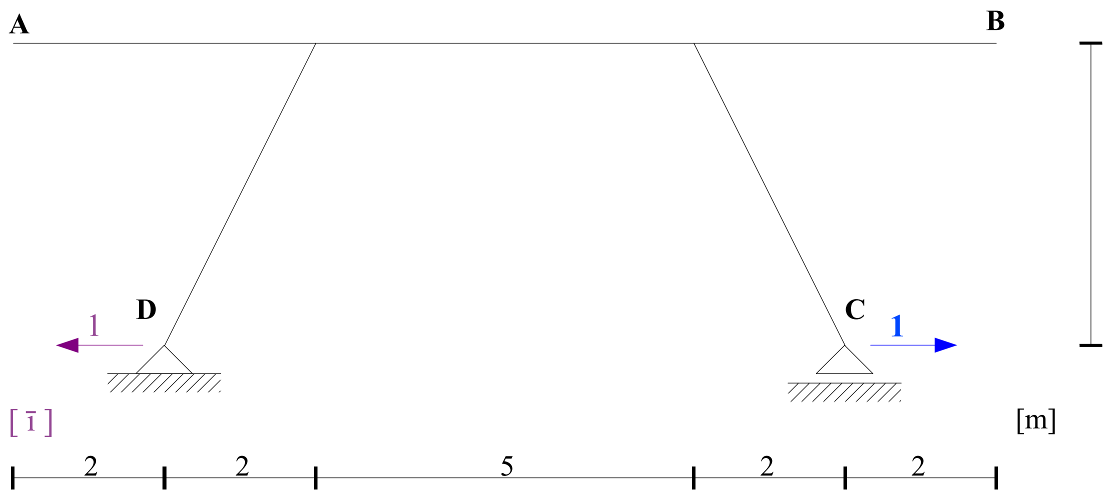


Wykres momentów zginających od jednostkowej siły w punkcie B

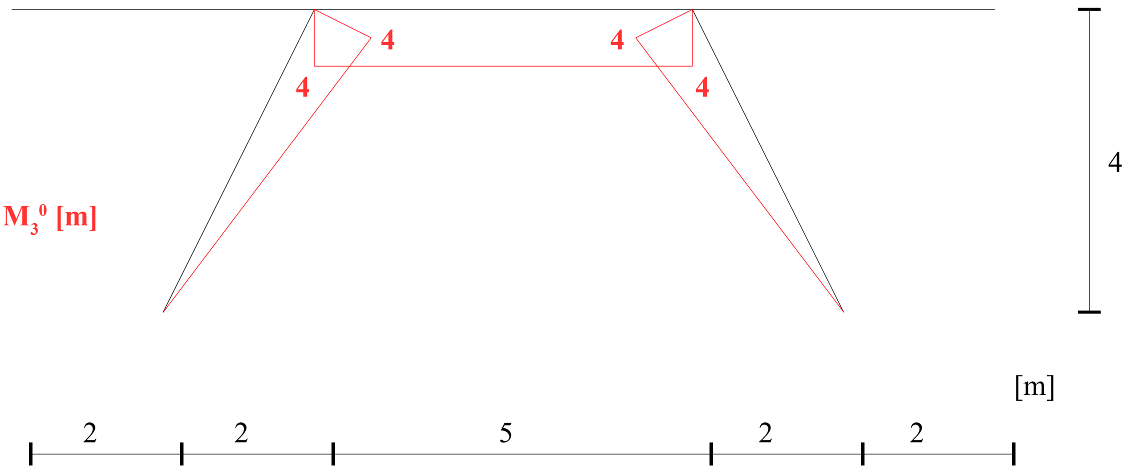



$$v_B=\frac{1}{EI}\cdot\left[\frac{1}{2}\cdot 5{,}386\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-0{,}444)+\frac{1}{2}\cdot 6{,}384\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-2{,}444)\right]$$
$$+\frac{1}{1{,}389\cdot EI}\cdot\left[\frac{1}{2}\cdot 5\cdot 7{,}190\cdot(-\frac{1}{3}\cdot 1{,}556-\frac{2}{3}\cdot 0{,}444)+\frac{1}{2}\cdot 5\cdot 31{,}360\cdot(\frac{2}{3}\cdot 1{,}556+\frac{1}{3}\cdot 0{,}444)\right.$$
$$\left.+\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\cdot(-\frac{1}{2}\cdot 1{,}556-\frac{1}{2}\cdot 0{,}444)+\frac{1}{2}\cdot 4\cdot 24{,}976\cdot\frac{2}{3}\cdot 4+\frac{2}{3}\cdot 4\cdot\frac{6\cdot 4^2}{8}\cdot\frac{1}{2}\cdot(-4)\right]$$
$$+\frac{1\cdot(-5{,}756)}{0{,}167\cdot EI}=\frac{-0{,}166}{EI}=-0{,}000027\ [m]$$
$$\left(\frac{0{,}166}{EI}\div\frac{31{,}360}{1{,}389\cdot EI}\right)\cdot 100\%=0{,}74\%$$

- obliczenie zerowego przemieszczenia pionowego $u_C$ dla nowego układu podstawowego:

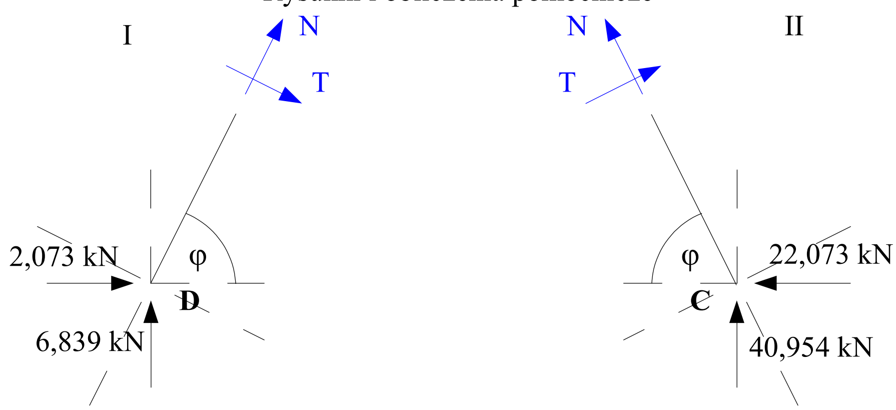


Wykres momentów zginających od jednostkowej siły w punkcie C



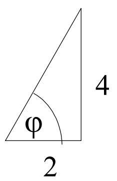

$$u_C=\frac{1}{EI}\cdot\left[\frac{1}{2}\cdot 5{,}386\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot 4+\frac{1}{2}\cdot 6{,}384\cdot\sqrt{4^2+2^2}\cdot\frac{2}{3}\cdot(-4)\right]$$
$$+\frac{1}{1{,}389\cdot EI}\cdot\left[4\cdot\left(\frac{1}{2}\cdot 5\cdot 7{,}190+\frac{1}{2}\cdot 5\cdot(-31{,}360)+\frac{2}{3}\cdot\frac{6\cdot 5^2}{8}\cdot 5\right)\right]=\frac{0{,}025}{EI}=0{,}000004\ [m]$$
$$\left(\frac{0{,}025}{EI}\div\frac{31{,}360}{1{,}389\cdot EI}\right)\cdot 100\%=0{,}11\%$$

Rysunki i obliczenia pomocnicze



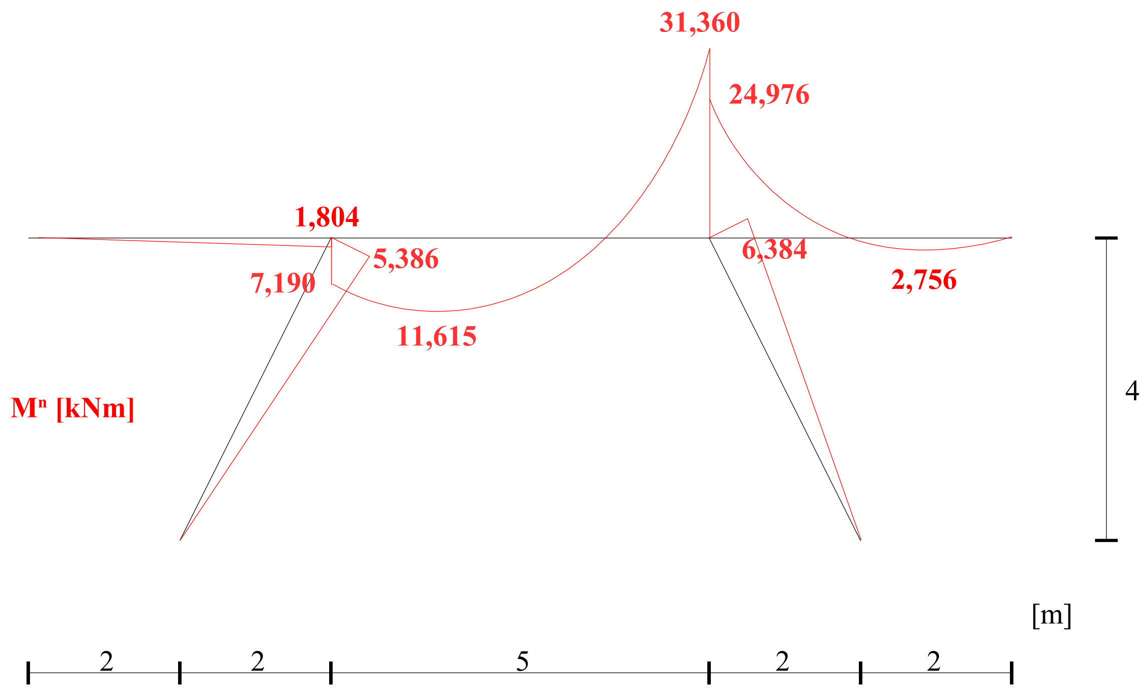

$$\text{dla części pierwszej }(I)$$
$$\sum\wedge:\quad T=-2{,}073\cdot\sin(\varphi)+6{,}839\cdot\cos(\varphi)$$
$$\sum\vee:\quad N=-2{,}073\cdot\cos(\varphi)-6{,}839\cdot\sin(\varphi)$$

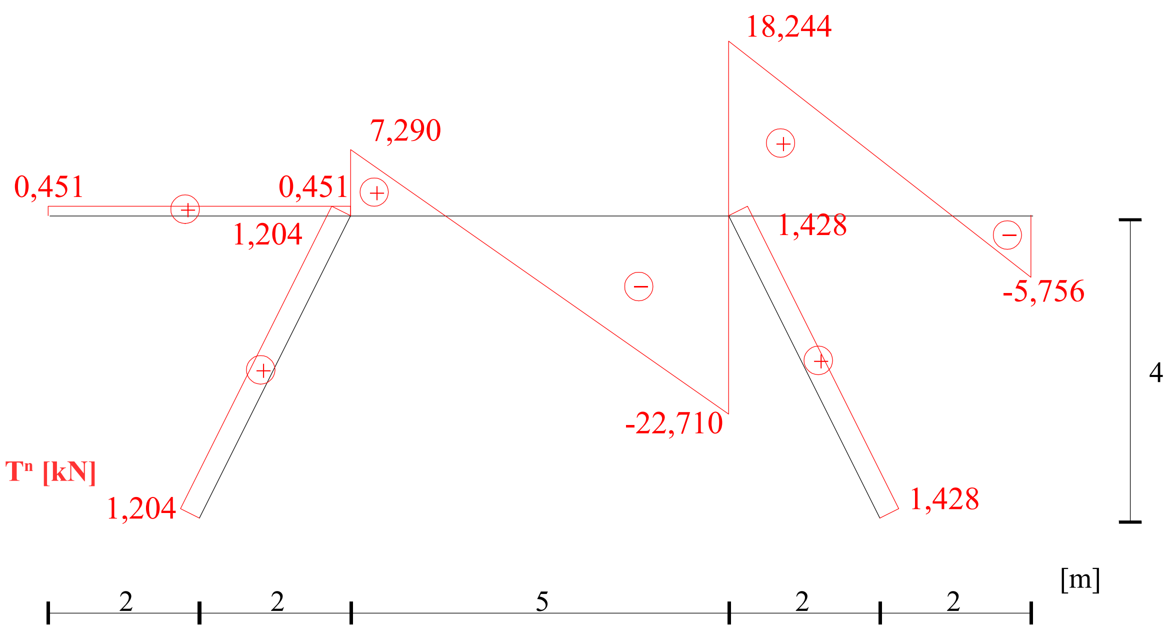
$$\text{dla części drugiej }(II)$$
$$\sum\wedge:\quad N=-40{,}954\cdot\sin(\varphi)-22{,}073\cdot\cos(\varphi)$$
$$\sum\vee:\quad T=-40{,}954\cdot\cos(\varphi)+22{,}073\cdot\sin(\varphi)$$

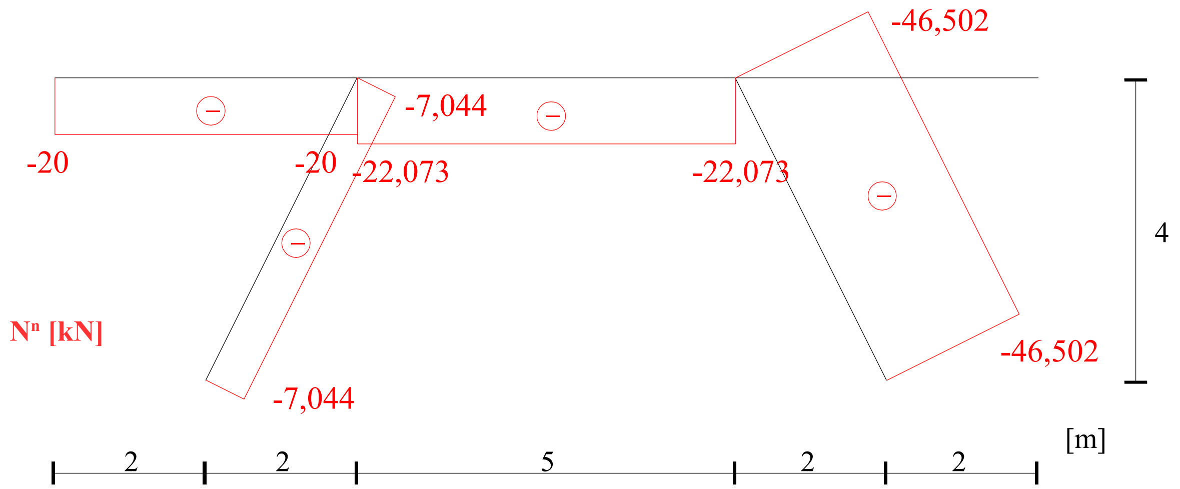



$$\sin(\varphi)=\frac{4}{\sqrt{4^2+2^2}}=0{,}89443$$
$$\cos(\varphi)=\frac{2}{\sqrt{4^2+2^2}}=0{,}44721$$

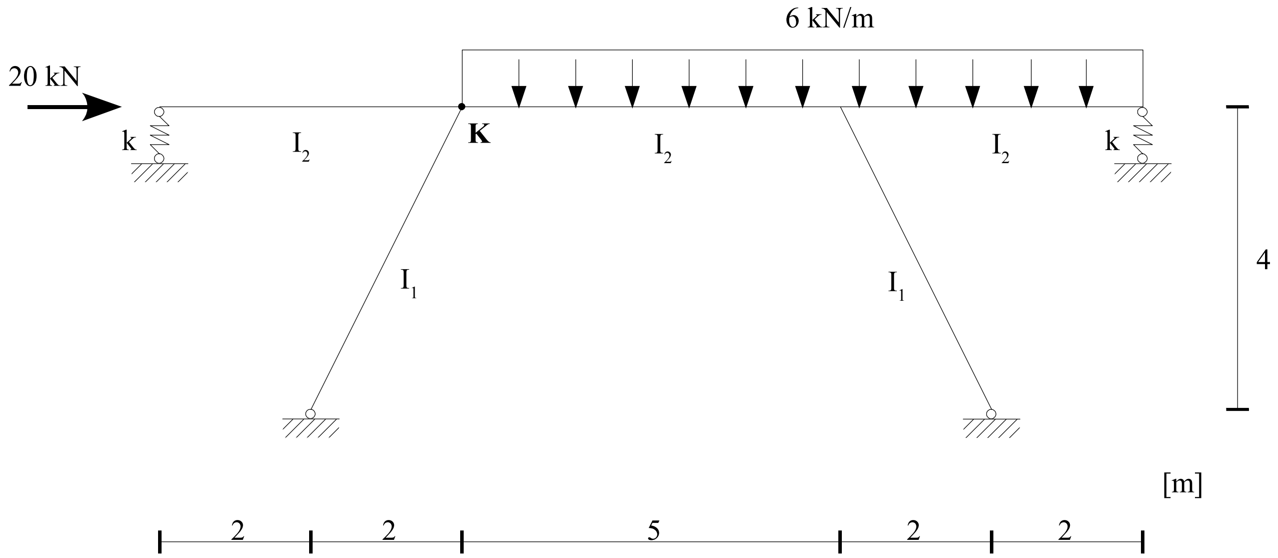
**Wykresy sił wewnętrznych w układzie statycznie niewyznaczalnym**



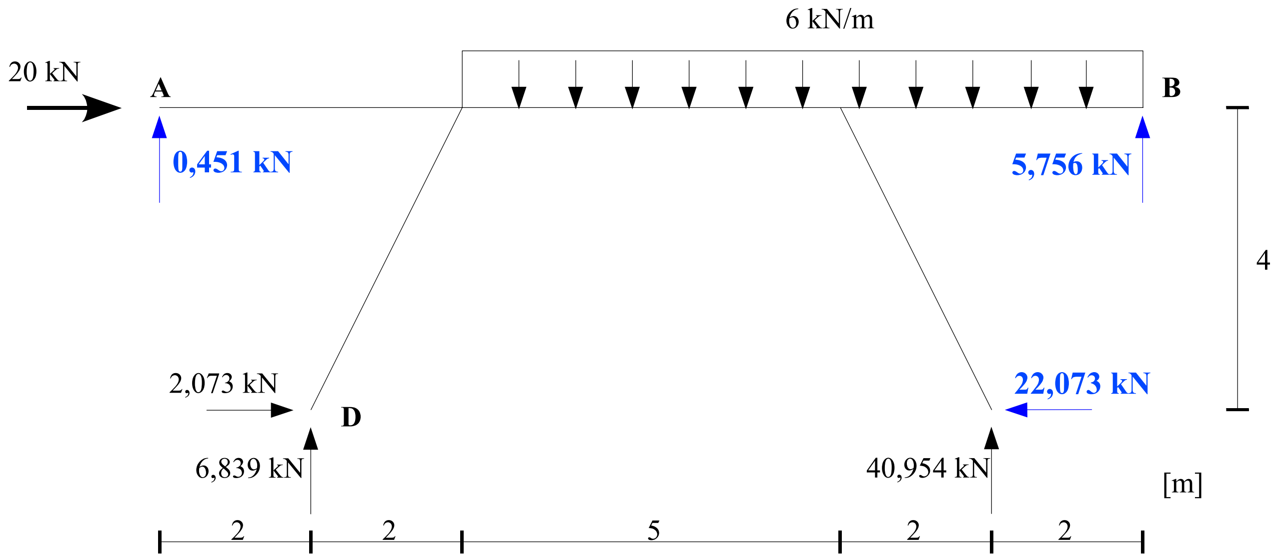

-46,502
-7,044
-20
-20
-22,073
-22,073
4
Nⁿ [kN]
-46,502
-7,044
[m]
2
2
5
2
2

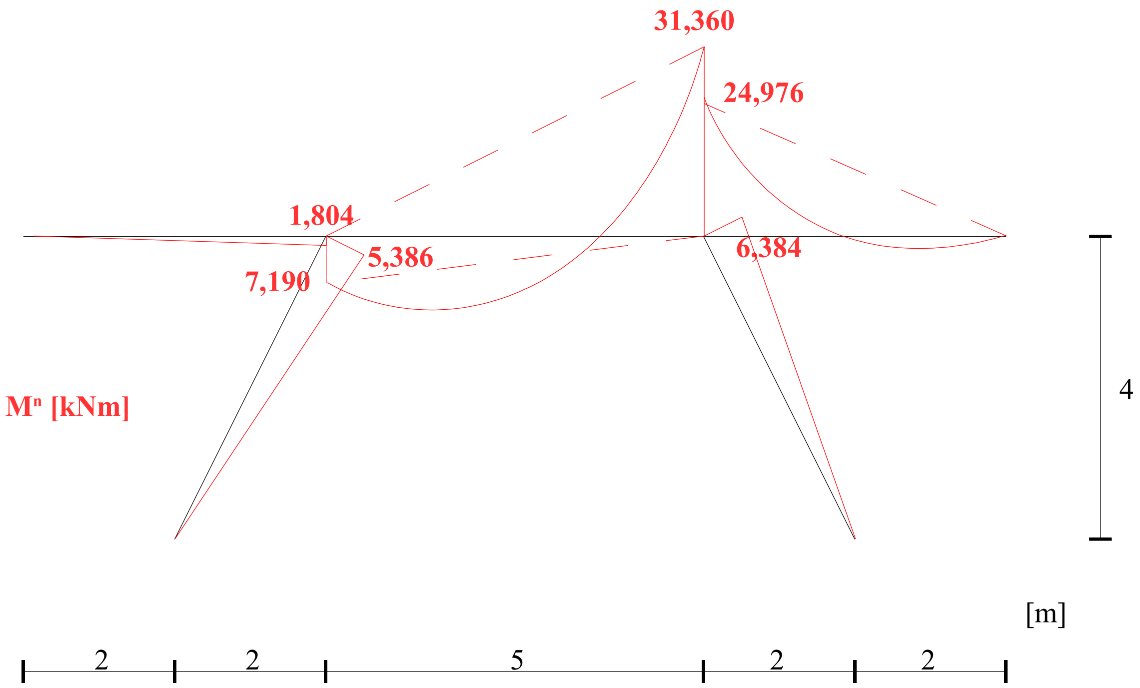
## Obliczenie przemieszczenia pionowego punktu K



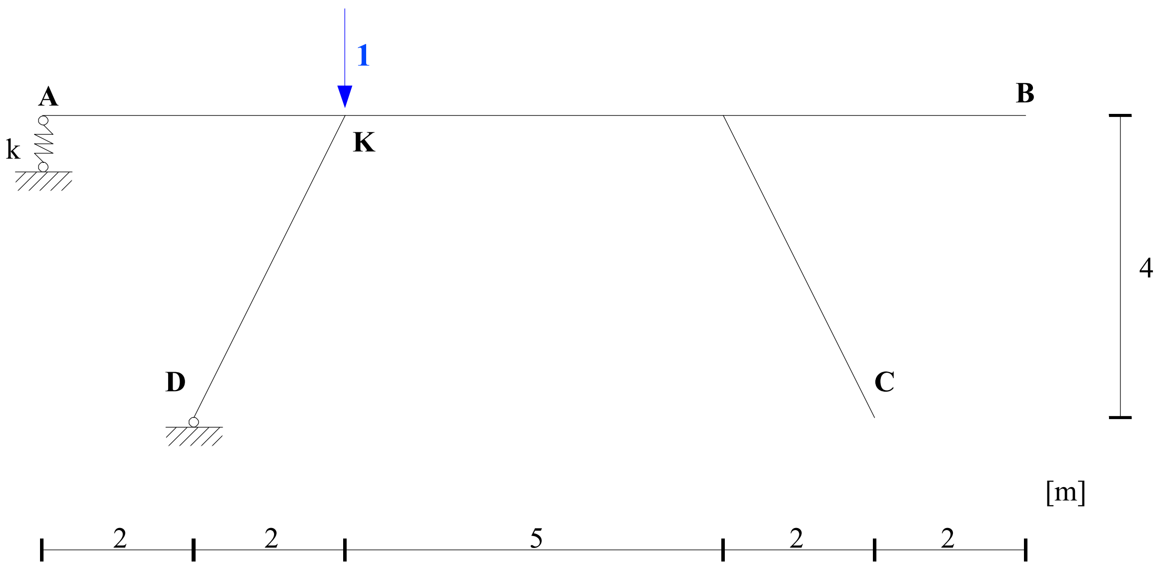

$$v_K(P)=\sum\int\frac{M^{(n)}\cdot\bar{M}^{(0)}}{EI}dx+\frac{\sum_m R_m^{(n)}\cdot\bar{R}_m^{(0)}}{k}$$

Korzystam z wcześniej wykonanych wykresów momentów zginających i obliczonych reakcji podpór dla układu statycznie niewyznaczalnego.

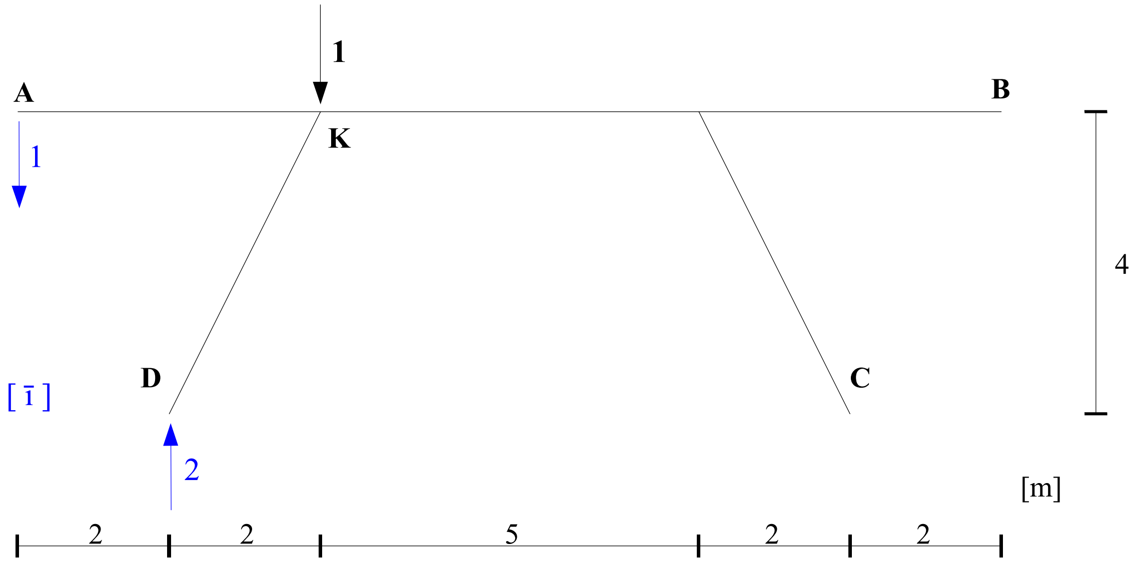




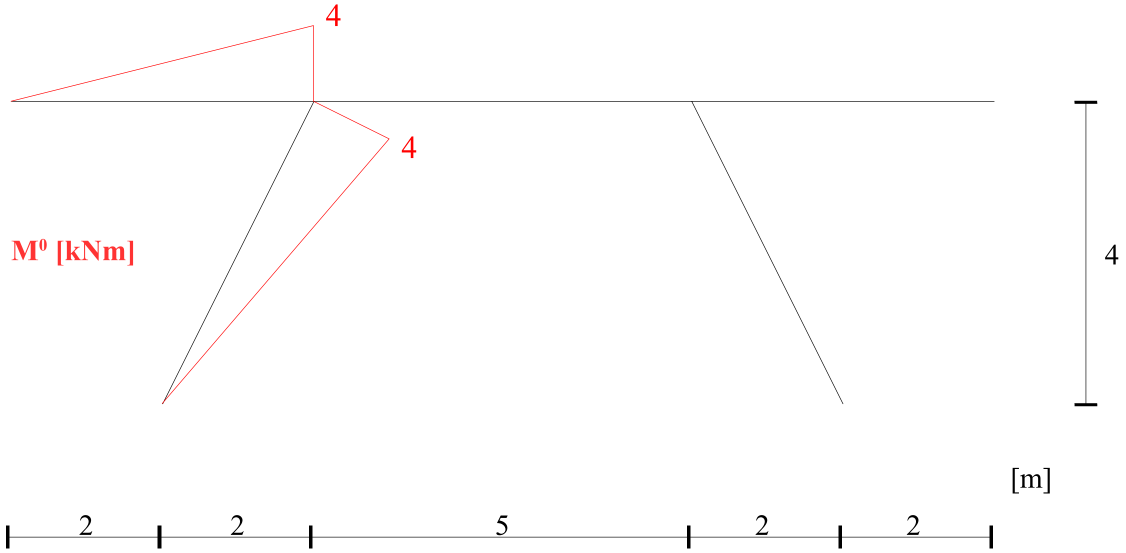

Przyjmuję układ podstawowy, dla którego obliczam wartości reakcji i wykonuję wykres momentów zginających od jednostkowej, wirtualnej siły pionowej przyłożonej w punkcie K.



Zestawienie wyników reakcji




Wykres momentów zginających




$$v_K(P)=\frac{1}{EI}\cdot\left(\frac{1}{2}\cdot\sqrt{4^2+2^2}\cdot 5{,}386\cdot\frac{2}{3}\cdot 4\right)+\frac{1}{1{,}389\cdot EI}\cdot\left(\frac{1}{2}\cdot 4\cdot 1{,}804\cdot\frac{2}{3}\cdot(-4)\right)$$
$$+\frac{0{,}451\cdot(-1)}{0{,}167\cdot EI}=\frac{22{,}483}{EI}=0{,}003584\ [m]$$