

APPLICATION OF LYAPUNOV EQUATION TO ANALYSIS OF RANDOM VIBRATION OF STRUCTURE WITH TUNED MASS DAMPERS

Roman LEWANDOWSKI, Justyna GRZYMISŁAWSKA

Institute of Structural Engineering, Poznan University of Technology

Piotrowo Street 5, 60-965 Poznan

e-mail: roman.lewandowski@ikb.poznan.pl, justyna.grzymislawska@ikb.poznan.pl

Abstract

In this paper the Lyapunov equation is used to analyse random vibration of building structure. The structures with mass dampers are considered. The excitation forces which are functions of fluctuations of wind velocity are treated as random forces. Lyapunov equation is used to determine root mean square of displacements. Results of example calculation are presented and briefly discussed.

Keywords: tuned mass damper (TMD), multiple-tuned mass damper (MTMD), Lyapunov equation

Introduction

Dynamic analysis of structure with tuned mass dampers (TMD) and multiple-tuned mass dampers (MTMD) have been studied for many years [1-3]. Mainly, the numerical integration have been used to calculate the root mean square of quantities which characterize the structures response.

In this paper it will be studied the possibility of application of Lyapunov equation to dynamic analysis of structure loaded by forces excited by wind pressure. Wind is treated as white noise random process. Some calculations were made for 20-story building. Results of calculations were compared with ones obtained in classical way. On this basis conclusions concerning effectiveness of using Lyapunov equation are formulated.

1. Designing of multiple tuned mass dampers

The parameters of tuned mass damper (or group of dampers) are chosen in such a way that the damper is tuned to the selected mode of vibration. It means, that the frequency of the damper (or a group of dampers) ω_d , treated as the one degree of freedom of the system, is close to selected vibration mode of structure ω_s ($\omega_d \approx \omega_s$). The optimal parameters of such damper (or group of dampers) can be determined from formulae given in paper [4]. The optimal frequency ratio is:

$$\frac{\omega_d^2}{\omega_s^2} = \frac{2 + \mu}{2(1 + \mu)^2}, \quad (1)$$

where

$$\mu = m_d / M_s, \quad \omega_s^2 = K_s / M_s, \quad \omega_d^2 = k_d / m_d. \quad (2)$$

Here M_s and K_s are the modal mass and modal stiffness of the structure of the s -th vibration mode, respectively.

If only one damper is tuned to frequency ω_s , then m_d is the mass of damper, and k_d is the stiffness coefficient of damper. However, if the group of dampers is tuned to the frequency ω_s , then m_d and k_d denote the mass and the stiffness coefficient of selected damper of this group, respectively. Assuming that the mass ratio μ is known, the damper frequency ω_d and the stiffness coefficient k_d can be determined from above formulae.

If excitation forces acting on structure have a random character and can be treated as the white noise process, the optimal value of non-dimensional damping coefficient of damper is determined from formula:

$$\gamma_{opt} = \sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}. \quad (3)$$

The value of damping coefficient c_d can be calculated in the following way:

$$c_d = 2\gamma_{opt}\omega_d m_d. \quad (4)$$

2. Equation of motion

Equation of motion of the system shown on Fig. 1 can be written in the form:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}(t) + \tilde{\mathbf{C}}\dot{\mathbf{q}}(t) + \tilde{\mathbf{K}}\mathbf{q}(t) = \tilde{\mathbf{P}}(t), \quad (5)$$

where $\tilde{\mathbf{M}}$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{K}}$ are the global matrices of mass, damping and stiffness of the system, $\mathbf{q}(t)$ is vector of displacement of the system $\mathbf{q}(t) = \text{col}(\mathbf{y}(t), \mathbf{x}(t))$, $\mathbf{y}(t)$ are horizontal displacements of frame, $\mathbf{x}(t)$ are horizontal displacements of dampers, $\tilde{\mathbf{P}}(t)$ is vector of excitation forces $\tilde{\mathbf{P}}(t) = \text{col}(\mathbf{P}(t), \mathbf{0})$.

It is assumed that the damping matrix of the structure has the form as follows: $\mathbf{C} = \alpha\mathbf{M} + \kappa\mathbf{K}$. Details concerning the mass and stiffness matrices of structure with multiple mass dampers are given in [7].

If Lyapunov equation is used to analyse random vibration of structure then it is desired to write the equation of motion (5) with a help of state-space variables. Introducing symbols

$$\mathbf{z} = \text{col}(\mathbf{q}^T(t), \dot{\mathbf{q}}^T(t)), \quad \mathbf{p}(t) = \tilde{\mathbf{P}}(t), \quad (6)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1} \end{bmatrix}, \quad (7)$$

where $\mathbf{z}(t)$ denotes the space-state vector we can rewrite (5) in the following form

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{p}(t), \quad (8)$$

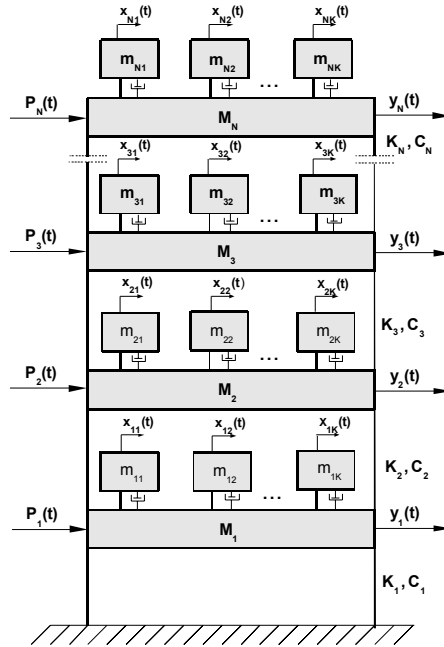


Fig.1. The system with MTMD

3. Modeling of wind load

It is assumed that load is the random, stationary process. Thus, the wind pressure in the arbitrary point of structure can be described in the following form:

$$P_i(t) = C_A A \rho U_i X^2 u_i(t), \quad (9)$$

where: C_A is the aerodynamic drag coefficient, A is the wind-exposed area, ρ is air density, X is the admittance function and $u_i(t)$ are the fluctuations of wind velocity on the level of floors.

Fluctuations of wind velocity are random process and, in this paper, are treated as white noise process. Taken into account the spatial correlation of fluctuations of wind velocity the matrix of spectral density of fluctuations of wind velocity $S_u(\lambda)$ can be calculated. Elements of the matrix $S_u(\lambda)$ are calculated from formula:

$$S_{lk}^u = \sqrt{S_{ll}^u(\lambda) S_{kk}^u(\lambda)} e^{-\Phi}, \quad (10)$$

where Φ denotes the correlation coefficient. In case of white-noise, elements $S_{ll}^u(\lambda)$ and $S_{kk}^u(\lambda)$ are independent from λ and then $S_{ll}^u(\lambda) = S_{l0} = const$, $S_{kk}^u(\lambda) = S_{k0} = const$.

The relationship between the spectral density matrix of load excited by wind pressure and the spectral density matrix of fluctuations of wind velocity is

$$\mathbf{S}_p = (C_A A \rho U)^2 X^2 \mathbf{S}_u . \quad (11)$$

4. Solution to the equation of motion

Solution to the equation of motion (8) has the following form (see [6]):

$$\mathbf{z}(t) - \bar{\mathbf{z}}(t) = \exp(\mathbf{A}(t - t_0))(\mathbf{z}(t) - \bar{\mathbf{z}}(t)) + \int_{t_0}^t \exp(\mathbf{A}(t - \tau)) \mathbf{B}(\mathbf{p}(\tau) - \bar{\mathbf{p}}(\tau)) d\tau . \quad (12)$$

The stochastic properties of response of randomly loaded structures is fully described by covariance matrix $\mathbf{Z}(t)$ and the mean value of response $\bar{\mathbf{z}}(t)$. Here, the mean value of structure displacements is zero because the mean value of excitation is equal zero. The covariance matrix is defined as follows:

$$\mathbf{Z}(t) = E[\mathbf{z}(t), \mathbf{z}^T(t)] . \quad (13)$$

Using the theory presented in [6] the covariance matrix can be calculated from the following Lyapunov equation:

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{Z}(t)\mathbf{A}^T + \mathbf{B}\hat{\mathbf{P}}(t)\mathbf{B}^T , \quad (14)$$

where $\hat{\mathbf{P}}(t) = 2T\check{\mathbf{P}}(t)$ and $\check{\mathbf{P}}(t)$ is the covariance matrix of excitation forces. Moreover, $\hat{\mathbf{P}}(t) = \check{\mathbf{P}} = P_0 \mathbf{I} = const$. Because wind forces are treated as the white noise random process it can be proved that $\mathbf{Z}(t) = \mathbf{Z} = const$ if $t \rightarrow \infty$. In this case the \mathbf{Z} matrix can be determined from the following linear algebraic Lyapunov equation:

$$\mathbf{A}\mathbf{Z} + \mathbf{Z}\mathbf{A}^T + \mathbf{B}\check{\mathbf{P}}\mathbf{B}^T = \mathbf{0} , \quad (15)$$

and the covariance matrix of structure response \mathbf{Z} is equal the correlation matrix \mathbf{R}_q .

5. Results of calculations

In this section the results of dynamic analysis of the structure using Lyapunov equation are discussed. It is considered structure with TMD and MTMD. Additionally, for comparison it has been made analysis in a classical way.

Parameters of building are given in Table 1. Non-dimensional damping coefficients of first and second vibration mode are equal 1% of critical damping. TMD (tuned to the first mode of vibration) and MTMD (tuned to first three modes of vibration) were located on the top floor. Parameters of dampers are shown in Table 2. The mean wind velocity i -th floor was calculated from formula:

$$U(z) = 2,5u_* \ln(z/z_0), \quad u_* = U(10)\sqrt{k} , \quad (16)$$

$U(10)$ is the mean wind velocity on the altitude 10m, k is the coefficient depended on type of area, z_0 is the roughness length and z is altitude. Moreover, the following data are used: $\rho = 1,226 \text{ kg/m}^3$, $U(10) = 30 \text{ m/s}^2$, $z_0 = 0,3$, $k = 12 \cdot 10^{-3}$.

story	mass [kg]	stiffness [N/m]
1	$2,83 \times 10^5$	$3,31 \times 10^8$
2 - 4	$2,76 \times 10^5$	$1,06 \times 10^9$
4 - 7	$2,76 \times 10^5$	$6,79 \times 10^8$
8 - 10	$2,76 \times 10^5$	$6,79 \times 10^8$
11 - 13	$2,76 \times 10^5$	$5,84 \times 10^8$
14 - 16	$2,76 \times 10^5$	$3,86 \times 10^8$
17 - 19	$2,76 \times 10^5$	$3,47 \times 10^8$
20	$2,92 \times 10^5$	$2,29 \times 10^8$

Table 1. Parameters of structure

number of mode	mass [kg]	stiffness [N/m]
TMD		
1	36214	472468
MTMD		
1	18107	238870
2	7956	722685
3	8550	2182386

Table 2. Parameters of dampers

The root mean square of i -th displacement of the structure is calculated from formula:

$$\sigma_q^2 = R_q, \quad (17)$$

where R_q is i -th element from diagonal of the \mathbf{R}_q matrix. Using above formulae the analysis of the structure without dampers, with conventional TMD and with MTMD were made. The classical method of calculation of root mean square of displacement are described in [7] Results of calculation are shown on Fig. 1.

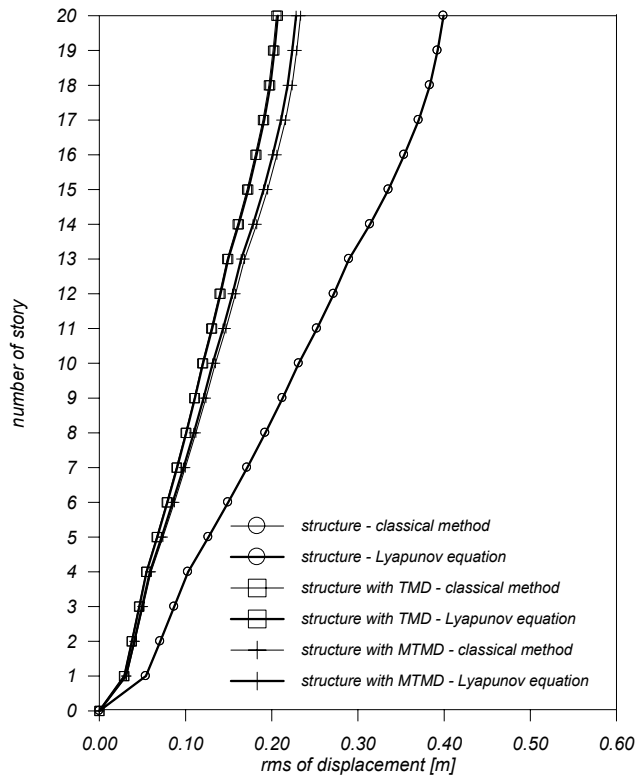


Fig. 1. Root mean square of displacements

It has been observed that in cases of structure without dampers and structure with TMD root mean square of displacements calculated using Lyapunov equation and calculated in a classical way are almost the same.

6. Conclusions

In this paper the possibility of application of Lyapunov equation to analysis of random vibration of structure with tuned mass dampers has been studied. The root mean square of displacement of structure were determined.

The proposed method which use the Lyapunov equation to dynamic analysis of structure can be alternative to the classical method of analysis of random vibrations of structures. However, currently the proposed method can be used only when wind is treated as the white noise process.

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