

DYNAMIC ANALYSIS OF STRUCTURE WITH MULTIPLE TUNED MASS DAMPERS

Roman Lewandowski, Justyna Grzymisławska

*Institute of Structural Engineering, Poznan University of Technology
Piotrowo Street 5, 60-965 Poznań, Poland*

e-mail: roman.lewandowski@ikb.poznan.pl, justyna.grzymislawska@ikb.poznan.pl

Abstract. The purpose of the paper is to analyse the possibility of vibration reduction of frame building structures. It is considered structures loaded by forces exposed to strong winds. The excitation forces which are functions of wind velocity fluctuations are treated as random forces. The spectral density functions of wind velocity fluctuations are assumed in the form proposed by Davenport. The correlation theory of random vibrations are used and right mean square of displacements and accelerations are determined. Several remarks concerning the effectiveness of multiple tuned mass dampers are formulated on the basis of results of calculations.

Keywords: tuned mass damper (TMD), multiple-tuned mass dampers (MTMD)

1. Introduction

Mass dampers have been used for vibration reduction of structures for many years [1]. They have been used among others to vibration reduction of building construction subjected to strong winds [2]. Tuned mass dampers (TMD) installed on the top floor have been studied, mainly. Tuned mass dampers have been designed to tune to the fundamental vibration mode of structure. In paper [3] the optimization method for various type of excitation forces was given by Warburton. It has been tried to reduce the displacements and accelerations of structure. The remaining ones have disadvantageous influences not only on building construction, but also on people who are inside.

In the 90-th the studies to application multiple tuned mass dampers (MTMD) for one-degree of freedom systems have been started [4]-[5]. It has been proved, that MTMD with distributed natural frequencies and total mass equal mass of conventional tuned mass damper are more effective than TMD. The studies of MTMD have been developed [6]-[7]. Later, it has been analysed multi-degree of freedom systems subjected to the seismic load [8]. MTMD were designed in this way to tune several vibration modes of structure. A number of dampers have been depended on considered vibration modes.

In this paper it will be analysed the possibility of vibration reduction of frame structure with help of MTMD. The structure is loaded with dynamic force excited by wind pressure. Wind is treated as random

and ergodic process. Some calculations were made for 20-story building and on this basis the effectiveness of MTMD was estimated.

2. Designing of multiple tuned mass dampers (MTMD)

Designing of multiple tuned mass dampers it aims to tune other vibration mode of structure by every dampers (or group of dampers). It means, that the frequency of damper (or group of dampers) ω_d is close to selected vibration mode of structure ω_s ($\omega_d \approx \omega_s$). The optimal parameters of such damper (or group of dampers) can be determined by formulae, which are given in paper [3]. The optimal frequency ratio of structure and damper can be formulated as:

$$\frac{\omega_d^2}{\omega_s^2} = \frac{2 + \mu}{2(1 + \mu)^2} \quad (1)$$

where:

$$\mu = \frac{m_d}{M_s}, \quad \omega_s^2 = \frac{K_s}{M_s}, \quad \omega_d^2 = \frac{k_d}{m_d} \quad (2)$$

M_s - the modal mass of structure connected with s number of vibration mode

K_s - the modal stiffness of structure connected with s number of vibration mode.

If only one damper is tuned to frequency ω_s , then m_d is the mass of damper, and k_d is the stiffness coefficient of damper. However, if the group

of dampers is designed to tune to frequency ω_s , then m_d and k_d denote the mass and the stiffness coefficient of selected damper of this group, respectively.

Assuming that the mass ratio μ is known, damper frequency and stiffness coefficient k_d can be determined from above formulae.

If excitation forces acting on structure have a random character and can be treated as the white-noise excitation, the optimal value of non-dimensional damping coefficient is determined from formula:

$$\gamma_{opt} = \sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}. \quad (3)$$

Value of damping coefficient c_d can be calculated In the following form:

$$c_d = 2\gamma_{opt}\omega_d m_d. \quad (4)$$

Using above formulae the parameters of MTMD were determined.

3. Equation of motion

Equation of motion of the system shown on Fig.1 can be described in the following form:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}(t) + \tilde{\mathbf{C}}\dot{\mathbf{q}}(t) + \tilde{\mathbf{K}}\mathbf{q}(t) = \mathbf{P}(t) \quad (5)$$

where:

$\tilde{\mathbf{M}}, \tilde{\mathbf{C}}, \tilde{\mathbf{K}}$ the global matrices of mass, damping and stiffness of the system,

$\mathbf{q}(t)$ - vector of displacement of the system,

$\mathbf{q}(t) = \text{col}(\mathbf{y}(t), \mathbf{x}(t))$,

$\mathbf{y}(t)$ - horizontal displacements of frame,

$\mathbf{x}(t)$ - horizontal displacements of dampers,

$\mathbf{P}(t)$ - vector of excitation forces, $\mathbf{P}(t) = \text{col}(\mathbf{P}(t), \mathbf{0})$.

Additionally, it is assumed that the damping matrix of structure has the form as follows: $\mathbf{C} = \alpha\mathbf{M} + \kappa\mathbf{K}$.

The matrix $\tilde{\mathbf{M}}$ has the form shown below (see Fig.1):

$$\tilde{\mathbf{M}} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{m} \end{bmatrix}, \quad (6)$$

where:

$$\mathbf{M} = \text{diag}[M_1, M_2, M_3, \dots, M_N],$$

$$\mathbf{m} = \text{diag}[m_{11}, m_{12}, \dots, m_{1K}, m_{21}, m_{22}, \dots, \dots, m_{2K}, m_{31}, m_{32}, \dots, m_{3K}, \dots, \dots, m_{1N}, m_{2N}, \dots, m_{NK}]$$

In above formulae, the symbols M_i , m_{ij} denote number of i -th floor mass and mass damper of the j -th group located on the i -th floor. The matrix $\tilde{\mathbf{K}}$ can be shown in the block form:

$$\tilde{\mathbf{K}} = \begin{bmatrix} \mathbf{K} + \mathbf{k}_1 & \mathbf{k}^* \\ \mathbf{k}^{*T} & \mathbf{k} \end{bmatrix},$$

where \mathbf{K} is the stiffness matrix of the structure

$$\mathbf{K} = \begin{bmatrix} K_1 + K_2 & -K_2 & 0 & 0 \\ -K_2 & K_2 + K_3 & -K_3 & 0 \\ 0 & -K_3 & \dots & -K_N \\ 0 & 0 & -K_N & K_N \end{bmatrix}.$$

The blocks $\mathbf{k}_1, \mathbf{k}^*$ have the following form:

$$\mathbf{k}_1 = \text{diag}[k_{11} + k_{12} + \dots + k_{1K}, k_{21} + k_{22} + \dots + k_{2K}, k_{31} + k_{32} + \dots + k_{3K}, \dots, k_{N1} + k_{N2} + \dots, \dots + k_{NK}]$$

$$\mathbf{k}^* = \begin{bmatrix} -k_{11} & -k_{12} & \dots & -k_{1K} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{21} & -k_{22} & \dots & k_{2K} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{31} & -k_{32} & \dots & -k_{3K} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_{N1} & -k_{N2} & \dots & -k_{NK} \end{bmatrix}.$$

The symbol k_{ij} denotes the stiffness coefficient of damper of the j -th group which is located on the i -th floor (see Fig.2). The block \mathbf{k} of the matrix $\tilde{\mathbf{K}}$ is the diagonal matrix in the following form:

$$\mathbf{k} = \text{diag}[k_{11}, k_{12}, \dots, k_{1K}, k_{21}, k_{22}, \dots, \dots, k_{2K}, k_{31}, k_{32}, \dots, k_{3K}, \dots, \dots, k_{1N}, k_{2N}, \dots, k_{NK}]$$

The form of matrix $\tilde{\mathbf{C}}$ is parallel as the matrix $\tilde{\mathbf{K}}$. Particular blocks of this matrix are defined below:

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C} + \mathbf{c}_1 & \mathbf{c}^* \\ \mathbf{c}^{*T} & \mathbf{c} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} C_1 + C_2 & -C_2 & 0 & 0 \\ -C_2 & C_2 + C_3 & -C_3 & 0 \\ 0 & -C_3 & \dots & -C_N \\ 0 & 0 & -C_N & C_N \end{bmatrix},$$

$$\mathbf{c}_1 = \text{diag}[c_{11} + c_{12} + \dots + c_{1K}, c_{21} + c_{22} + \dots + c_{2K}, c_{31} + c_{32} + \dots + c_{3K}, \dots, c_{N1} + c_{N2} + \dots + c_{NK}]$$

$$\mathbf{c}^* = \begin{bmatrix} -c_{11} & -c_{12} & \dots & -c_{1K} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_{21} & -c_{22} & \dots & c_{2K} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{31} & -c_{32} & \dots & -c_{3K} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_{N1} & -c_{N2} & \dots & -c_{NK} \end{bmatrix}$$

In above formulae the symbol c_{ij} denotes the damping coefficient of damper of the j -th group which is located on the i -th floor (see Fig.2). The block \mathbf{c} of the matrix $\tilde{\mathbf{C}}$ is the diagonal matrix in the following form:

$$\mathbf{c} = \text{diag}[c_{11}, c_{12}, \dots, c_{1K}, c_{21}, c_{22}, \dots, \dots, c_{2K}, c_{31}, c_{32}, \dots, c_{3K}, \dots, \dots, c_{1N}, c_{2N}, \dots, c_{NK}]$$

The frame shown in Fig.1 is the model of the building structure. The mass of the structure is concentrated on the level of building floors and the beams of frame are infinite stiff. Horizontal displacements of floors are the dynamic degrees of freedom. The fluctuations of wind velocity forces are load the frame and the forces are applied on the level of building floors.

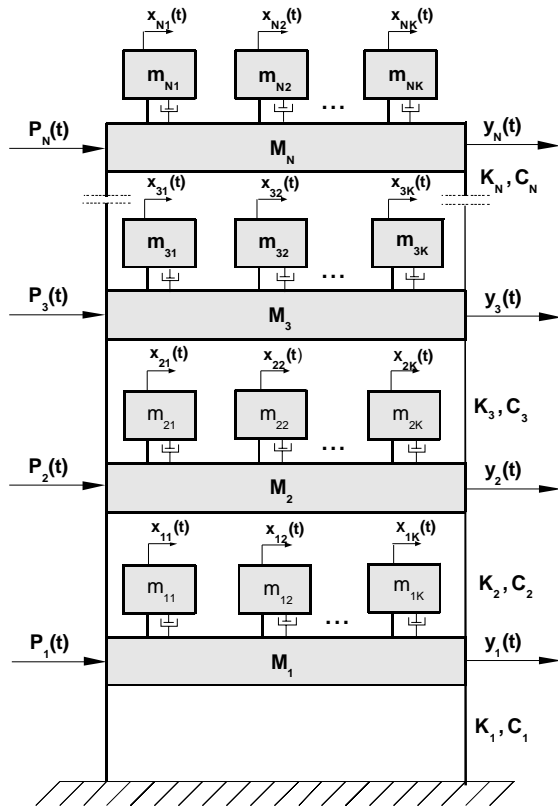


Fig 1 The system with MTMD

Taking into account that the matrices of mass, stiffness and damping have the form (6), the equation of motion (5) can be rewritten in the following form:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + (\mathbf{C} + \mathbf{c}_1)\dot{\mathbf{y}}(t) - \mathbf{c}^{*T}\dot{\mathbf{x}}(t) + (\mathbf{K} + \mathbf{k}_1)\mathbf{y}(t) - \mathbf{k}^{*T}\mathbf{x}(t) = \mathbf{P}(t) \quad (7a)$$

$$\mathbf{m}\ddot{\mathbf{x}}(t) + \mathbf{c}^*\dot{\mathbf{y}}(t) + \mathbf{c}\dot{\mathbf{x}}(t) + \mathbf{k}^*\mathbf{y}(t) + \mathbf{k}\mathbf{x}(t) = \mathbf{0} \quad (7b)$$

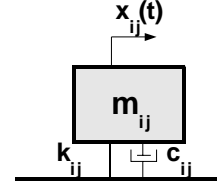


Fig.2 Diagram of damper

4. Modeling of wind load

It is assumed that load is the random, stationary process. Thus, the wind pressure in the arbitrary point of structure can be described in the following form:

$$P_i(t) = C_A A \rho U_i X^2 u_i(t), \quad (8)$$

where:

C_A - the aerodynamic drag coefficient,

A - the wind-exposed area,

ρ - air density.

The admittance function X describes influence of building on forces of wind pressure and there is always that $X \leq 1$. According to Holmes [13] the admittance function is connected with correlation coefficient Φ , which is used to determine matrix elements of the spectral density function. Determining of the admittance function is troublesome. Moreover, in many cases this value is unknown. Therefore, often $X = 1$ is assumed in calculations.

For multi-degree of freedom systems the correlation matrix of the fluctuations of wind velocity is formulated as:

$$\mathbf{R}_u(\tau) = E(\mathbf{u}, \mathbf{u}^T), \quad (9)$$

where $\mathbf{u} = \text{col}\{u_i(t)\}$ is the vector of the fluctuations of wind velocity.

Using the Fourier transform it obtains the following expression of the spectral density function:

$$\mathbf{S}_u(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{R}_u(\tau) e^{-i\lambda\tau} d\tau \quad (10)$$

In this paper is used the spectral density function proposed by Davenport [11-13]. Elements of this matrix are calculated from formula:

$$S_{lk}^u = \sqrt{S_{ll}^u(\lambda) S_{kk}^u(\lambda)} e^{-\Phi} = \sqrt{S_u(\lambda, z_l) S_u(\lambda, z_k)} e^{-\Phi} \quad (11)$$

$S_u(\lambda, z_l)$ and $S_u(\lambda, z_k)$ are the elements taken from the main diagonal of the matrix $\mathbf{S}_u(\lambda)$. They are calculated with help of the spectral density function for particular stories. The symbol Φ denotes the correlation coefficient, which takes into consideration the spatial correlation of the fluctuations of wind velocity. According to Chmielewski [9] this coefficient is calculated by homogeneous random field and it can be determined from formula:

$$\Phi = \frac{2\lambda C_z |z_l - z_k|}{U(z_l) + U(z_k)} \quad (12)$$

where λ is the force frequency, whereas C_z is the empirical constant. The expression $\frac{U(z_l) + U(z_k)}{2}$ is the mean wind velocity on the level of considered story. If it assumed that the fluctuations of wind velocity are totally correlated then $e^{-\Phi} = 1$. Whereas, the correlation is disregarded then the matrix $\mathbf{S}_u(\lambda)$ is the diagonal matrix.

The correlation matrix of forces excited of wind pressure can be written in the following form:

$$\mathbf{R}_p(\tau) = E(\mathbf{P}, \mathbf{P}^T), \quad (13)$$

where $\mathbf{P} = \text{col}\{P_i(t)\}$.

Substituting (8) and (9) to (13) it obtains:

$$\mathbf{R}_p(\tau) = (C_A A \rho U)^2 X^2 \mathbf{R}_u(\tau). \quad (14)$$

Using the Fourier transform it obtains from equation (14):

$$\begin{aligned} \mathbf{S}_p(\lambda) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{R}_p(\tau) e^{-i\lambda\tau} d\tau = \\ (C_A A \rho U)^2 X^2 &\frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{R}_u(\tau) e^{-i\lambda\tau} d\tau = \\ (C_A A \rho U)^2 X^2 &\mathbf{S}_u(\lambda). \end{aligned} \quad (15)$$

This is the relationship between the spectral density matrix of load excited by wind pressure and the spectral density matrix of fluctuations of wind velocity.

Using above formulae the excitation forces acted on the structure are determined.

5. Solution to the equation of motion

Solution to the equation of motion (5), with initial conditions $t=0$, $\dot{\mathbf{q}}(t)=0$, $\mathbf{q}(t)=0$, can be written in the following form:

$$\mathbf{q}(t) = \int_0^t \mathbf{h}(t-\tau) \mathbf{P}(\tau) d\tau, \quad (16)$$

where:

$\mathbf{h}(t-\tau)$ - the matrix of impulse transfer function,

$\mathbf{P}(\tau)$ - the vector of excitation forces.

If random load acted on the structure has stationary character then also dynamic response of the system is stationary process. Thus, the correlation matrix of responses of the structure can be written as:

$$\mathbf{R}_q(t_1, t_2) = E(\mathbf{q}(t_1), \mathbf{q}^T(t_2)). \quad (17)$$

Substituting (16) to (17), it obtains:

$$\begin{aligned} \mathbf{R}_q(t_1, t_2) &= \\ E \left(\int_0^{t_1} \int_0^{t_2} \mathbf{h}(t_1 - \tau_1) E(\mathbf{P}(\tau_1) \mathbf{P}^T(\tau_2)) \mathbf{h}^T(t_2 - \tau_2) d\tau_1 d\tau_2 \right) \end{aligned} \quad (18)$$

Taken into account that

$$\mathbf{R}_p(\tau) = \int_{-\infty}^{+\infty} \mathbf{S}_p(\lambda) e^{i\lambda\tau} d\lambda, \quad (19)$$

$$E(\mathbf{P}(t), \mathbf{P}(t-\tau)) = \int_{-\infty}^{+\infty} \mathbf{S}_p(\lambda) e^{i\lambda\tau} d\lambda, \quad (20)$$

the equation (18) can be rewritten in the following form:

$$\mathbf{R}_q(\tau) = \int_{-\infty}^{+\infty} \mathbf{H}(\lambda) \mathbf{S}_p(\lambda) \overline{\mathbf{H}}^T(\lambda) e^{i\lambda\tau} d\lambda, \quad (21)$$

where

$$\tilde{\mathbf{H}} = (\tilde{\mathbf{K}} - \lambda^2 \tilde{\mathbf{M}} - i\lambda \tilde{\mathbf{C}}), \quad (22)$$

and $\overline{\mathbf{H}}$ is the conjugate matrix. Substituting $\tau=0$ to equation (21) it obtains the correlation matrix of displacements $\mathbf{R}_q(0)$ and on this basis the root mean square of displacements can be determined

$$\mathbf{R}_q(0) = \int_{-\infty}^{+\infty} \mathbf{S}_q(\lambda) d\lambda, \quad (23)$$

where

$$\mathbf{S}_q(\lambda) = \mathbf{H}(\lambda) \mathbf{S}_p \overline{\mathbf{H}}^T(\lambda). \quad (24)$$

The integral in the formula (23) is calculated numerically.

6. Results of example calculations

In this section the results of dynamic analysis of the structure with installed MTMD are discussed. Additionally, for comparison it has been made analysis for the system with one tuned mass damper (TMD), which is tuned to the first vibration mode of structure. TMD was located on the top floor.

Parameters of building were calculated on the basis of paper [3] and they are given in table 1.

story	mass [kg]	stiffness [N/m]
1	2,83 x10 ⁵	3,31 x10 ⁸
2 - 4	2,76 x10 ⁵	1,06 x10 ⁹
4 - 7	2,76 x10 ⁵	6,79 x10 ⁸
8 - 10	2,76 x10 ⁵	6,79 x10 ⁸
11 - 13	2,76 x10 ⁵	5,84 x10 ⁸
14 - 16	2,76 x10 ⁵	3,86 x10 ⁸
17 - 19	2,76 x10 ⁵	3,47 x10 ⁸
20	2,92 x10 ⁵	2,29 x10 ⁸

Table 1. Parameters of structure

Parameters of dampers were designed from formulae (1)-(4), by assuming, that it aims to tune three vibration mode of the structure (Fig.3). In this case it has been assumed that three groups of dampers are installed on the structure. One damper was designed in each group of dampers. Nondimensional damping coefficients of first and second vibration mode were assumed as 1% of critical damping.

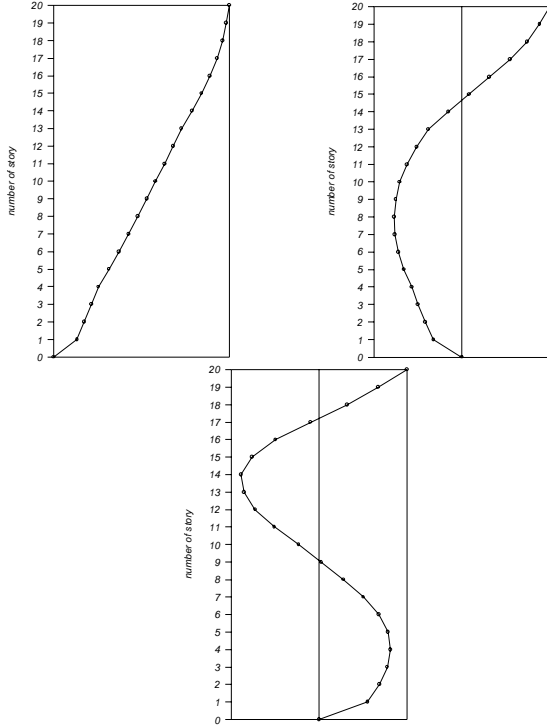


Fig. 3 Three vibration mode of the structure

MTMD were located on the top floor. Parameters of dampers and location on the structures are shown in table 2. Calculations were made by assuming that the mass of TMD is equal total mass of MTMD.

number of mode/placement	mass [kg]	stiffness [N/m]
TMD		
1/20	36214	472468
MTMD		
1/20	18107	238870
2/20	7956	722685
3/20	8550	2182386

Table 2. Parameters of dampers

Calculating the spectral density function it used the function proposed by Davenport [11]-[13]

$$S_u(n) = \frac{4u_*^2 f^2(n)}{n[1 + f^2(n)]^{4/3}}, \quad (25)$$

where

$$f(n) = \frac{1200n}{U(10)}, \quad (26)$$

and n denotes frequency in Hz.

The mean wind velocity i -th floor was calculated from formula:

$$U(z) = 2,5u_* \ln\left(\frac{z}{z_0}\right) \quad (27)$$

where

$$u_* = U(10)\sqrt{k}, \quad (28)$$

$U(10)$ - the mean wind velocity on the altitude 10m,

k - the coefficient depended on type of area,

z_0 - the roughness length,

z - altitude.

It assumed the following parameters to calculations:

$$\rho = 1,226 \frac{kg}{m^3}, \quad U(10) = 30 \frac{m}{s}, \quad C_A = 10, \quad z_0 = 0,3,$$

$$k = 12e^{-3}.$$

The dynamic response of the structure is stationary and ergodic process. In this case the root mean square of displacement q_i and acceleration \ddot{q}_i are calculated from formulae:

$$\sigma_q^2 = \int_{-\infty}^{+\infty} S_q(\lambda) d\lambda = \int_{-\infty}^{+\infty} S_{ii}^q(\lambda) d\lambda, \quad (29)$$

$$\sigma_{\ddot{q}}^2 = \int_{-\infty}^{+\infty} \lambda^4 S_q(\lambda) d\lambda = \int_{-\infty}^{+\infty} \lambda^4 S_{ii}^q(\lambda) d\lambda, \quad (30)$$

where S_{ii}^q is the diagonal element of the matrix $S_q(\lambda)$. Using above formulae the analysis of the structure without dampers, with installed conventional TMD and with installed MTMD were made. Results of these analysis is shown on Figs 4 and 5. On Fig.4 the root mean square of structure displacements has been shown. It has been observed that displacements reduction with installed MTMD is a little smaller than in case where TMD was installed on the structure. In comparison to the system without installed dampers, the maximum reduction of root mean square of structure displacements (top floor) is 30% for TMD and 25% for MTMD, respectively. Taken into account the total sum of floors displacements, TMD reduces the root mean square about 30% and MTMD about 25%. As it was considered previously, the results concerned accelerations were elaborated (Fig. 5). It has been observed that using MTMD accelerations reduction is bigger only below eleventh floor than using TMD. Above eleventh floor it has been observed lesser accelerations reduction in comparison with TMD. The maximum root mean square of acceleration (top floor) are almost equal. The total sum of root mean square of acceleration is 38% for MTMD and 40% for TMD comparing with the structure without installed dampers.

7. Concluding remarks

In this paper the analysis of vibration building construction with installed MTMD tuned three vibration mode has been studied. The root mean square of displacement and accelerations of structure with MTMD were determined.

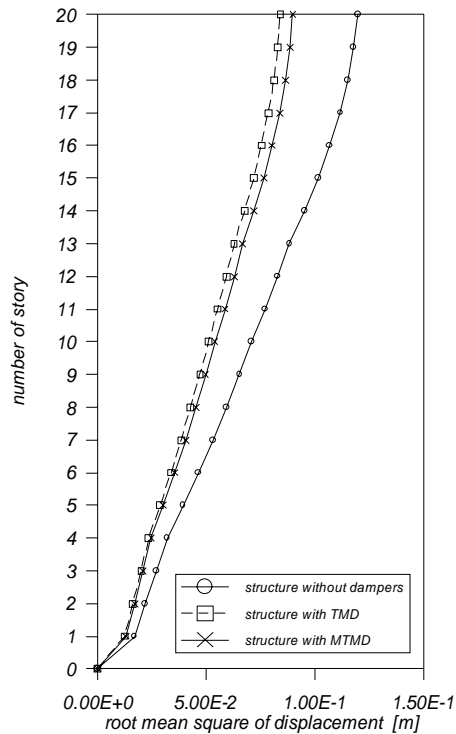


Fig.4 Root mean square of displacement

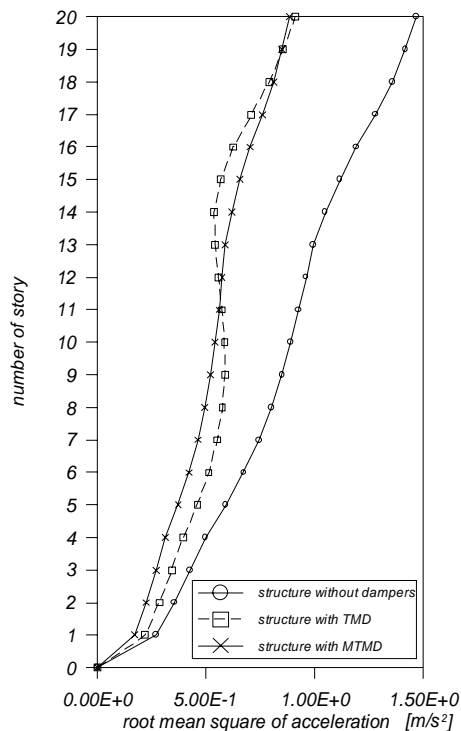


Fig.5 Root mean square of acceleration

These calculations were compared to the root mean square of displacement and acceleration of the same structure with installed conventional TMD. On the basis of the results of calculations the following conclusions were formulated:

- MTMD reduce to the similar extent displacements and accelerations of structure ,
- MTMD reduce to the large extent accelerations on lower floors of structure.

These are the first results of calculations and therefore below conclusions can not be treated as definitive. The effectiveness of MTMD is similar as the effectiveness of TMD. However, MTMD have series of advantages which can be followed as:

- MTMD are smaller then conventional they occupy much smaller space for installation,
- MTMD are necessary accelerations reduction of structures, besides this reduction is on lower floors when comparing with TMD.

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