

Optimization of the Location and Damping Constants of Viscous Dampers

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Abstract

A simple and efficient method of optimization of the location of dampers and distribution of damper factors is presented. Viscous dampers only are considered in the paper. The objective function in the form of a sum of weighted, non-dimensional modal damping ratios is assumed and maximized in the optimization procedure. The sequential linearization method together with the simplex procedure is used to solve the optimization problem. The results of a typical calculation concerning optimal distribution of dampers on a building structure, modeled as a planar, shear frame, are presented and briefly discussed. A comparison of results with those obtained by means of the sequential optimization method is also presented.

Keywords: optimization, damper location, damper constants, viscous dampers.

1 Introduction

There has been observed a constant tendency to build structures using advanced materials with more efficient mechanical properties. However, these structures are flexible, their inherent damping is quite low and they are sensitive to dynamic loads. In many cases there is an urgent need to reduce the dynamic response of structures, as measured by displacements, internal forces, or accelerations. Many systems; classified in three main groups, i.e. passive, active and semi-active systems, may be used to achieve the desired reduction of vibrations. A good description and discussion of the advantages and disadvantages of the above mentioned methods are given in [1 - 3].

Passive systems consist of different kinds of mechanical devices, mounted on structures which dissipate a portion of energy introduced by excitation forces acting on the structures. Different kinds of mechanical devices called viscous dampers, viscoelastic dampers, tuned mass dampers, or base isolation systems are used in the

passive systems. In contrary to the active systems, the passive ones do not require a large amount of energy to operate and online measurements of the dynamical state of the structure are not necessary.

The optimal distribution of the damping properties of dampers and optimal positioning of dampers are important from a designer's point of view. The optimal positioning of a single viscous damper based on energy criterion is considered in [4]. The optimal location of dampers for the vibration control of aerospace structures is presented in [5]. In a series of papers [6 – 8] Takewaki and co-workers used a gradient-based approach for the optimal placement of dampers by minimizing the norm of the response transfer function calculated for the undamped fundamental frequency of structure. In the paper [9], Singh and Moreschi used a gradient-based optimization procedure to obtain the optimal distribution of viscous dampers. Moreover, in the paper [10], the genetic algorithm is used to find the optimal size and location of viscous and viscoelastic dampers. Tsuji and Nakamura [11] describe a method to find the optimal storey stiffness distribution and the optimal damper distribution for structures subjected to a set of earthquakes. Several parametric studies on the effects of damper distribution on the behaviour of structures subjected to earthquakes are presented in the paper [12]. A sequential search algorithm is presented by Garcia and Soong in the paper [13] for the design of optimal damper configuration. In [14], the authors consider the optimal distribution of viscous dampers used for the rehabilitation of an existing building with soft storeys. The method of simultaneous optimal distribution of stiffness and damping for the rehabilitation of existing buildings is also proposed in the paper [15]. The objective function of this method combines the displacement, absolute acceleration and base shear transfer function. In [16], Lavan and Levy proposed a method for the optimal design of viscous dampers based on a global damage index. In several papers [17 - 20], the active control theory is used to optimize the size and location of dampers. For example, the H_2 method and the H_∞ methods are used in [17] while the LQR method is used in the paper [20] to determine damper allocations. Interesting studies concerning the allocation and sizing of viscous dampers are presented by Main and Krenk [21] and by Engelen et al. [22].

In this paper, the problem of optimal distribution of viscous dampers is investigated using the objective function in the form of a sum of weighted non-dimensional modal damping ratios. This type of the objective function is particularly interesting and often employed in the design of dampers when the excitation mechanisms are not well understood and the significance of modes of vibration is known. The constraint on the total sum of the damping factors of added dampers is also introduced. The presented method considers the damper damping factors as design variables. The simple, iterative and efficient method based on a sequential linearization of the objective function is proposed to solve the considered problem. Linearization of the objective function and the linearity of constraints with respect to increments in design variables enable us to use the standard simplex procedure for solving the optimization problem. The results of example calculation concerning optimal distribution of dampers on the building structure modeled as the planar, shear frame are presented and briefly discussed. A comparison of the results with those obtained by means of the sequential optimization method is also presented.

2 Formulation and solution to the optimization problem

Due to limitations resulting from the building functionality and manufacturing constraints, the positions of dampers can not be freely chosen. Therefore, it is reasonable to assume that during the building design process some, say m , places in the building are chosen as acceptable damper locations.

In this paper, we assume the damper damping factors are continuous design variables. However, in practical applications, the damper capacity and size can be found only from a set of actually manufactured dampers.

Dampers are fixed to a structure with the help of braces which are treated here as rigid elements.

The considered optimization problem is formulated as follows.

For a given set of m possible damper locations, find the positions of dampers and their damping factors c_{de} which maximize the following objective function

$$I(\mathbf{c}) = \sum_{i=1}^n w_i \gamma_i(\mathbf{c}) = \mathbf{w}^T \mathbf{g}(\mathbf{c}) , \quad (1)$$

where $\mathbf{g}(\mathbf{c}) = \text{col}(\gamma_1, \dots, \gamma_i, \dots, \gamma_n)$ and $\mathbf{w} = \text{col}(w_1, \dots, w_i, \dots, w_n)$ is the vector of modal damping ratios and the vector of weight factors, respectively, γ_i denotes the non-dimensional modal damping factors of the i th mode of vibration and w_i is the corresponding weight factor. The symbol n is the number of degrees of freedom of structures and the $\mathbf{c} = \text{col}(c_{d1}, \dots, c_{de}, \dots, c_{dm})$ vector collects all damper factors.

Moreover, the following constraints must be fulfilled:

$$\sum_{e=1}^m c_{de} = C_t , \quad \mathbf{c} \geq \mathbf{0} , \quad (2)$$

where C_t is the total amount of damping factors.

The weight factor w_i could be chosen more or less arbitrarily or, which is more reasonable, it could reflect the influence of the i th mode of vibration on a chosen quantity which characterizes the dynamic response of structures. The displacement or acceleration in a chosen point of the structures could be an example of the above-mentioned quantity. In the case of structures excited by earthquakes, weight factors can be determined using the method presented in [23].

The considered non-linear optimization problem is solved using the version of the sequential linearization method [24] that was briefly described above. The proposed method consists of some incremental steps. The total amount of damping factors C_t is divided into increments ΔC_t . In a typical incremental step i the current total amount of damping factors C_{ti} is equal to $C_{t(i-1)} + \Delta C_t$ and it is assumed that we have an admissible solution denoted as the \mathbf{c}_i vector. The admissible solution could easily be obtained using the relation

$$\tilde{\mathbf{c}}_i = \mathbf{c}_{i-1} + \delta \mathbf{c} , \quad (3)$$

where $\delta\mathbf{c} = \text{col}(\delta c_{d1}, \dots, \delta c_{de}, \dots, \delta c_{dm})$ is the vector of initial increments of damping factors fulfilling the following condition:

$$\sum_{e=1}^m \delta c_{de} = \Delta C_t . \quad (4)$$

A good initial distribution of ΔC_t is a uniform distribution of ΔC_t in all possible damper places, i.e.

$$\delta c_{de} = \Delta C_t / m . \quad (5)$$

The objective function (1) is linearized in the vicinity of the current values of damping factors given in \mathbf{c}_i . Expanding (1) into the Fourier series and taking into account only two elements in this expansion, the following temporal objective function is obtained:

$$\Delta I(\Delta\mathbf{c}) = I(\tilde{\mathbf{c}}_i + \Delta\mathbf{c}) - I(\tilde{\mathbf{c}}_i) = \sum_{i=1}^n \sum_{e=1}^m w_i \frac{\partial \gamma_i(\mathbf{c})}{\partial c_{de}} \Delta c_e = \mathbf{w}^T \mathbf{G}(\tilde{\mathbf{c}}_i) \Delta\mathbf{c} , \quad (6)$$

where $\Delta\mathbf{c} = \text{col}(\Delta c_{d1}, \dots, \Delta c_{de}, \dots, \Delta c_{dm})$ is the vector of increments of damping factors and the sensitivity matrix $\mathbf{G}(\tilde{\mathbf{c}}_i)$, of which the values are determined at $\tilde{\mathbf{c}}_i$, is defined as

$$\mathbf{G}(\mathbf{c}_i) = \begin{bmatrix} \frac{\partial \gamma_1}{\partial c_{d1}} & \dots & \frac{\partial \gamma_1}{\partial c_{de}} & \dots & \frac{\partial \gamma_1}{\partial c_{dm}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\partial \gamma_i}{\partial c_{d1}} & \dots & \frac{\partial \gamma_i}{\partial c_{de}} & \dots & \frac{\partial \gamma_i}{\partial c_{dm}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\partial \gamma_n}{\partial c_{d1}} & \dots & \frac{\partial \gamma_n}{\partial c_{de}} & \dots & \frac{\partial \gamma_n}{\partial c_{dm}} \end{bmatrix} . \quad (7)$$

The element $\partial \gamma_i / \partial c_{de}$ of the above matrix is the sensitivity of the i th damping ratio with respect to the damping factor of the e th damper.

The constraints (2) are now rewritten in the form:

$$\sum_{e=1}^m \Delta c_{de} = 0 , \quad \mathbf{c}_i + \Delta\mathbf{c} \geq \mathbf{0} . \quad (8)$$

Moreover, additional constraints in the form:

$$-\Delta\mathbf{c}_{\max} \geq \Delta\mathbf{c} \geq \Delta\mathbf{c}_{\max} , \quad (9)$$

are added to the previous ones because the first order sensitivity used in the temporal objective function (6) requires sufficiently small increments of damping forces for a

good approximation of the objective function (1). In relation (9) $\Delta \mathbf{c}_{\max}$ is the assumed vector of maximal increments of damping factors.

Relations (6), (8) and (9) define the linear optimization problem, with Δc_{de} as the optimization variables, which can easily be solved using the simplex method.

Having the solution to the linear optimization problem, the current vector of damping factors is obtained from

$$\mathbf{c}_i = \tilde{\mathbf{c}}_i + \Delta \mathbf{c} . \quad (10)$$

At the end of the incremental procedure we can perform some artificial incremental steps with $\delta \mathbf{c} = \mathbf{0}$ and $\Delta C_i = 0$. Due to changes of the sensitivity matrix values we can additionally improve the obtained solution to the non-linear optimization problem. The local maximum of the objective function (1) is obtained when the solution to the linear optimization problem is $\Delta \mathbf{c} = \mathbf{0}$.

3 Description of structures and dampers

3.1 Assumptions and equation of motion

The structure with dampers is treated as an elastic, damped discrete system with n degrees of freedom. The motion equation of such system is in the well-known form:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{P}(t) + \mathbf{s}(t) , \quad (11)$$

where t is the time, the $\mathbf{q}(t)$ symbol denotes the $(n \times 1)$ vector of nodal displacements, $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are the $(n \times n)$ mass, damping and stiffness matrices, respectively. Moreover, $\mathbf{P}(t)$ and $\mathbf{s}(t)$ are the vector of nodal excitation forces and the vector of nodal damping forces, respectively.

In the local coordinate system the dynamic behaviour of the typical viscous damper could be described with the help of the following equation:

$$u_e(t) = c_{de} (\dot{\tilde{q}}_3 - \dot{\tilde{q}}_1) , \quad (12)$$

where $u_e(t)$ is the damper force, c_{de} is the damping factor, the symbols $\dot{\tilde{q}}_3, \dot{\tilde{q}}_1$ denote nodal displacement velocity and $e = 1, 2, \dots, E$ and E is the total number of dampers (see Figure 1). In the global coordinate system, Equation (12) could be rewritten in the form:

$$\mathbf{s}_e(t) = -\mathbf{C}_{de} \dot{\mathbf{q}}_e(t) , \quad (13)$$

where (see Fig.1)

$$\mathbf{s}_e(t) = \text{col}(s_1(t), s_2(t), s_3(t), s_4(t)), \quad \mathbf{q}_e(t) = \text{col}(q_1(t), q_2(t), q_3(t), q_4(t)),$$

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{0} . \quad (17)$$

The solution to Equation (17) is in the form:

$$\mathbf{q}(t) = \mathbf{a} \exp(st) , \quad (18)$$

which, after being introduced into Equation (17), gives us the following quadratic eigenvalue problem:

$$(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K})\mathbf{a} = \mathbf{0} , \quad (19)$$

where the symbols \mathbf{a} and s are the eigenvector and eigenvalue respectively.

The eigenvalue problem (19) has $2n$ solutions denoted by \mathbf{a}_i and s_i ($i=1,2,\dots,2n$). It is assumed that eigenvalues s_i are distinct. If the structure is under-critically damped the eigenvalues are a set of pairs of complex and conjugate numbers, i.e.

$$s_i = \mu_i + j\eta_i , \quad s_{i+n} = \mu_i - j\eta_i , \quad j = \sqrt{-1} . \quad (20)$$

If the structure is over-damped then at least one pair of eigenvalues, say, s_k and s_{k+n} , are real numbers.

Following [25] we introduce the natural frequency of damped structures ω_j and the modal damping ratio γ_j . These quantities fulfil the relations:

$$s_i = -\gamma_i\omega_i + j\omega_i\sqrt{1-\gamma_i^2} , \quad s_{i+n} = -\gamma_i\omega_i - j\omega_i\sqrt{1-\gamma_i^2} , \quad (21)$$

$$s_k = -\gamma_k\omega_k + \omega_k\sqrt{\gamma_k^2-1} , \quad s_{k+n} = -\gamma_k\omega_k - \omega_k\sqrt{\gamma_k^2-1} , \quad (22)$$

for the under- and over-damped structure, respectively.

It is easy to verify the following relations;

$$-2\gamma_i\omega_i = s_i + s_{i+n} , \quad \omega_i^2 = s_i s_{i+n} , \quad (23)$$

which are valid in both of the considered cases.

3.2.2 Design sensitivities of eigenvalues, frequencies and damping ratios

The design sensitivities of eigenvalues with respect to the design parameter p could be determined by the method described in [25]. Using that method and taking into account that the parameter p does not influence the mass or stiffness matrices, the following simple formula

$$\frac{\partial s_i}{\partial p} = -s_i \mathbf{a}_i^T \frac{\partial \mathbf{C}_d}{\partial p} \mathbf{a}_i , \quad (24)$$

is obtained. Here, the symbol $\partial s_i / \partial p$ denoted the above mentioned sensitivity of eigenvalue with respect to the change of a design parameter. Moreover, it was assumed that the eigenvector is normalized in the following way:

$$\mathbf{a}_i^T (2s_i \mathbf{M} + \mathbf{C}) \mathbf{a}_i = 1 . \quad (25)$$

The sensitivity of frequency ω_i can be obtained by differentiating Equation (23.2) with respect to p , from which we obtain:

$$\frac{\partial \omega_i}{\partial p} = \frac{1}{2\omega_i} \left(s_{i+n} \frac{\partial s_i}{\partial p} + s_i \frac{\partial s_{i+n}}{\partial p} \right) . \quad (26)$$

The sensitivity of the modal damping ratio, obtained by differentiating Equation (23.1), is given by the formula:

$$\frac{\partial \gamma_i}{\partial p} = -\frac{1}{2\omega_i} \left(\frac{\partial s_i}{\partial p} + \frac{\partial s_{i+n}}{\partial p} \right) + \frac{\gamma_i}{\omega_i} \frac{\partial \omega_i}{\partial p} . \quad (27)$$

The formula (27) can be simplified in a case when the damper constant is the design parameter (i.e. $p = c_{dk}$). It is well known that the sensitivity of frequency ω_i in respect of damper constant is usually weak, which means that $\partial \omega_i / \partial c_{dk} \approx 0$ and Equation (27) could be rewritten in the form:

$$\frac{\partial \gamma_i}{\partial c_{dk}} \approx -\frac{1}{2\omega_i} \left(\frac{\partial s_i}{\partial c_{dk}} + \frac{\partial s_{i+n}}{\partial c_{dk}} \right) . \quad (28)$$

4 Determination of weight factors

Four versions of objective functions, each with different weight factors, are chosen. In the first case, the weight factors are chosen arbitrarily. The values of weight factors of the second, third and fourth objective functions are equal to the values of appropriately defined modal contribution factors \bar{r}_i .

The concept of modal contribution factors is presented in [23]. Strictly speaking, this concept is well defined when the structure is proportionally damped and excitation can be described in the form:

$$\mathbf{P}(t) = \mathbf{f} p(t) . \quad (29)$$

Since, in the case in question, the structure is non-proportionally damped the modal contribution factors must be understood as an approximation of appropriate values of the weights of the objective function (1).

Because of the relative importance of the different modes of vibrations in the determination of the response of structure the modal contribution factor \bar{r}_i , measuring the contribution of the i th mode to a generic response quantity r , is defined as follows [23]:

$$\bar{r}_i = \frac{r_{i,st}}{r_{st}} , \quad (30)$$

where r_{st} is the static value of the response of quantity r due to static forces $\mathbf{f} = \mathbf{M}\mathbf{i}$, $r_{i,st}$ is the static value of the response of quantity r due to static forces $\mathbf{f}_i = \varepsilon_i \mathbf{M}\bar{\mathbf{a}}_i$. The $\bar{\mathbf{a}}_i$ vector is the eigenvector corresponding to the i th natural frequency of the undamped system and it fulfils the following equation:

$$\mathbf{K}\bar{\mathbf{a}}_i = \omega_i^2 \mathbf{M}\bar{\mathbf{a}}_i . \quad (31)$$

Moreover,

$$\varepsilon_i = \frac{\bar{\mathbf{a}}_i^T \mathbf{M}\mathbf{i}}{\bar{\mathbf{a}}_i^T \mathbf{M}\bar{\mathbf{a}}_i} . \quad (32)$$

The modal contribution factors defined above are dimensionless, independent of how the eigenvectors $\bar{\mathbf{a}}_i$ are normalized, and their sum over all modes is a unity.

In the fourth case, some global quantity r , i.e., elastic energy of structures is chosen. The elastic energy of a structure is given by

$$E_s = \frac{1}{2} \mathbf{q}_{st}^T \mathbf{K}\mathbf{q}_{st} , \quad (33)$$

where, the \mathbf{q}_{st} vector fulfils the following equation:

$$\mathbf{K}\mathbf{q}_{st} = \mathbf{f} . \quad (34)$$

Additionally, we introduce the vector of modal displacements $\mathbf{q}_{i,st}$ which is given by:

$$\mathbf{q}_{i,st} = \mathbf{K}^{-1} \mathbf{f}_i \equiv \varepsilon_i \mathbf{K}^{-1} \mathbf{M}\bar{\mathbf{a}}_i . \quad (35)$$

Moreover, it can be easy to check the following identity:

$$\mathbf{q}_{st} = \sum_{i=1}^n \mathbf{q}_{i,st} . \quad (36)$$

Premultiplying Equation (31) by $\bar{\mathbf{a}}_j^T \mathbf{M}\mathbf{K}^{-1}$ and using the orthogonality conditions we obtain, for $j \neq i$, another useful relation:

$$\omega_i^2 \bar{\mathbf{a}}_j^T \mathbf{M}\mathbf{K}^{-1} \mathbf{M}\bar{\mathbf{a}}_i = 0 . \quad (37)$$

Taking into account relations (35), (36) and (37) we can rewrite Equation (33) in the following form:

$$E_s = \frac{1}{2} \sum_{i=1}^n \mathbf{q}_{i,st}^T \mathbf{K}\mathbf{q}_{i,st} = \sum_{i=1}^n E_{s,i} , \quad (38)$$

which means that the total elastic energy is a sum of modal elastic energies $E_{s,i}$.

Finally, if the elastic energy is treated as a generic quantity r then the contribution factors \bar{r}_i , which are also the weight factors, is given by

$$\bar{r}_i = \frac{E_{s,i}}{E_s}, \quad (39)$$

and has all of the above mentioned properties.

5 Results of example calculation

A ten storey building is considered as an example. A shear, plane frame is used to model the building structure (see Figure 2). The data are taken from [26]. The mass of each storey is 2070.0 kg . The stiffness and damping matrices are tri-diagonal ones. The stiffnesses of the stories are different and they are: $k_1 = k_2 = 68710.0 \text{ kN/m}$, $k_3 = k_4 = 54010.0 \text{ kN/m}$, $k_5 = k_6 = 42170.0 \text{ kN/m}$, $k_7 = k_8 = 28660.0 \text{ kN/m}$, $k_9 = k_{10} = 16450.0 \text{ kN/m}$.

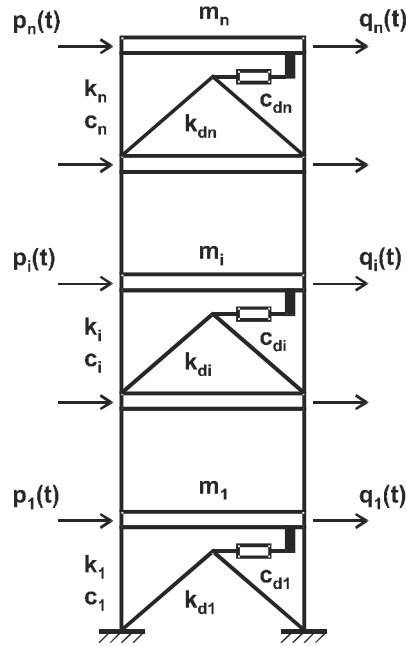


Figure 2: The mechanical model of structure

The five natural frequencies of frame vibrations are: $\omega_1 = 22.69 \text{ rad/s}$, $\omega_2 = 56.54 \text{ rad/s}$, $\omega_3 = 91.91 \text{ rad/s}$, $\omega_4 = 127.47 \text{ rad/s}$, $\omega_5 = 151.77 \text{ rad/s}$.

The damping properties of a structure are defined with the help of the damping factors of the stories. The adopted values of stories damping factors are: $c_1 = c_2 = 4.76 \text{ kNs/m}$, $c_3 = c_4 = 3.73 \text{ kNs/m}$, $c_5 = c_6 = 2.91 \text{ kNs/m}$, $c_7 = c_8 = 1.98 \text{ kNs/m}$, $c_9 = c_{10} = 1.44 \text{ kNs/m}$. The values of five non-dimensional damping ratios of structures without dampers are: $\gamma_1 = 0.000801$, $\gamma_2 = 0.002164$, $\gamma_3 = 0.003496$, $\gamma_4 = 0.004744$, $\gamma_5 = 0.006123$.

The aim of the optimization procedure is to optimally distribute some viscous dampers, of which the total damping capacity, as measured by the total damping factor, is $C_t = 400.0 \text{ kNs/m}$. The damper located on the k th storey is connected with consecutive slabs, as shown in Figure 2.

Four versions of the objective functions, each with different weight factors, are chosen. In the first case, the non-dimensional damping factor of the first vibration mode is maximized, i.e., $w_1 = 1.0$ and $w_i = 0.0$ for $i = 2, 3, \dots, n$.

In the second case, the values of weight factors are equal to the values of modal contribution factors of the ten storey displacement whereas, in the third case, the values of weight factors are equal to the values of modal contribution factors of the shear force on the first storey. The calculated values of the above mentioned weight factors are:

$$\begin{aligned} w_1 &= 1.062580, & w_2 &= -0.073652, & w_3 &= 0.014094, & w_4 &= -0.004486, \\ w_5 &= 0.001744, & w_6 &= -0.000342, & w_7 &= 0.000061, & w_8 &= -0.000004, \\ w_9 &= w_{10} = 0.000000, \end{aligned}$$

in the second case and

$$\begin{aligned} w_1 &= 0.757917, & w_2 &= 0.130242, & w_3 &= 0.049456, & w_4 &= 0.024853, \\ w_5 &= 0.008884, & w_6 &= 0.013180, & w_7 &= 0.005345, & w_8 &= 0.003629, \\ w_9 &= 0.003285, & w_{10} &= 0.003209, \end{aligned}$$

in the third case.

In the fourth case, the values of weight factors, which take into account the global characteristic of the structure, are:

$$\begin{aligned} w_1 &= 0.967679, & w_2 &= 0.026786, & w_3 &= 0.003848, & w_4 &= 0.001005, \\ w_5 &= 0.000254, & w_6 &= 0.000260, & w_7 &= 0.000081, & w_8 &= 0.000040, \\ w_9 &= 0.000027, & w_{10} &= 0.000020. \end{aligned}$$

The results of the optimization procedure are presented in Table 3 for all of the considered cases. Moreover, the numbers in brackets are the results obtained using the method of sequential optimization [26]. According to the procedure of sequential optimization method, the total damping factor C_t is divided into increments ΔC_t , which represents the damper factor of a single damper. For each possible location of one damper the values of non-dimensional modal damping factors and the value of the objective function are calculated. The right fixed location of a damper is the position for which the maximal value of the objective function is obtained. When the first damper location is determined the procedure described above is repeated until all locations for the dampers are found.

The presented optimal solutions have some interesting features. All dampers are located on only two stories, i.e. on the seventh and ninth stories in cases 1 and 4, on the fifth and seventh stories in case 2 and on the ninth and tenth stories in case 3. Moreover, the results obtained by the present method and the sequential optimization method are identical if we compare dampers locations, and they are similar if the distribution of damper factors are compared.

| Storey | Optimal values of damper factors [kNs/m] | | | |
|--------|--|---------------|---------------|---------------|
| | Case 1 | Case 2 | Case 3 | Case 4 |
| 1 – 4 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5 | 0.0 | 108.0 (100.0) | 0.0 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 |
| 7 | 272.5 (280.0) | 292.0 (300.0) | 0.0 | 215.0 (220.0) |
| 8 | 0.0 | 0.0 | 0.0 | 0.0 |
| 9 | 127.5 (120.0) | 0.0 | 245.0 (250.0) | 185.0 (180.0) |
| 10 | 0.0 | 0.0 | 155.0 (150.0) | 0.0 |

Table 1 Results of optimization

The non-dimensional damping ratios for all modes of vibration and for all of the considered cases are shown in Figures 3 – 6. An interesting feature of the optimal solution is the very high values of non-dimensional damping ratios of the selected modes of vibration. The damping ratio of the seventh and fifth modes in case 1, the damping ratio of the seventh mode in case 2, the damping ratio of the third and fifth modes in case 3 and the damping ratio of the fifth and seventh mode in case 4, are several times as high as the other ones. Moreover, except case 2, the non-dimensional damping ratios of the highest modes are low.

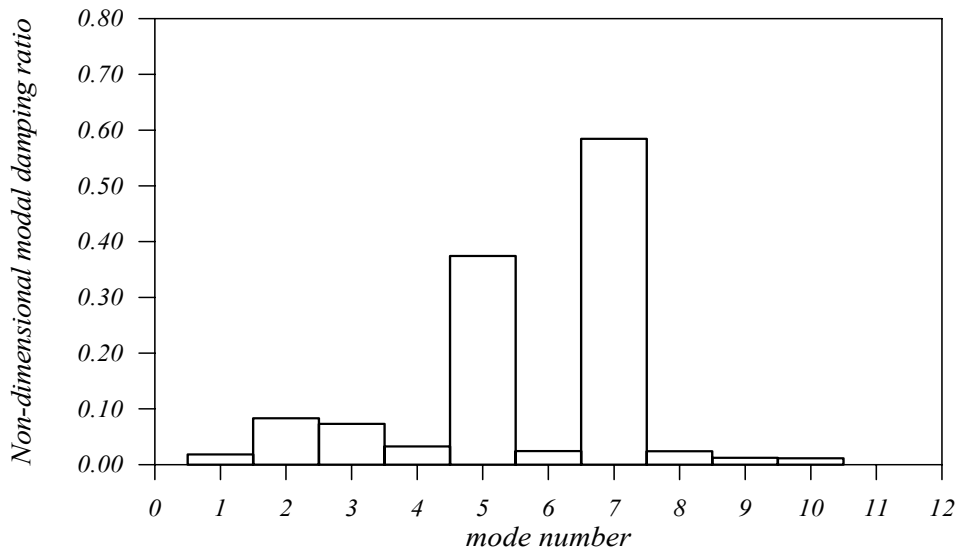


Figure 3: Non-dimensional damping ratios – first version of objective function

For comparison with the previous results, in Figure 7 the non-dimensional damping ratios are presented when dampers are uniformly distributed on the structure, i.e. the damper with $c_t = 40.0 \text{ kNs/m}$ is present on each storey. Now

distribution of the non-dimensional damping ratio is more uniform. It is evident that optimization procedure redistributes dampers in such a way that the damping ratio of particular, not necessarily lower, modes of vibrations significantly grows.

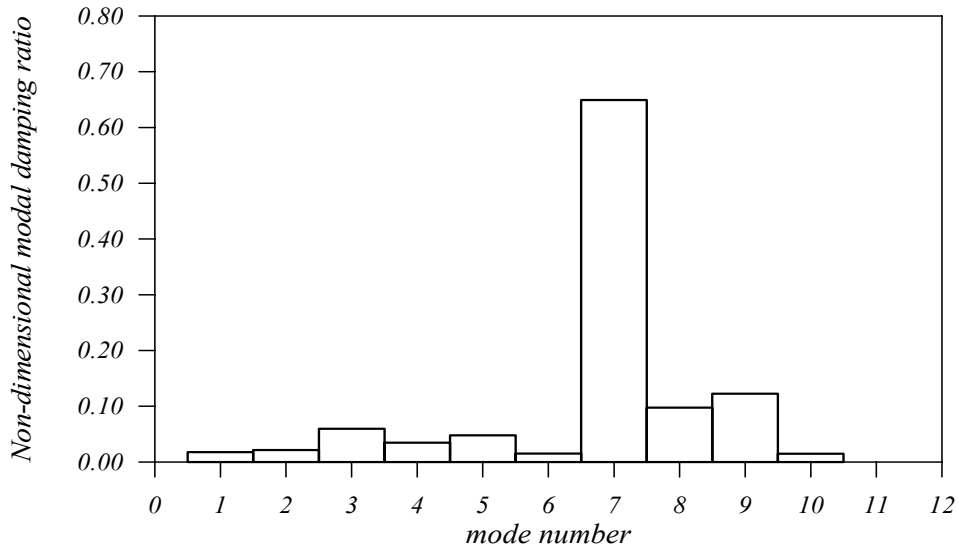


Figure 4: Non-dimensional damping ratios – second version of objective function

The value of damping ratio of the fundamental mode of vibration is usually very important quantities. For the considered structure we obtain: $\gamma_1 = 0.01833$, $\gamma_1 = 0.01757$, $\gamma_1 = 0.01212$ and $\gamma_1 = 0.01812$ in case 1 – 4, respectively and $\gamma_1 = 0.012295$ when dampers are uniformly distributed on structure.

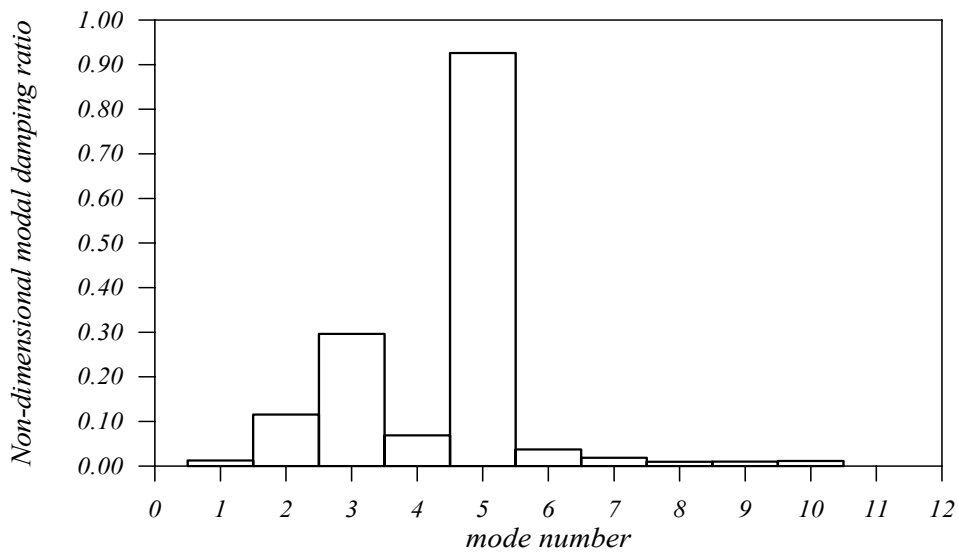


Figure 5: Non-dimensional damping ratios – third version of objective function

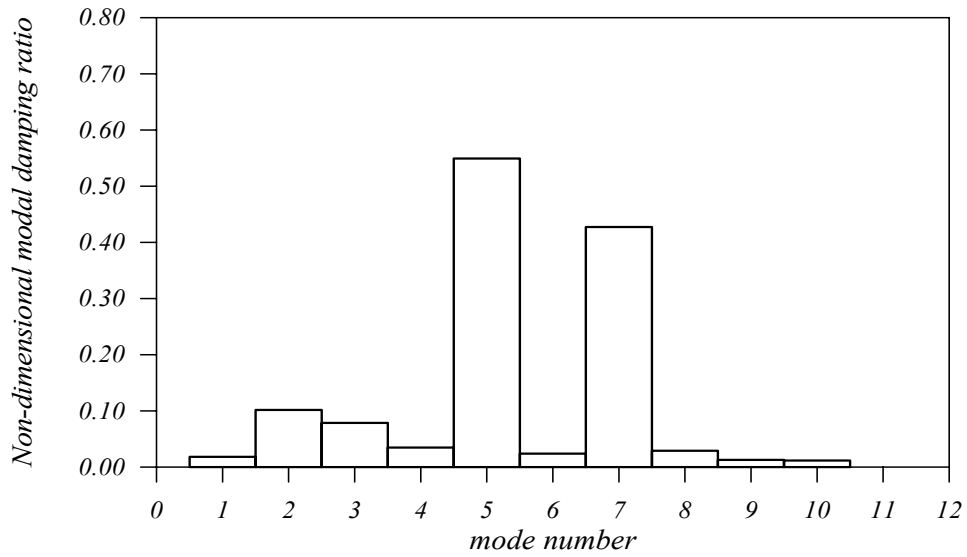


Figure 6: Non-dimensional damping ratios – fourth version of objective function

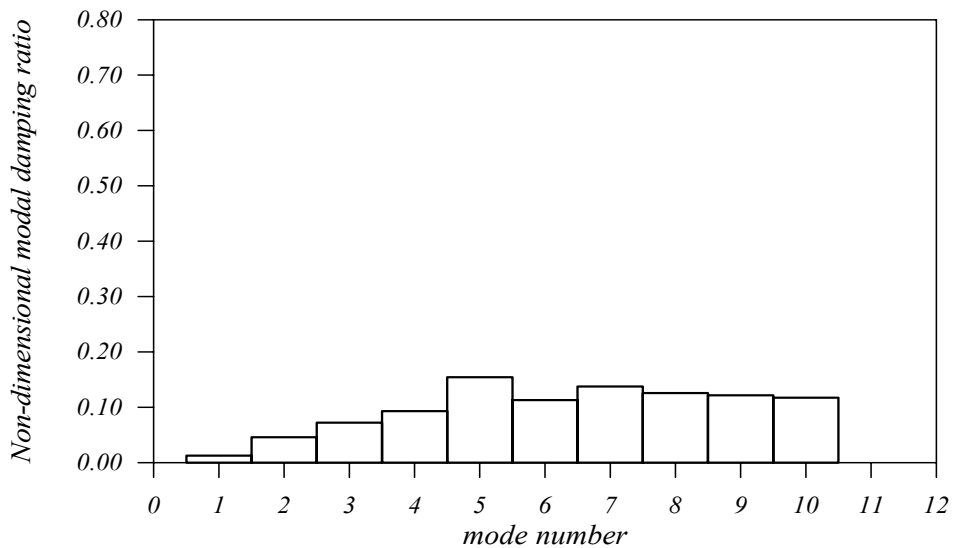


Figure 7: Non-dimensional modal damping ratios – uniform distribution of dampers

6 Concluding remarks

A simple and efficient method of optimization of the location of dampers and distribution of damper factors is presented in the paper. The solution is obtained by maximizing the objective function that represents the sum of weighted non-dimensional modal damping factors. In the numerical calculation only the well-known simplex procedure is used to obtain the solutions of the considered optimization problem. Solutions for four set of values of weighing factors resulting from different criteria used for determination of the values of weighting factors are presented and compared with results obtained by means of the often used sequential

optimization procedure. All solutions obtained show that the dampers must be located on a few, two in this case, appropriately chosen building stories. The optimal locations of dampers and damper factors strongly depend on the values of weighting factors.

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