IDENTIFICATION OF PARAMETERS OF THE FRACTIONAL MAXWELL MODEL

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Abstract

The passive dampers are often modeled using the either classical or fractional rheological models. An important problem, bounded with the fractional models, is an estimation of the model parameters from the experimental data. The process of parameter identification is an inverse problem which is underdetermined and can be ill conditioned. The new method of parameters identification of the fractional Maxwell model is proposed. The parameters are estimated using results obtained from dynamical tests. Results of example calculation based on artificial and experimental data are presented.

Keywords: dampers, identification of parameters, fractional Maxwell model

Introduction

Viscoelastic (VE) dampers have been often used for control vibration of structures to reduce oscillations of building structures induced by earthquakes and strong winds. Many applications of VE dampers in civil engineering are listed in [1]. The VE dampers could be divided broadly into the fluid and the solid VE dampers. Analysis of structures supplemented with VE dampers requires the good description of the dynamical behaviour of dampers. The dampers behaviour depends mainly on the rheological properties of the viscoelastic material from which dampers are made.

In a classical approach, the mechanical models consisting of the springs and dashpots are used to describe the rheological properties of VE dampers [2, 3]. A good description of VE dampers requires mechanical models build from a set of appropriately connected springs and dashpots. In this approach the dynamic behaviour of a single damper is described by a set of differential equation, (see [3]) what considerably complicate the dynamic analysis of structures with dampers because the large set of motion equation must be solve. Moreover, the nonlinear regression procedure, described for example in [4], must be used to determine parameters of the mentioned above models.

The rheological properties of VE dampers are also described using the fractional mechanical models. Currently this approach received considerable attention and has been used in modeling the rheological behaviour of linear viscoelastic materials [5 – 7]. The fractional models have an ability to correctly describe the behaviour of viscoelastic material using a small number of parameters. A single equation is enough to describe the VE damper dynamics, what is an important advantage of the discussed models. In this case the VE damper equation of motion is the fractional differential equation. The fractional models of VE fluid dampers are proposed in [8, 9].
An important problem, bounded with fractional models is an estimation of model parameters from the experimental data. In the past, many different methods have been tested for estimation of model parameters [2, 10, 11]. The process of parameter identification is an inverse problem which can be ill conditioned (see, [4, 11]).

The aim of this paper is to describe a new method of parameters identification of the fractional Maxwell model. The parameters are estimated using results obtained from dynamical tests. Results of example calculation are further presented.

1. Fractional Maxwell model equation of motion and their steady state solution

In order to construct the fractional models equation of motion, we introduce the fractional element called also the springpot which obey the following equation:

\[\ddot{q}(t) + \tau \dot{q}(t) + c \alpha q(t) = \tau^\alpha D^\alpha q(t) + c \tau^\alpha D^\alpha q(t),\]

where \(c = \tau^\alpha\) and \(\alpha, (0 < \alpha \leq 1)\), are the springpot parameters and \(D^\alpha q(t)\) is the fractional derivative of order \(\alpha\) with respect to time \(t\). There are a few definitions of fractional derivatives which coincide under certain conditions. Here, symbols like \(D^\alpha q(t)\) means the Riemann-Liouville fractional derivatives with the lower limit \(-\infty\) (see [12]). The considered element can be understood as an interpolation between the spring element (\(\alpha = 0\)) and the dashpot element (\(\alpha = 1\)).

The fractional Maxwell model is build from the spring and the springpot connected in series as it is shown schematically on Fig 1. For the considered model we can write:

\[u(t) = \tau^\alpha D^\alpha q(t) + c \tau^\alpha D^\alpha q(t),\]

where \(\tau = \tau^\alpha / k = c / k\).

Eliminating \(q(t)\) from above relations we get the motion equation in the form:

\[u(t) + \tau^\alpha D^\alpha u(t) = k \tau^\alpha D^\alpha q(t),\]

The considered model has three real and positive value parameters: \(k\), \(c\) and \(\alpha\).

Fig. 1 Scheme of fractional Maxwell model

In a case of harmonically excitation, the steady state solution to motion equation of the Maxwell fractional model is assumed in the following form:
where \( \tau \) is the excitation frequency.

Introducing relations (4) into equation (3) we obtain that the coefficients \( q', q, u' \) and \( u \) are interrelated in the following way:

\[
q = \phi_1 u - \phi_3 u', \quad q = \phi_2 u + \phi_3 u',
\]

where

\[
\phi_1 = \frac{1}{k(\tau \lambda)} \left[ (\tau \lambda) \sin(\alpha \pi / 2) \right], \quad \phi_2 = \frac{1}{k(\tau \lambda)} \sin(\alpha \pi / 2).
\]

2. Identification of parameters of the fractional Maxwell model

In the proposed method, for the given frequency of excitation \( \lambda \), the experimentally measured damper force \( u(t) \) and the experimentally measured damper displacement \( q(t) \) are approximated by

\[
\tilde{u}(t) = \tilde{u}_a, \quad \tilde{q}(t) = \tilde{q}_a, \quad \tilde{u}(t) = \tilde{u}_a, \quad \tilde{q}(t) = \tilde{q}_a,
\]

where quantities \( \tilde{u}_a, \tilde{u}_a, \tilde{q}_a \) and \( \tilde{q}_a \) are determined using the last-square method. When the experimental data concerning damper force are considered the values of \( \tilde{u}_a, \tilde{u}_a \) are obtained from the following equations:

\[
I_{\tilde{u}} \tilde{u} + I_{\tilde{u}} \tilde{u} = I_{\tilde{u}}, \quad I_{\tilde{u}} \tilde{u} + I_{\tilde{u}} \tilde{u} = I_{\tilde{u}},
\]

where

\[
I_{\tilde{u}} = \int \tilde{u}_a(t) \cos \lambda t dt, \quad I_{\tilde{u}} = \int \sin \lambda t dt, \quad I_{\tilde{u}} = \int \sin \lambda t \cos \lambda t dt, \quad I_{\tilde{u}} = \int \tilde{u}_a(t) \sin \lambda t dt.
\]

In a similar way the parameters \( \tilde{q}_a, \tilde{q}_a \) are determined.

Next, we assume that the quantities \( \tilde{u}_a, \tilde{u}_a, \tilde{q}_a \) and \( \tilde{q}_a \) obtained from the experimental data approximately fulfils equations (5) i.e. we can rewrite (5) in the form:

\[
\phi_1 \tilde{u} - \phi_3 \tilde{u} = \tilde{q}_a, \quad \phi_2 \tilde{u} + \phi_3 \tilde{u} = \tilde{q}_a,
\]

where \( \phi_1 = \phi_1(\lambda) \) and \( \phi_2 = \phi_2(\lambda) \). From above equations we have

\[
\phi_1 = \frac{\tilde{q}_a \tilde{u} - \tilde{q}_a \tilde{u}}{\tilde{u} + \tilde{u}}, \quad \phi_2 = \frac{\tilde{q}_a \tilde{u} - \tilde{q}_a \tilde{u}}{\tilde{u} + \tilde{u}}.
\]
For a given set of excitation frequencies $\lambda_i$ ($i = 1, 2, ..., n$), used in experiments, two sets of values of $\phi_i$ and $\phi_j$ are obtained.

The model parameters $c$, $k$ and $\alpha$ will be determined using the last-square method. The error functional which will be minimized is chosen in the form:

$$J(\bar{c}, \bar{k}, \alpha) = \sum_{i=1}^n (r_i^2 + s_i^2),$$  \hspace{1cm} (13)

where $\bar{k} = 1/k$, $\bar{c} = 1/c$ and

$$r_i = \bar{k} + \bar{c} \lambda_i^\alpha \cos \frac{\alpha \pi}{2} - \phi_i, \hspace{1cm} s_i = \bar{c} \lambda_i^\alpha \sin \frac{\alpha \pi}{2} - \phi_j.$$  \hspace{1cm} (14)

If we assume that parameter $\alpha$ is known, the stationary conditions of functional (13) with respect to $\bar{k}$ and $\bar{c}$ give us the following system of equations:

$$n\bar{k} + \bar{c} \sum_{i=1}^n \lambda_i^\alpha \cos \frac{\alpha \pi}{2} = \sum_{i=1}^n \phi_i,$$

$$\bar{k} \sum_{i=1}^n \lambda_i^\alpha \cos \frac{\alpha \pi}{2} + \bar{c} \sum_{i=1}^n \lambda_i^{2\alpha} = \sum_{i=1}^n \left( \phi_i \cos \frac{\alpha \pi}{2} + \phi_j \sin \frac{\alpha \pi}{2} \right),$$  \hspace{1cm} (15)

For particular values of $\alpha$ the values of damper parameters, resulting from (15), could be negative. These solutions haven’t physical meaning and must be rejected. The right value of $\alpha$ are obtained using the method of systematic searching. The values of $\alpha$, $\bar{k}$ and $\bar{c}$ for which (13) has a minimal value are the searched parameters of the model.

3. Results of example identification of Maxwell model parameters

First the method is applied to the artificially generated experimental data. The artificial solutions are calculated on a base of steady state solution given by relations (4) and (5). The following data are used: $n = 9$, $q_i = 0$, $q_s = 0.001 m$, $\alpha = 0.6$, $k = 290.0 kN/m$, $c = 68.0 kNs/m$, $\lambda_1 = 0.5 Hz$, $\lambda_2 = 1.0 Hz$, $\lambda_3 = 2.0 Hz$, $\lambda_4 = 4.0 Hz$, $\lambda_5 = 6.0 Hz$, $\lambda_6 = 8.0 Hz$, $\lambda_7 = 10.0 Hz$, $\lambda_8 = 12.5 Hz$, $\lambda_9 = 15.0 Hz$. The obtained data are modified applying random perturbations. After application of the identification procedure the following results are obtained: $\alpha = 0.61$, $k = 284.543 kN/m$, $c = 68.096 kNs/m$, when 3 percent noises are randomly introduced to artificial data.

On Fig. 2 the plot of functional (13) versus the $\alpha$ parameter is shown for three levels of noises (3, 5 and 10%). In a range of values of $\alpha$ parameter we have one minima of functional. Results of calculation performed for the artificial data shown that if noises are not too much the results obtained using suggested method of identification is not sensitive to noises. Errors of values of parameters obtained are of order of noises level.
The next step is to apply the identification procedure to real experimental data. The experimental data presented by Makris and Constantinou [9] are chosen and used in this example. Makris and Constantinou were using the damper manufactured by GERB Schwingungsisolierungen GmbH & Co. KG in their investigations. Similar dampers are often used in piping systems or in machine foundations. The following parameters of fractional Maxwell model are determined: $$\alpha = 0.77$$, $$k = 503.350 \text{ kN/m}$$, $$c = 13.823 \text{ kN.s/m}$$ . On Figs. 3 and 4 the comparison of experimental and approximated storage modulus $$K'$$ and loss modulus $$K''$$ are presented. These quantities can be calculated from:

$$K' = \frac{k(\tau \lambda)^{\alpha}(\tau \lambda)^{\alpha} + \cos(\alpha \pi / 2)}{1 + (\tau \lambda)^{2\alpha} + 2(\tau \lambda)^{\alpha}\cos(\alpha \pi / 2)}$$, $$K'' = \frac{k(\tau \lambda)^{\alpha}\sin(\alpha \pi / 2)}{1 + (\tau \lambda)^{2\alpha} + 2(\tau \lambda)^{\alpha}\cos(\alpha \pi / 2)}$$ . 

(16)

The three-parameter fractional Maxwell model satisfactory well describes dynamic properties of the considered damper.

Fig. 2 Error functional (13) of Maxwell model versus the $$\alpha$$ parameter

Fig. 3 Storage modulus

Fig. 4 Loss modulus
4. Concluding remarks

The presented model is working satisfactory. The identification procedure of parameters of the fractional Maxwell model is simply, well applicable and efficient. After few modifications this procedure can be used to determine parameters of other fractional models, for example, to determine parameters of the fractional Kelvin-Voight model.

However the three-parameter fractional Maxwell has some limitations. There are materials (used in VE dampers) to which this model cannot be fitted in satisfactory way. The other restriction is an impossibility to analyze very low and medium frequencies together. The results are going worse when experimental results for very low frequencies are included.

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