

Identification of parameters of the fractional rheological model of viscoelastic dampers

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Abstract

An identification method for determination of parameters of the rheological model of damper made of viscoelastic material is presented in this paper. The rheological model of damper has four parameters and the model equation of motion contains derivatives of the fractional order. The identification procedure has two main parts. Results of dynamical experiments are approximated using the trigonometric function in the first part of the procedure while the model parameters are determined in the second part of the procedure as the solution to an optimization problem. The particle swarm optimization method is used to solve the optimization problem. Efficiency and accuracy of the proposed method are proof on an example where the parameters of the rheological model are determined on the basis of artificially generated experimental data with measured noises.

Keywords: viscoelastic dampers, fractional rheological model, identification procedure

1. Introduction

Fractional rheological models of viscoelastic (VE) dampers are becoming more and more popular. The reason is their ability to correctly describe the behavior of VE dampers using a small number of parameters. A single equation is enough to describe the VE damper dynamics. An important problem, connected with the fractional models, is the estimation of model parameters using experimental data. The process of parameter identification is an inverse problem which can be ill conditioned. The identification procedures for the three parameters fractional Kelvin-Voigt model and the fractional Maxwell model are proposed in [1]. The problem of parameters identification of rheological models with fractional derivatives is also discussed by Pritz in [2].

A new method for identification of the parameters of the fractional model of VE dampers with four parameters is presented in this paper. The results of static and dynamical test are used to identify the parameters of a damper model. The identification procedure comprises two main steps. The experimental results are approximated by a simple harmonic function in the time domain in the first step while model parameters are determined in the second stage of the identification procedure. The validity, accuracy and effectiveness of the procedures have been tested using artificial experimental data.

2. Description of the rheological model and a steady state vibration of the model

The equation of motion of the considered rheological model is in the following form:

$$u(t) + \tau^\alpha D_t^\alpha u(t) = k_0 q(t) + k_\infty \tau^\alpha D_t^\alpha q(t) \quad (1)$$

where $u(t)$ denotes the dampers force, $q(t)$ is the dampers displacement, k_0 , k_∞ , τ and α are models parameters. Moreover, a symbol such as $D_t^\alpha q(t)$, is the Riemann-Liouville fractional derivative of the order α of consecutive function, here $q(t)$, with respect to time t (please, consult [3] for details concerning fractional derivatives).

Based on the results presented by Lion in [4], it can be demonstrated that this model fulfils the second law of thermodynamics for $0 \leq \alpha \leq 1$, $\tau > 0$ and $k_\infty > k_0 > 0$.

Equation (1) can be understood as the equation of motion of two mechanical models shown in Figures 1 and 2. These models consists of springs and springpot elements connected in parallel or in series. The springpot element can be seen as an interpolation between the spring ($\alpha = 0$) and the dashpot ($\alpha = 1$). The springpot element satisfies the following constitutive equation (see [1] for details):

$$u(t) = c D_t^\alpha q(t) . \quad (3)$$

The parameters of mechanical models are related to parameters of the considered fractional model in the following way:

$$k_0 = k_1 , \quad k_\infty = k_1 + k_2 , \quad \tau^\alpha = c_2 / k_2 , \quad (3)$$

$$k_0 = k_1 k_2 / (k_1 + k_2) , \quad k_\infty = k_1 , \quad \tau^\alpha = c_2 / (k_1 + k_2) , \quad (4)$$

for the first and the second mechanical model, respectively.

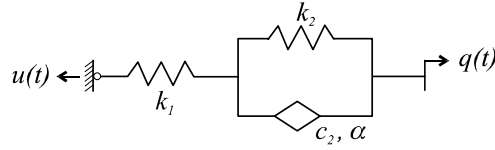


Figure 1. The first mechanical models of VE dampers

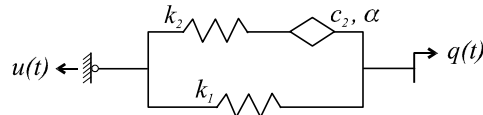


Figure 2. The second mechanical models of VE dampers

If the damper executes harmonic oscillations then the damper's steady state vibration is described by

$$u(t) = u_c \cos \lambda t + u_s \sin \lambda t , \quad q(t) = q_c \cos \lambda t + q_s \sin \lambda t , \quad (5)$$

and the parameters shown above fulfil the following relationships

$$u_c = z_1 q_c + z_2 q_s , \quad u_s = -z_2 q_c + z_1 q_s , \quad (6)$$

where

$$z_1 = \frac{k_0 + (k_0 + k_\infty)(\tau\lambda)^\alpha \cos(\alpha\pi / 2) + k_\infty (\tau\lambda)^{2\alpha}}{1 + 2(\tau\lambda)^\alpha \cos(\alpha\pi / 2) + (\tau\lambda)^{2\alpha}} , \quad (7)$$

$$z_2 = \frac{(k_\infty - k_0)(\tau\lambda)^\alpha \sin(\alpha\pi / 2)}{1 + 2(\tau\lambda)^\alpha \cos(\alpha\pi / 2) + (\tau\lambda)^{2\alpha}} . \quad (8)$$

3. Description of identification method

The identification procedure consists of two main steps. In the first step the experimental results are approximated by a simple harmonic function in the time domain while the model parameters are determined in the second stage of the identification procedure.

In the first step, experimentally measured displacements $q_e(t)$ of the damper are approximated using the function:

$$\tilde{q}(t) = \tilde{q}_c \cos \lambda t + \tilde{q}_s \sin \lambda t , \quad (9)$$

The least-square method is used to determine parameters \tilde{q}_c and \tilde{q}_s of function (9). This method requires minimization of the following functional:

$$J_1(\tilde{q}_c, \tilde{q}_s) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [q_e(t) - \tilde{q}(t)]^2 dt , \quad (10)$$

where the symbols t_1 and t_2 denote the beginning and the end of the time range in which the damper's displacements were measured. Part of the measuring results relating to a steady state vibration is used as data in this step. From the stationary conditions of the functional (10), the following system of equations is obtained:

$$I_{cc}\tilde{q}_c + I_{sc}\tilde{q}_s = I_{cq} , \quad I_{sc}\tilde{q}_c + I_{ss}\tilde{q}_s = I_{sq} \quad (11)$$

from which the parameters \tilde{q}_c and \tilde{q}_s are obtained and where:

$$I_{cc} = \int_{t_1}^{t_2} \cos^2 \lambda t dt , \quad I_{ss} = \int_{t_1}^{t_2} \sin^2 \lambda t dt , \quad I_{cs} = I_{sc} = \int_{t_1}^{t_2} \sin \lambda t \cos \lambda t dt \quad (12)$$

$$I_{cq} = \int_{t_1}^{t_2} q_e(t) \cos \lambda t dt , \quad I_{sq} = \int_{t_1}^{t_2} q_e(t) \sin \lambda t dt . \quad (13)$$

Similarly, the experimentally measured dampers force $u_e(t)$ is approximated by

$$\tilde{u}(t) = \tilde{u}_c \cos \lambda t + \tilde{u}_s \sin \lambda t . \quad (14)$$

Proceeding to a description of the second step of identification method, it is assumed that a set of results of the above-described first step of procedure given by $\tilde{u}_i(t)$, \tilde{u}_{ci} , \tilde{u}_{si} , $\tilde{q}_i(t)$, \tilde{q}_{ci} and \tilde{q}_{si} and relating to the different excitation frequencies λ_i ($i = 1, 2, \dots, n$) is known. If the considered rheological model is able to correctly simulate the VE damper behavior then the relationships (6) must approximately be fulfilled by the above-mentioned results of the first step identification procedure, i.e.:

$$\tilde{u}_{ci} = \tilde{z}_{1i}\tilde{q}_{ci} + \tilde{z}_{2i}\tilde{q}_{si} , \quad \tilde{u}_{si} = -\tilde{z}_{2i}\tilde{q}_{ci} + \tilde{z}_{1i}\tilde{q}_{si} , \quad i = 1, 2, \dots, n . \quad (15)$$

Solving Equations (15) with respect to \tilde{z}_{1i} and \tilde{z}_{2i} the following is obtained:

$$\tilde{z}_{1i} = \frac{\tilde{u}_{ci}\tilde{q}_{ci} + \tilde{u}_{si}\tilde{q}_{si}}{\tilde{q}_{ci}^2 + \tilde{q}_{si}^2}, \quad \tilde{z}_{2i} = \frac{\tilde{u}_{ci}\tilde{q}_{si} - \tilde{u}_{si}\tilde{q}_{ci}}{\tilde{q}_{ci}^2 + \tilde{q}_{si}^2}. \quad (16)$$

If the rheological model perfectly fits the experimental data then $z_{1i} - \tilde{z}_{1i} = 0$ and $z_{2i} - \tilde{z}_{2i} = 0$ for $i=1,2,\dots,n$, where $z_{1i} = z_1(\lambda_i)$, $z_{2i} = z_2(\lambda_i)$ are calculated using formulas (7) and (8). In practice some differences usually exist and parameters, k_0 , k_∞ , τ and α of the rheological model are determined as the solution of the appropriately defined optimization problem. However, here it is assumed that the parameter k_0 is known and determined previously using the experimental data taken from the static tests.

In the paper the optimization problem mentioned above is formulated as follows.

Find the values of k_∞ , τ and α which minimize the functional

$$J(k_\infty, \tau, \alpha) = \sum_{i=1}^n \left\{ [z_{1i}(k_\infty, \tau, \alpha) - \tilde{z}_{1i}]^2 + [z_{2i}(k_\infty, \tau, \alpha) - \tilde{z}_{2i}]^2 \right\}, \quad (17)$$

and fulfil the following constraints:

$$0 < \alpha \leq 1, \quad \tau > 0, \quad k_\infty > k_0 > 0. \quad (18)$$

The above optimization problem is solved with the help of the particle swarm optimization method described, for example in [5] and briefly in the following Section.

4. Description of the adopted version of the particle swarm optimization method

The particle swarm optimization (PSO) method is a population based optimization technique inspired by the social behavior of animals. The populations consist of possible solutions (referred to as particles) and the search for optimal solutions is performed by updating the subsequent positions of particles. Each particle explores the problem space being drawn to current optimal solutions. Moreover, each particle keeps its best values of functional (17) achieved so far (along with the associated solution $\mathbf{p}^{(j)}(k) = \text{col}(p_1^{(j)}(k), p_2^{(j)}(k), p_3^{(j)}(k))$, where k is the number of the current time instance, the superscript j is the number of the current particle; ($j = 1, 2, \dots, m$)) and the best fitness and corresponding solution achieved in the particle's neighborhood $\mathbf{p}^{(b)}(k) = \text{col}(p_1^{(b)}(k), p_2^{(b)}(k), p_3^{(b)}(k))$. It was shown that using global neighborhood (all particles are fully aware of other particles' fitness) minimizes the median number of iterations needed to converge. On the other hand, the neighborhood of size 2 gives the highest resistance to local minima.

At each time instances k of the PSO, the velocities of the particles are changed (accelerated) towards the $\mathbf{p}^{(j)}(k)$ and the $\mathbf{p}^{(b)}(k)$ and the particles are moved to new positions according to the following formulas:

$$v_i^{(j)}(k+1) = w(k+1)v_i^{(j)}(k) + c_1 r_{1i}^{(j)}(k+1) [p_i^{(j)}(k) - x_i^{(j)}(k)] / \Delta t + c_2 r_{2i}^{(j)}(k+1) [p_i^{(g)}(k) - x_i^{(j)}(k)] / \Delta t, \quad (19)$$

$$x_i^{(j)}(k+1) = x_i^{(j)}(k) + v_i^{(j)}(k+1)\Delta t. \quad (20)$$

where $\Delta t = 1$, $v_i^{(j)}(k)$ and $x_i^{(j)}(k)$ are the i -th element of the velocity and the position vectors of the j -th particle, respectively; $w(k+1)$ is the inertia factor providing balance between exploration and exploitation, c_1 is the individuality constant, and c_2 is the sociality constant. To speed up convergence, the inertia weight was linearly reduced from $w_{\max} = 0.9$ to $w_{\min} = 0.1$. In our experiments we have used $m = 10$ particles, a maximum number of iterations $i_{\max} = 400$ and $c_1 = c_2 = 2.0$. A size 4 neighborhood was used as a tradeoff between fast convergence and resistance to local minima. Moreover, $r_{1i}^{(j)}$ and $r_{2i}^{(j)}$ are random numbers taken from the range from 0 to 1. More information on the selection of the algorithm parameters, constraints handling and selecting the starting vectors $\mathbf{x}^{(j)}(0)$ and $\mathbf{v}^{(j)}(0)$ can be found in [5].

2. Results of demonstration applications of identification method

A typical calculation is performed using the artificially generated data. At the beginning, the artificial data without noises for the second mechanical model are calculated using formulas (3) and (6) and assuming that: $n = 14$, $k_1 = 600.0$ kN/m, $k_2 = 400.0$ kN/m, $c = 150.0$ kNs/m, $q_{si} = 0.01$ m, $q_{ci} = 0.005$ m and $\alpha = 0.6$. The chosen values of the excitation frequency are taken from the range 0.5–13.5 Hz with the frequency increment $\Delta\lambda = 1.0$ Hz. After applying the identification procedure and assuming that $k_0 = 240.0$ kN/m is known from the static test, the following results, very close to the exact ones, are obtained: $k_{1,iden} = 601.5$ kN/m, $k_{2,iden} = 399.3$ kN/m, $c_{iden} = 149.9$ kNs/m, $\alpha_{iden} = 0.5968$.

Moreover, the random noises are added to the artificial data using the formulas: $\hat{u}_{ci} = (1 + \tilde{r}_{1i}\varepsilon)\tilde{u}_{ci}$, $\hat{u}_{si} = (1 + \tilde{r}_{2i}\varepsilon)\tilde{u}_{si}$, $\hat{q}_{ci} = (1 + \tilde{r}_{3i}\varepsilon)\tilde{q}_{ci}$, $\hat{q}_{si} = (1 + \tilde{r}_{4i}\varepsilon)\tilde{q}_{si}$ (20) where ε is the noise level, \tilde{r}_{1i} , \tilde{r}_{2i} , \tilde{r}_{3i} and \tilde{r}_{4i} are random numbers taken from the range from 0 to 1.

The calculation is made for $\varepsilon = 0.02$. After several runs of the identification procedure the following median solution is obtained: $k_{1,iden} = 608.7$ kN/m, $k_{2,iden} = 396.2$ kN/m, $c_{iden} = 151.6$ kNs/m and $\alpha_{iden} = 0.5866$. It is easy to find that the accuracy of the obtained values of model parameters is of the order of noises introduced.

A comparison of the storage modulus resulting from the artificially generated data with noises (small crosses) and from the rheological model (solid curve) is presented in Fig. 3. It is evident that both approaches are in good agreement.

3. Concluding remarks

The proposed identification method can be effectively used to determine parameters of the rheological model with the fractional derivatives. The mentioned rheological model

can be used to modelling the dynamic behaviour of VE dampers. The identification problem is reduced to the nonlinear optimization problem which is solved by means of the particle swarm optimization method. Based on the demonstration calculation, it was found that the proposed method is not sensitive to any noises introduced during the measurements.

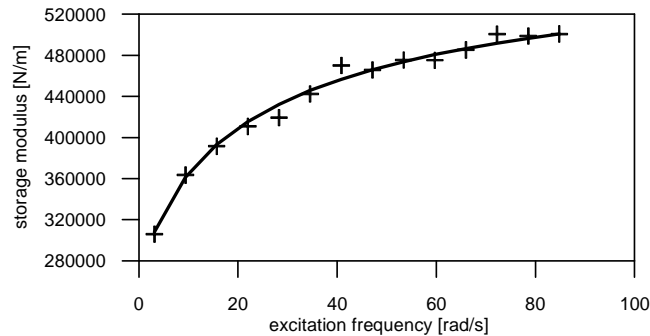


Figure 3. Comparison of the storage modulus

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Identyfikacja parametrów ułamkowego modelu reologicznego modelującego zachowanie tłumika lepkosprężystego

W pracy omawia się metodę identyfikacji parametrów modelu reologicznego tłumika wykonanego z materiału o właściwościach lepkosprężystych. Model reologiczny tłumika ma cztery parametry, a równanie ruchu modelu zawiera pochodne ułamkowego rzędu. Procedura identyfikacji składa się z dwóch części. W pierwszej części aproksymuje się wyniki badań doświadczalnych za pomocą funkcji trygonometrycznych, a w drugiej części wyznacza się parametry modelu jako rozwiązanie pewnego zadania optymalizacji. Do rozwiązania zadania optymalizacji użyto metody roju cząstek. Efektywność i dokładność zaproponowanej metody identyfikacji wykazano wyznaczając parametry ułamkowego modelu reologicznego na podstawie sztucznie wygenerowanych danych doświadczalnych z szumami pomiarowymi.