REMARKS ON MODELLING OF PASSIVE VISCOELASTIC DAMPERS

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Abstract. This paper concentrate on review of the results of realized experimental tests of viscoelastic dampers. The most popular and well-known models frequently used to describing these dampers are here discussed. Experimental tests of designed by authors VE dampers and the results of that tests are presented. The generalized Maxwell model and the Zener model were used to represent the behaviour of VE dampers. The constants of above models were determined by using the proposed method of fitting mentioned models to experimental data.

Keywords: viscoelastic (VE) dampers, rheological models, modelling of parameters, experimental tests

1. Introduction

Going beyond new limits in designing modern buildings and structures is carrying along the necessity of including (in absolutely new dimension) loads that were tilled now even ignored. It is for example in case of loads generated by winds. The significations of them were appeared in tall buildings. Dynamic growth of modern agglomerations and shrinking of free areas coerce location of buildings on the earthquake-threaten terrains. The evolution of new technologies and necessity of keeping various manufacturing rigours is causing that bigger attention is focused on dynamic loads which have an influence on production lines, machines and workers. The passive viscoelastic dampers (VE dampers) are very often used in all mentioned cases. There are two main types of these dampers. First type is viscoelastic liquid damper, which can have the shape of cylinder filled with silicone gel (e.g. GERB dampers). The second type is viscoelastic solid damper, which consist of e.g. three steel plates and two layers of viscoelastic polymer (e.g. ISD Polymer 111 or 111).

Application of passive dampers demands exact description of behaviour of these dampers. The simplest models, like the Maxwell model, the Kelvin-Voight model [1,2,3] or the Burgers model are very often used to describing VE dampers. These models are able to represent correctly mechanical properties of the dampers only for individual frequencies. Because of that, their application in structure designing is quite narrow.

Unfortunately the mechanical properties of the major number of polymers are strongly frequency-dependent. As it was mentioned, the simplest models allow describing the behaviour of the VE dampers only for selected frequency of excitation. Standard rheological models like the generalized Maxwell or generalized Voigt models are often used to describe properties of many polymers. The model parameters are determined on a base of experimental results. Standard Mechanical Models and their practical application in describing the VE dampers were presented by S.W. Park [4,5]. The vulnerability of these models is generating, in certain cases, negative (unphysical) values of searched parameters (e.g. stiffness) of the VE dampers. Authors have experienced similar problems during the identification of parameters of their own VE dampers. In some situations finding of proper relaxation times for calculations become troublesome. It should be concerned that relaxation times are generally specified *a priori* [4,6] and this is the main source of problems.

The next group of models describing the behaviour of VE dampers are the fractional derivative models. These models are able to reproduce the VE damper nature in frequency and time domains in an effective way. Using fractional calculus a number of fractional models can be developed, e.g. fractional Kelvin-Voight model [7], fractional Zener model [8,9], fractional Jeffreys model [10] or fractional Maxwell model. The Maxwell model and the Zener model were tested by authors. The problem to solve with these models is about good and efficient fitting them to experimental data. Otherwise there are materials which behaviour exceeds beyond mentioned models.

This work concentrate on review of existing models of viscoelastic dampers, their advantages and disadvantages. It is an introduction to develop mathematical model of the damper, which parameters will not depend on frequency of excitation. The aim of the future investigations will be developing of possible the most universal model describing the behaviour of VE damper in a wide range of frequencies (for instance from 0 to 500 rad/s).

2. Exemplary experimental results and their characteristic properties

The static and the dynamic tests are used to identify properties of the VE dampers. The dynamic tests are the most effective way to catch the complex relationships between parameters of the damper and the frequency of excitation, amplitude of excitation and temperature. In this test, the VE damper is exposed to the harmonic variable excitation, e.g. displacement. Generally, the excitation changes harmonically with time [11 - 15]. The mentioned type of experimental tests face with some problems in test system limits form. They can be performed only for a relatively narrow range of frequencies. As wider range of frequencies of excitations you want to overview, the smaller amplitudes by test system can be realize. The range of available frequencies of excitation (which test stations can generate) is dramatically reduced for large amplitudes of displacements.

The properties of viscoelastic materials are often presented by the concept of the complex parameter:

$$X^* = X' + iX'', \tag{2.1}$$

where: X' is the real part of this parameter (stiffness), X'' is the imaginary part (damping) and $i = \sqrt{-1}$ is the imaginary unit. Specifying, X^* can be e.g. the complex stiffness (if force-displacement relation is analyzed) or the complex shear modulus [4]:

$$G^* = G' + iG'', \tag{2.2}$$

where: G' is the storage modulus and G'' is the loss modulus. The quotient of loss modulus and storage modulus is called the loss factor η, i.e.:

$$\eta = G''/G'. \tag{2.3}$$

In Figs. 2.1 – 2.12, the chosen, exemplary graphs of experimental data, taken from literature, are presented.

Presented plots are showing how different properties of viscoelastic material can be. Observation of the loss factor plots (Figs 2.2, 2.6, 2.8 and 2.12) gives following remarks. The loss factor curve can be quite constant (Fig. 2.2), can grow with the frequency excitation (Fig. 2.6) or can decrease as it is shown on Figs 2.8 and 2.12. The forecasting of the behaviour of VE damper beyond the measured frequencies is very difficult. The storage and loss modulus or stiffness have generally the same character and in all cases these quantities increase with excitation frequency. It can be observe, that the major part of forgoing experimental data covers a relatively narrow range of frequencies. It gives us a question about further course of discussed parameters in the frequency domain.

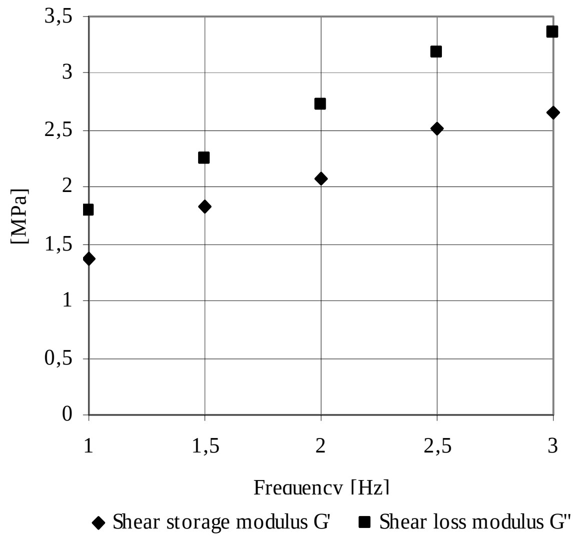

Fig 2.1 Shear storage and loss modulus of VE solid damper with viscoelastic material manufactured by 3M (Shen, Soong, Chang, Lai, 1995 [15])

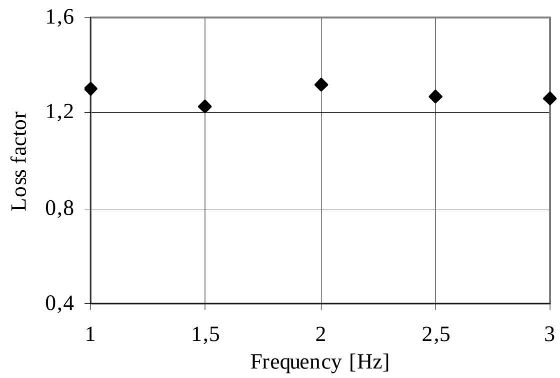

Fig 2.2 Loss factor η of VE solid damper (Shen, Soong, Chang, Lai, 1995 [15])

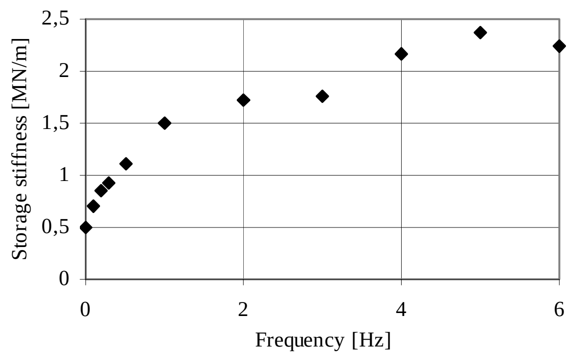

Fig 2.3 Storage stiffness K' of VE solid damper (Min, Kim, Lee, 2004 [11])

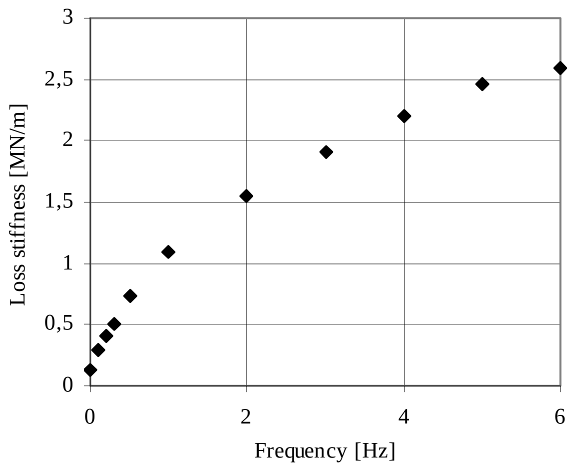

Fig 2.5 Loss stiffness *K”* of VE solid damper (Min, Kim, Lee, 2004 [11])

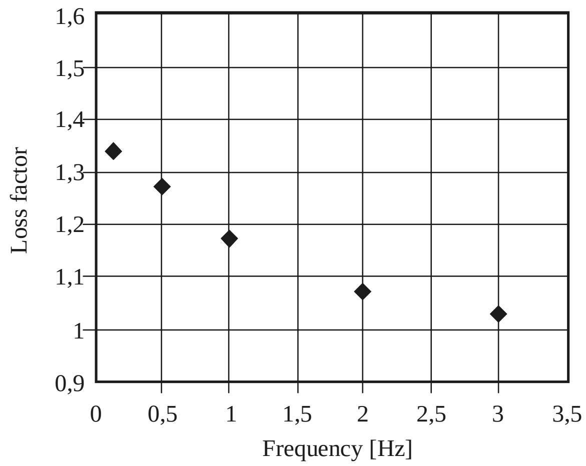

Fig 2.8 Loss factor η of VE solid damper at 20% strain (Aprile, Inaudi, Kelly, 1997 [16])

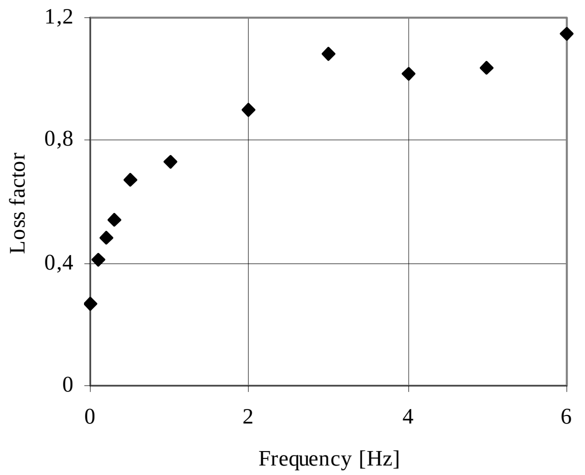

Fig 2.6 Loss factor η of VE solid damper (Min, Kim, Lee, 2004 [11])

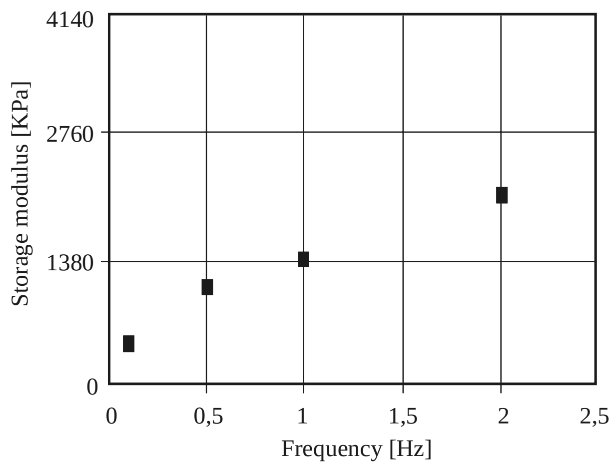

Fig 2.9 Shear storage modulus *G’* of VE solid damper at 24°C and 5% strain (Higgins, Kasai, 1998 [12])

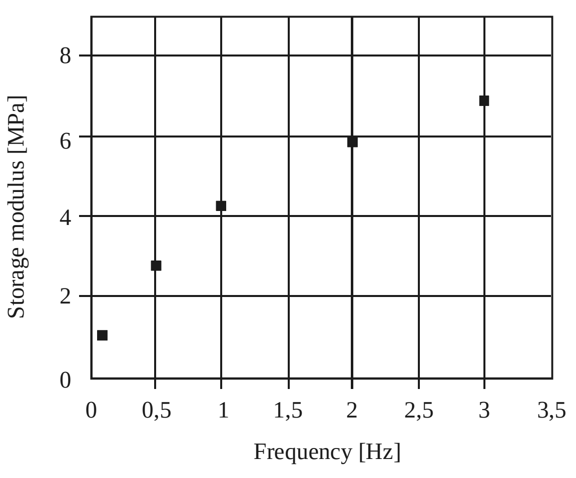

Fig 2.7 Shear storage *G’* modulus of VE solid damper at 20% strain (Aprile, Inaudi, Kelly, 1997 [16])

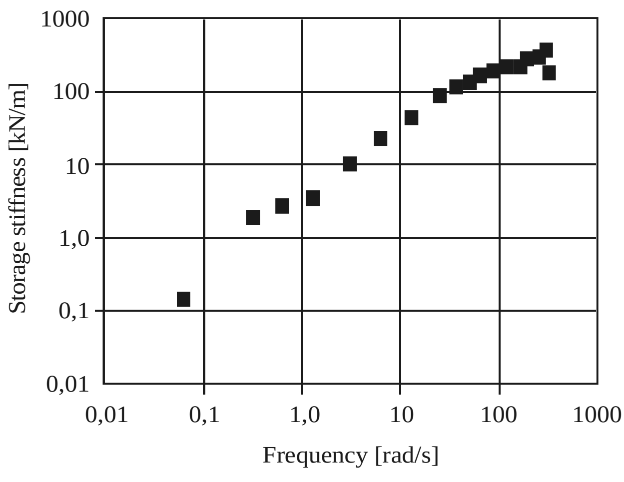

Fig 2.10 Storage stiffness *K’* of VE liquid damper (S.W. Park based on Makris tabular experimental data, 2001 [4])

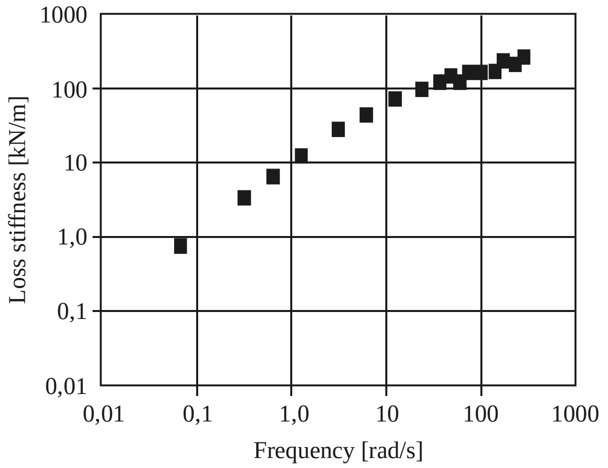

Fig 2.11 Loss stiffness *K”* of VE liquid damper (S.W. Park based on Makris tabular experimental data, 2001 [4])

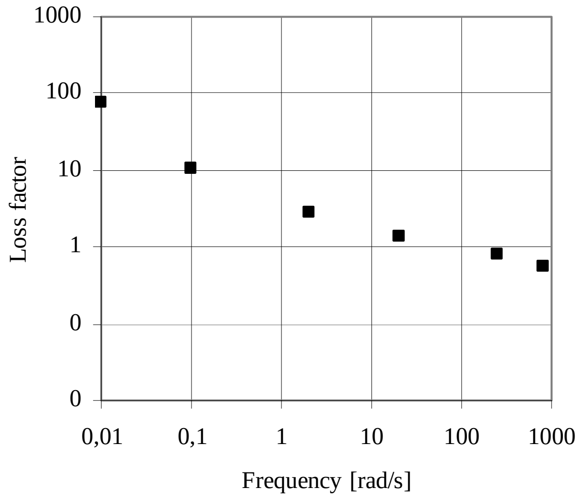

Fig 2.12 Loss factor η of VE liquid damper (data from [4])

3. Analysis of usage possibilities of rheological models to description of dynamic behaviour of VE dampers

In this section the dynamic properties of several rheological models will be determined and theirs ability to model qualitatively the dynamic behaviour of VE dampers will be considered.

3.1. The simple Kelvin model

The Kelvin model, which consists of spring and dashpot connected in parallel (see Fig. 3.1a) is governed by the equation

$$u(t) = k\big(q(t) + \tau\,\dot{q}(t)\big)\ , \qquad (3.1)$$

where t is time, $u(t)$ is the dampers force, $q(t)$ is the dampers relative displacement, $\tau = c/k$ is the relaxation time. Moreover, $(\dot{\circ}) = d(\circ)/dt$, c and k are the dampers stiffness and damping factor, respectively.

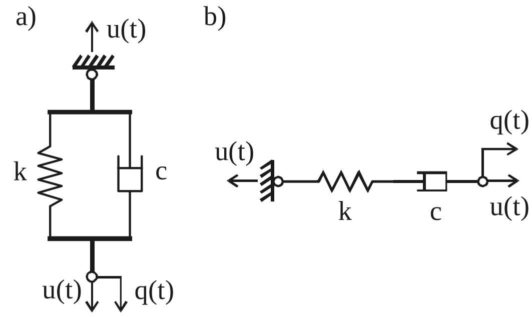

Fig 3.1 The Kelvin and Maxwell rheological models

If the damper is harmonically excited, i.e.

$$q(t) = q_0 \exp(i\lambda t)\ , \qquad (3.2)$$

where $i = \sqrt{-1}$ and λ is the excitation frequency the damper force in the case of steady state vibration can be described as

$$u(t) = u_0 \exp(i\lambda t)\ . \qquad (3.3)$$

Moreover, it is easy to show that

$$u_0 = k(1 + i\,\tau\,\lambda)q_0\ , \qquad (3.4)$$

what means that the storage modulus $K' = k$ is the constant function of λ, while the loss modulus $K'' = k\tau\,\lambda$ and the loss factor $\eta = K''/K' = \tau\,\lambda$ are linear functions of λ.

It is obvious that the simple Kelvin model is not able to model correctly the dynamic behaviour of VE damper (see also Figs. 3.2 - 3.4).

The Kelvin model is used in [2, 18, 19] to analyze the dynamic behaviour of buildings with the viscoelastic dampers.

3.2. The simple Maxwell model

The equation of Maxwell model, shown in Fig. 3.1b, is given by

$$u(t) + \tau\,\dot{u}(t) = k\tau\,\dot{q}(t)\ . \qquad (3.5)$$

After introducing Eqs (3.2) and (3.3) into Eqn (3.5) we obtain

$$u_0 = k\tau\,\lambda \frac{\tau\,\lambda + i}{1 + \tau^2\lambda^2} q_0\ , \qquad (3.6)$$

what means that

$$K' = k\frac{\tau^2\lambda^2}{1+\tau^2\lambda^2}\ ,\quad K'' = k\frac{\tau\,\lambda}{1+\tau^2\lambda^2}\ ,\quad \eta = \frac{1}{\tau\,\lambda}\ . \qquad (3.7)$$

Figures 3.2-3.4 show how the storage and loss modulus and the loss factor vary with the excitation frequency for both the Kelvin (dashed line) and the Maxwell model (solid line). According to the Maxwell model the storage modulus gradually growth with λ and the value of storage modulus is approximately equal k

for large values of $\tau\lambda$. The function of loss modulus $K^{"}(\lambda)$ has extremum, what can be observed in some experiments (see [17]). In this model extremum is at $\lambda_e = 1/\tau$ and $K^{"}(\lambda_e) = k/2$. According to the Maxwell model the loss factor $\eta(\lambda)$ always decreases with λ what is consistent with some experimental data (see Figs. 2.8 and 2.12). However, for $\lambda \to 0$, $\eta(\lambda) \to \infty$ what is not observed experimentally. Moreover, $K^{'}(0) = 0$ what is not confirmed in some experiments (see Fig. 2.3 and 2.7).

The Maxwell model is used in [1, 20, 21] to analyze the dynamic response of buildings.

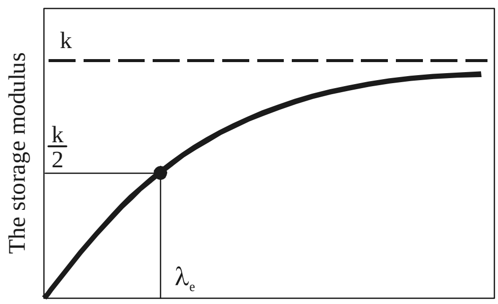

Fig 3.2 The storage modulus for the Kelvin and Maxwell model versus excitation frequency

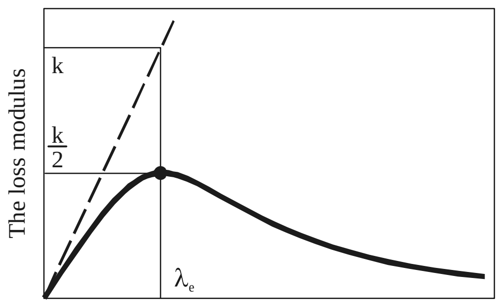

Fig 3.3 The loos modulus for the Kelvin and Maxwell model versus excitation frequency

3.3. Advanced rheological model of dampers

In this subsection the model, shown in Fig. 3.5, in which elastic spring connected in parallel with n elements of Maxwell type is considered. In a case $n = 1$ we have the well-known standard model. If we remove the elastic spring from this model (i.e. when $k_0 = 0$) we obtain the model used by Park [4] to describe behaviour of the fluid damper. In paper [22] the mentioned above model is used to analyze the dynamic response of building equipped with dampers.

The behaviour of considered model can be described with a help of the following system of equations:

$$u(t) = \sum_{i=0}^{n} u_i(t) \ , \qquad u_0(t) = k_0 q(t) \ , \qquad (3.8)$$

$$u_i(t) + \tau_i \, \dot{u}_i(t) = k_i \tau_i \, \dot{q}(t) \ , \qquad (3.9)$$

where $u(t)$ is the total damper force, $u_0(t)$ is the force in elastic spring, $u_i(t)$, ($i = 1,2,...,n$) are the forces in the Maxwell elements, $\tau_i = c_i / k_i$. Moreover, k_0 is the stiffness of elastic spring and c_i, k_i are the stiffness and damping factor of the i-th Maxwell element, respectively.

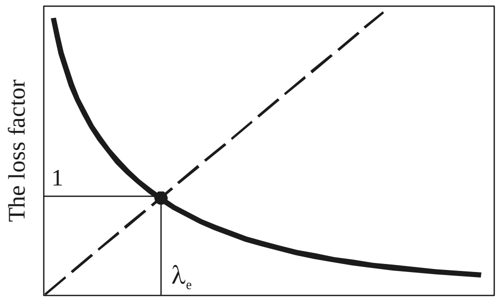

Fig 3.4 The loss factor for the Kelvin and Maxwell model versus excitation frequency

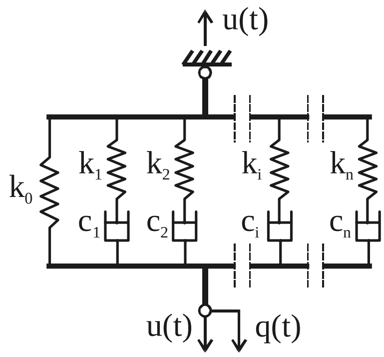

Fig 3.5 Scheme of the advanced rheological model

If damper is harmonically excited, the steady state response of damper can be described by

$$u(t) = u_0 \exp(i\lambda t) \ , \quad u_0(t) = \tilde{u}_0 \exp(i\lambda t) \ , \qquad (3.10)$$

$$u_i(t) = \tilde{u}_i \exp(i\lambda t) \ , \quad (i = 1,2,...,n) \ . \qquad (3.11)$$

Introducing Eqn (3.2), (3.10) and (3.11) into Eqns (3.8) and (3.9) we obtain

$$\tilde{u}_0 = k_0 q_0 \ , \quad \tilde{u}_i = k_i \tau_i \lambda \frac{\tau_i \lambda + i}{1 + (\tau_i \lambda)^2} q_0 \ , \qquad (3.12)$$

$$u_0 = \left(k_0 + \sum_{i=1}^{n} k_i \tau_i \lambda \frac{\tau_i \lambda + i}{1 + (\tau_i \lambda)^2} \right) q_0 \ . \qquad (3.13)$$

It means that the functions of storage and loss modulus and the function of loss factor are:

$$K^{'}(\lambda) = k_0 + \sum_{i=1}^{n} k_i \frac{(\tau_i \lambda)^2}{1+(\tau_i \lambda)^2} \ , \tag{3.14}$$

$$K^{''}(\lambda) = \sum_{i=1}^{n} k_i \frac{\tau_i \lambda}{1+(\tau_i \lambda)^2} \ , \tag{3.15}$$

$$\eta(\lambda) = \frac{\sum_{i=1}^{n} \left\{ k_i \tau_i \lambda \prod_{j=1,\, j\neq i}^{n} \left[1+(\tau_j \lambda)^2\right] \right\}}{k_0 \prod_{j=1}^{n} \left[1+(\tau_j \lambda)^2\right] + \sum_{i=1}^{n} \left\{ k_i (\tau_i \lambda)^2 \prod_{j=1,\, j\neq i}^{n} \left[1+(\tau_j \lambda)^2\right] \right\}} \tag{3.16}$$

Many particular models can be obtained if different Maxwell elements are taken into account. These models have much more free parameters then simple models but the qualitative properties of generalized models are similar to the simple Maxwell model or standard model.

Now, the properties of standard model will be briefly discussed. For $n=1$, from Eqns (3.14) – (3.16) we obtain results for the standard model. In this case the function $K^{''}(\lambda)$ is similar as one for the Maxwell model, but now we have the initial stiffness because $K^{'}(0)=k_0$ what is in agreement with experimental results shown in Figs. 2.3 and 2.7. The function of storage modulus is identical as for the Maxwell model. However the function of loss factor $\eta(\lambda)$ is different, especially for small values of $\tau_1 \lambda$ because $\eta(0)=0$. Figures 3.6 show plots of function $\eta(\lambda)$ for different stiffness ratio of $a = k_0 / k_1$.

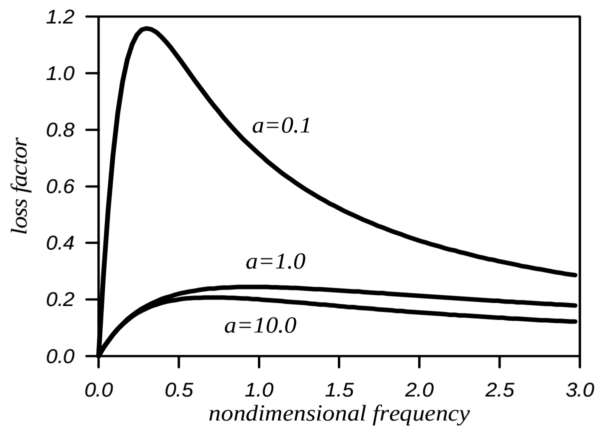

Fig. 3.6 The loss factor versus nondimensional frequency – standard model

3.4. Fractional Maxwell model of dampers

Recently, the new rheological models based on fractional calculus have been proposed to describe VE dampers behaviour [8,17,23]. Several fractional rheological models were proposed [8,23]. One of them is the fractional Maxwell model. The equation of the four parameters fractional Maxwell model is given by (see [8])

$$u(t) + \tau^{\alpha} \frac{d^{\alpha} u(t)}{dt^{\alpha}} = k \tau^{\beta} \frac{d^{\beta} q(t)}{dt^{\beta}} \ , \tag{3.17}$$

The model parameters $k,\ \tau,\ \alpha,\ \beta$ must be positive numbers, $0<\alpha,\ \beta \le 1$ and $\alpha \ge \beta$. Makris *et al.* [23] developed the fractional Maxwell model in which parameters are complex numbers. Symbols like

$$\frac{d^{\alpha} u(t)}{dt^{\alpha}}$$

denote the fractional derivatives of order α of $u(t)$ with respect to time. Some valuable information about fractional calculus can be found in [24].

Introducing Eqns (3.2) and (3.3) into Eqn (3.17) and taking into account that

$$\frac{d^{\alpha} \exp(i\lambda t)}{dt^{\alpha}} = (\lambda\, i)^{\alpha} \exp(i\lambda t) \ , \tag{3.18}$$

$$(\,i)^{\alpha} = \cos\frac{\alpha\pi}{2} + i \sin\frac{\alpha\pi}{2} \ , \tag{3.19}$$

we obtain

$$u_0 = \left(K^{'} + iK^{''}\right) q_0 = K^{'}(1+i\eta)\, q_0 \ , \tag{3.20}$$

where

$$K^{'} = k(\tau\,\lambda)^{\beta} \frac{\cos\frac{\beta\pi}{2} + (\tau\,\lambda)^{\alpha} \cos\frac{(\beta-\alpha)\pi}{2}}{1+(\tau\,\lambda)^{2\alpha} + 2(\tau\,\lambda)^{\alpha} \cos\frac{\alpha\pi}{2}} \ , \tag{3.21}$$

$$K^{''} = k(\tau\,\lambda)^{\beta} \frac{\sin\frac{\beta\pi}{2} + (\tau\,\lambda)^{\alpha} \sin\frac{(\beta-\alpha)\pi}{2}}{1+(\tau\,\lambda)^{2\alpha} + 2(\tau\,\lambda)^{\alpha} \cos\frac{\alpha\pi}{2}} \ , \tag{3.22}$$

$$\eta = \frac{k^{''}}{k^{'}} = \frac{\sin\frac{\beta\pi}{2} + (\tau\,\lambda)^{\alpha} \sin\frac{(\beta-\alpha)\pi}{2}}{\cos\frac{\beta\pi}{2} + (\tau\,\lambda)^{\alpha} \cos\frac{(\beta-\alpha)\pi}{2}} \ . \tag{3.23}$$

In literature also exist the fractional Maxwell model with three parameters. In this case $\alpha=\beta$ and

$$K^{'} = k(\tau\,\lambda)^{\alpha} \frac{(\tau\,\lambda)^{\alpha} + \cos\frac{\alpha\pi}{2}}{1+(\tau\,\lambda)^{2\alpha} + 2(\tau\,\lambda)^{\alpha} \cos\frac{\alpha\pi}{2}} \ , \tag{3.24}$$

$$K^{''} = k(\tau\,\lambda)^{\alpha} \frac{\sin\frac{\alpha\pi}{2}}{1+(\tau\,\lambda)^{2\alpha} + 2(\tau\,\lambda)^{\alpha} \cos\frac{\alpha\pi}{2}} \ , \tag{3.25}$$

$$\eta = \frac{\sin\dfrac{\alpha\pi}{2}}{(\tau\,\lambda)^{\alpha} + \cos\dfrac{\alpha\pi}{2}} \; . \qquad (3.26)$$

The properties of three parameters model, for different values of α parameter are shown on Figs 3.7 – 3.9. On these figures the nondimensional frequency is defined as $\tau\,\lambda$.

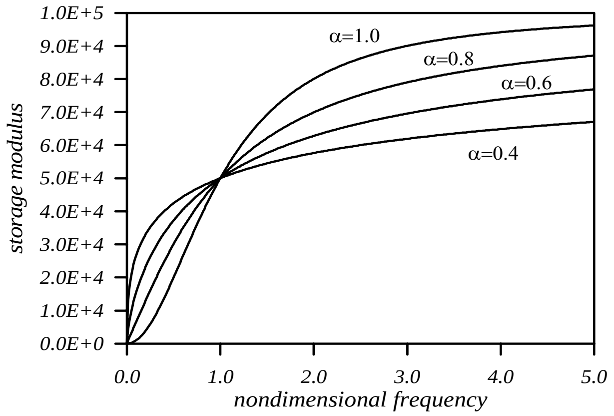

Fig. 3.7 Storage modulus of fractional Maxwell model for
different values of α parameter

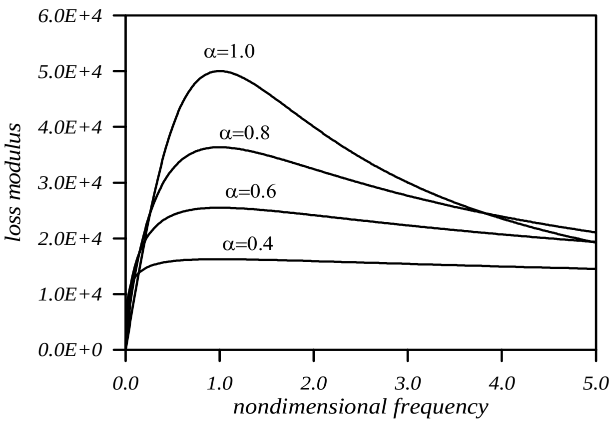

Fig. 3.8 Loss modulus of fractional Maxwell model for different values of α parameter

Now, the qualitative agreement of the function $K^{'}(\lambda)$ with experimental results is observed. The function of loss modulus could be very flat (for instant when $\alpha = 0.4$) what agree with results shown in Fig. 2.2 or (for $\alpha > 0.4$) can well approximate results shown on Figs. 2.5 and 2.11. On the other hand, the function of loss factor $\eta(\lambda)$ doesn't agree with the data shown in Fig. 2.6 but it is in qualitative agreement with the values of experimental loss factor presented in Fig. 2.12. Moreover, $\eta(0) = tg(\beta\pi/2)$ what means that now the loss factor has a finite value for $\lambda = 0$. It is important difference in comparison with the loss factor function of the simple Maxwell model which values go to infinity if λ goes to zero. In conclusion, the tree parameter fractional Maxwell model is in qualitative agreement with the experimental data given in [4].

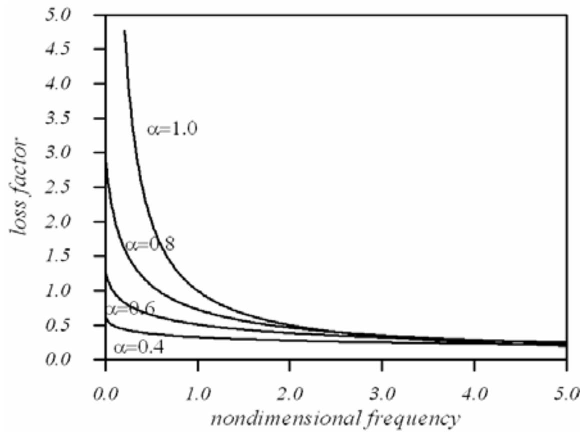

Fig. 3.9 Loss factor of fractional Maxwell model for different values of α parameter

Results of the fourth parameter fractional Maxwell model (for $\alpha = 0.8$ and chosen values of β parameter) are shown on Figs. 3.10 – 3.12. The results are qualitatively similar to the ones obtained for the three parameter fractional Maxwell model. One important exception is that values of the loss modulus function and the loss factor function can be negative (unphysical) if the difference between values of α and β parameters are sufficiently large.

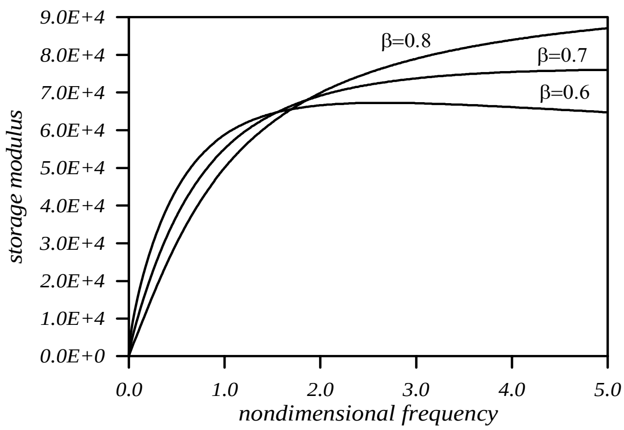

Fig. 3.10 Storage modulus of fractional Maxwell model for different values of β parameter ($\alpha = 0.8$)

4. The parameters identification of composite models of viscoelastic damper

4.1. Description of dampers and experimental test

In the experimental tests a VE dampers made of VHB 4959 material (viscoelastic material was manufactured by 3M) were used. The dampers were fabricated from two layers of viscoelastic material and three steel plates. Cyclic test were performed in a MTS 810 workstation. The sampling frequency was 800 Hz and the dampers were exposed to a harmonic variable displacement.

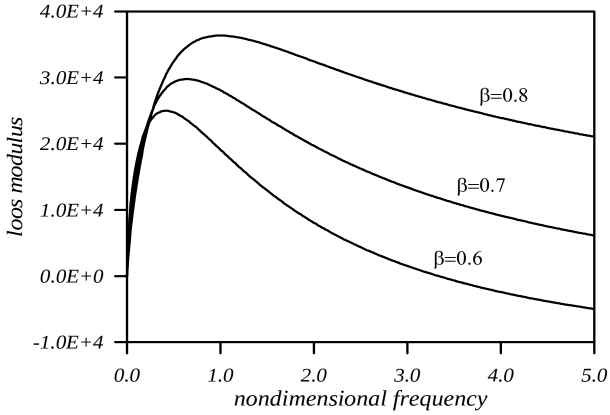

Fig. 3.11 Loss modulus of the fractional Maxwell model for different values of β parameter ($\alpha = 0.8$)

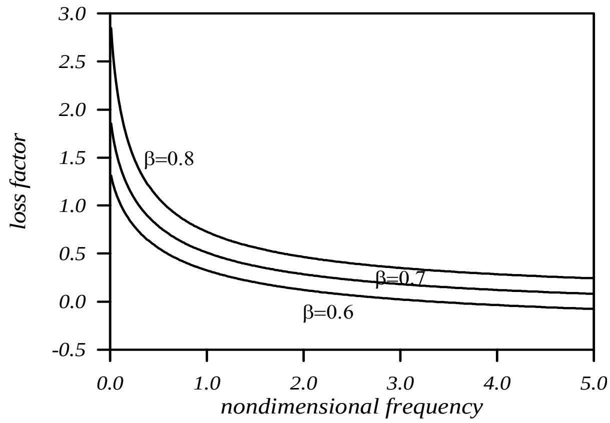

Fig. 3.12 Loss factor of the fractional Maxwell model for different values of β parameter ($\alpha = 0.8$)

During tests, two output functions u(t) (function of forces in a time domain) and q(t) (function of displacements in a time domain) were obtained. The data were collected for various amplitudes and various frequencies of excitation. The identification procedure consists of two steps.

The first step of identification procedure of the VE damper parameters (described briefly below) was applied to each frequency of excitation. For the given frequency of excitation the simple Maxwell model was used to represent the damper behaviour. The measured displacements of the damper were approximated by using the least-square method. The parameters c_d (the damping coefficient of damper) and k_d (the damper stiffness) of the simple Maxwell model were determined with a help of the **least-square method**. In details this procedure is described in [25]. The storage stiffness, loss stiffness and the loss factor were calculated and theirs frequency-dependences are shown in Figs 4.1 and 4.2.

The next section contains the detailed description of the second step of identification procedure.

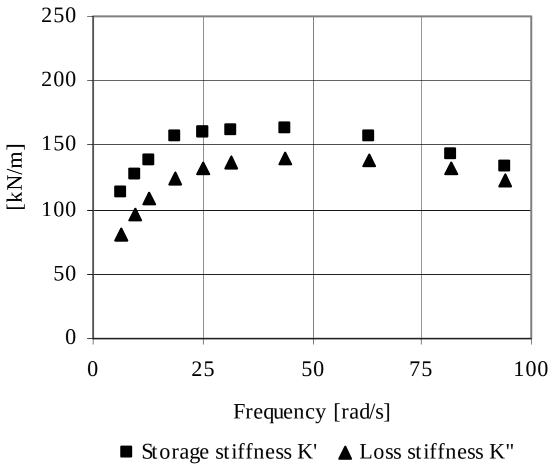

Fig 4.1 The storage stiffness and the loss stiffness from the experiments

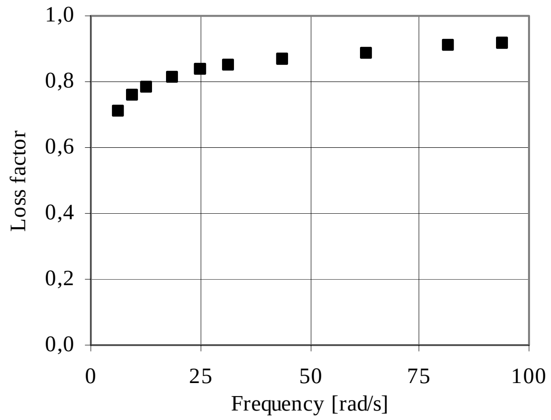

Fig 4.2 The loss factor from the experiments

4.2. Fitting models to experimental data

Having, from experiment and for different excitation frequencies, the values of storage stiffness, the loss stiffness and the loss factor the generalized Maxwell model and the generalized Zener model were used to describe the overall behaviour of VE dampers**.** At the beginning of the second step of identification procedure the initial values of relaxation times are assumed. The finding of proper relaxation times will be described later in the this section. The least-square method was used to fit model parameters. In both cases, the following functional was used:

$$J = \sum_{k=1}^{N} \left\{ \left(\frac{K'(\lambda_k)}{K'_e(\lambda_k)} - 1 \right)^2 + \left(\frac{K''(\lambda_k)}{K''_e(\lambda_k)} - 1 \right)^2 \right\}, \quad (4.1)$$

where: $K'(\lambda_k)$ and $K''(\lambda_k)$ are the searched storage and loss stiffness of the model, $K'_e(\lambda_k)$ and $K''_e(\lambda_k)$ are the storage and the loss stiffness calculated from experimental data, N is the number of frequencies, for which experimental data are known.

From stationary conditions of functional (4.1) following system of equations is determined:

$$\mathbf{A}\mathbf{k} = \mathbf{g} \ , \qquad (4.2)$$

where: $\mathbf{A}$ is the matrix of α_{rj} – coefficient, $\mathbf{k}$ is the vector of searched model parameters k_j and $\mathbf{g}$ is the vector of γ_r – free terms. The expressions for α_{rj} and γ_r are given as follow:

$$\alpha_{rj} = \sum_{k=1}^{N} \frac{\lambda_k^2 \tau_r \tau_i}{(1+\lambda_k^2 \tau_r^2)(1+\lambda_k^2 \tau_i^2)} \left(\frac{\lambda_k^2 \tau_r \tau_i}{K'^{\,2}_e(\lambda_k)} + \frac{1}{K''^{\,2}_e(\lambda_k)} \right), \qquad (4.3)$$

$$\gamma_r = \sum_{k=1}^{N} \frac{\lambda_k \tau_r}{1+\lambda_k^2 \tau_r^2} \left(\frac{\lambda_k \tau_r}{K'_e(\lambda_k)} + \frac{1}{K''_e(\lambda_k)} \right). \qquad (4.4)$$

The constants of the VE damper model are determined by solving the system of equations (4.2). The value of functional (4.1) can be also calculated. In this procedure the appropriate relaxation times τ and number of Maxwell elements must be assumed *a priori*. If they are wrong, the system of equations gives us negative (unphysical) values of stiffness of Maxwell elements.

Now the minimum of functional (4.1) with respect to the relaxation times must be determined. It is not so simple, because sometimes functional reaches its minimum for negative values of k_j. Authors used the Solver software (available in Excel), to search the minimum of functional (4.1) with respect to τ_i, but this method does not guarantee faultless results and the minimization procedure must be repeated starting from different set of initial values of relaxation times.

The computed constants can be used to draw the hysteresis plots, which can be compared with similar plots obtained from experimental data. The agreement of generalized Maxwell model with the experimental data is shown in Figs 4.3 - 4.5.

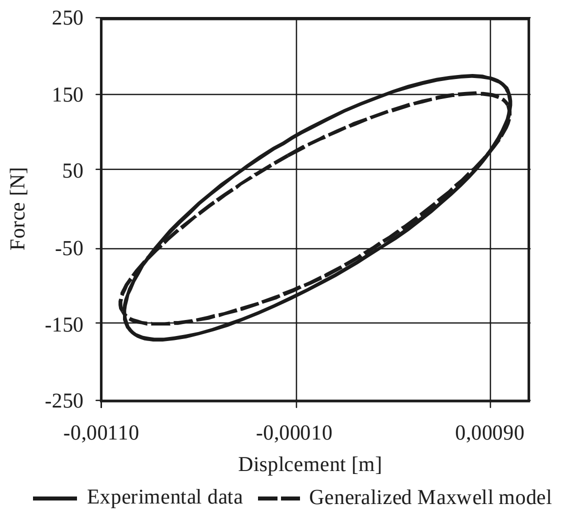

Fig 4.3 Generalized Maxwell model fitted to experimental data (VE damper excited by 1,0 mm displacement at 2 Hz)

It should be mentioned that generalized models could well approximate the behaviour of VE damper only in some range of frequencies, (see Fig. 4.6 for details).

5. Conclusions

The VE dampers, which can significantly reduce vibration of structures, are usually modelled with a help of Kelvin-Voight or Maxwell rheological models [1, 18-21]. These models are simple but our own experimental results and results previously published shown different properties of VE dampers. However, papers that describe results of experimental testing of VE dampers are rather rare and the experimental data are given for the restricted range of excitation frequencies. The properties of viscoelastic materials used in the experimental test are demonstrating significant differences. For example, some tests shown that the loss factor decrease with the excitation frequency while results of others tests show an opposite relation between the loss factor and the excitation frequency.

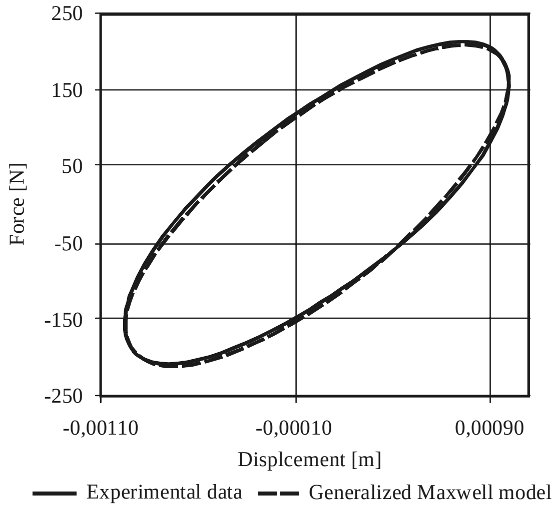

Fig 4.4 Generalized Maxwell model fitted to experimental data (VE damper excited by 1,0 mm displacement at 5 Hz)

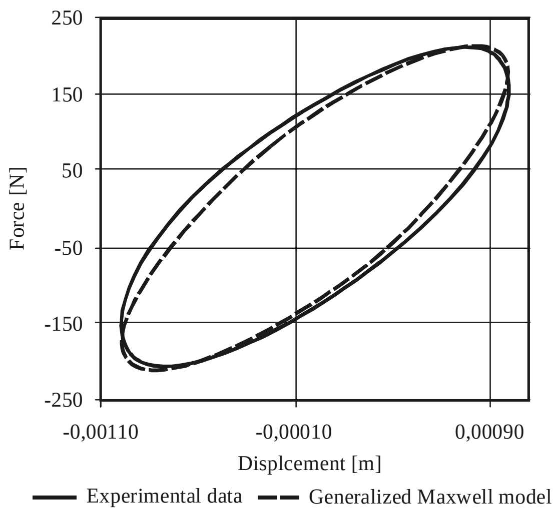

Fig 4.5 Generalized Maxwell model fitted to experimental data (VE damper excited by 1,0 mm displacement at 10 Hz)

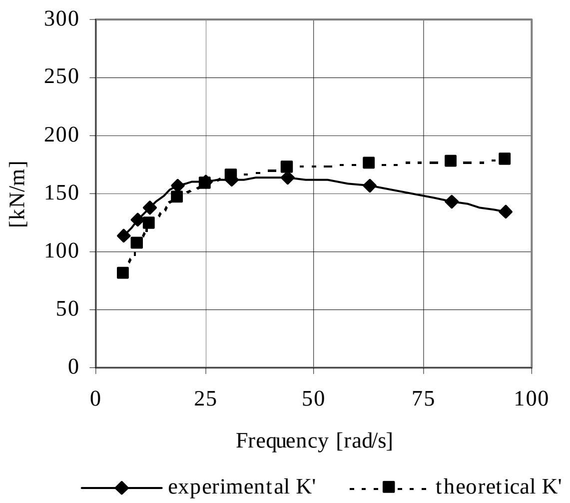

Fig 4.6 Comparison of experimental storage stiffness and theoretical storage stiffness delivered by generalized Maxwell model (VE damper excited by 1,0 mm)

Considering that, different models can be used to describe the VE dampers. The worst description of the VE damper seems to be given by the Kelvin-Voight model. The generalized Maxwell model or the Zener model is the better choice. The best and most adequate qualitative description of behaviour of VE dampers could be done with a help of the fractional derivative model. The reported experimental results shown that the storage stiffness decrase with the excitation frequency for $\lambda \geq 50Hz$. This fact needs further investigation.

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