

Description of Dynamics Behaviour of Frame Structures with Fractional Viscoelastic Dampers

Roman LEWANDOWSKI*, Zdzisław PAWLAK*

*Poznan University of Technology, ul. Piotrowo 5, 60-965 Poznan, Poland
E-mails: roman.lewandowski@put.poznan.pl; zdzislaw.pawlak@put.poznan.pl

Abstract. In this paper, frame structures with viscoelastic dampers mounted on them are considered. Viscoelastic (VE) dampers are modelled using two, three-parameter, fractional rheological models. The structures are treated as elastic linear systems. The equation of motion of the whole system (structure with dampers) is written in terms of state-space variables. The resulting matrix equation of motion is the fractional differential equation. Moreover, results of typical calculations are presented.

1 Introduction

In civil engineering, VE dampers are successfully used to reduce vibrations caused by winds and earthquakes. It was found that incorporation of VE dampers into a structure leads to a significant reduction of unwanted vibrations [1]. In the past, several rheological models were proposed to describe the dynamic behaviour of VE materials and dampers. Both the classical and the so-called fractional models of dampers are available. Descriptions of these models are given in [2 – 5].

In this paper, the frame structures with VE dampers mounted on it are considered. Viscoelastic dampers are modelled using three-parameter, fractional rheological models. The structures are treated as elastic linear systems. The equation of motion of the whole system (structure with dampers) is written in terms of state-space variables. The resulting matrix equation of motion is the fractional differential equation.

Moreover, the paper is dedicated to determination of the dynamics characteristics of the considered structures. The nonlinear eigenvalue problem is formulated, from which the dynamics characteristics of a system can be determined. The continuation method is used to solve the above-mentioned nonlinear eigenvalue problem.

The results of typical calculations are also presented and briefly discussed. In particular, the influence of the key parameter, which describes the order of fractional derivative, on the dynamic characteristics of frame with VE dampers is shown.

2 The equations of motion of frame with VE dampers

2.1 The rheological models of dampers

In this paper, two fractional rheological models, i.e., the fractional Kelvin model and the fractional Maxwell model (see Fig.1), are used to describe the dynamic behaviour of VE dampers. The considered models of a typical damper, i , have three parameters: stiffness $k_{d,i}$, damping factor $c_{d,i}$, and fractional parameter α_i , ($0 < \alpha_i \leq 1$).

The motion equation of the Kelvin model (see Fig. 1a) could be written in the form:

$$u_i = k_{d,i}q_{d,i} + c_{d,i}D_t^{\alpha_i}q_{d,i} , \quad (1)$$

where u_i is the damper force and $q_{d,i}$ is the relative damper displacement. Moreover, $D_t^{\alpha_i}(\bullet)$ denotes the Riemann-Liouville fractional derivative of the order α_i with respect to time, t . For a precise definition of the Riemann-Liouville fractional derivative, [6] may be consulted.

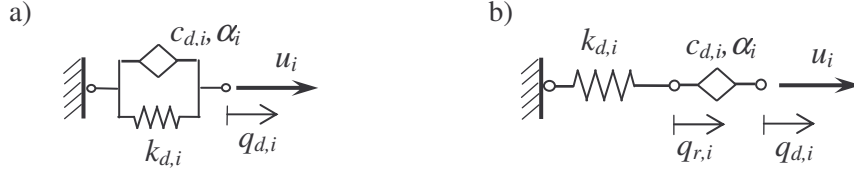


Figure 1: Rheological models of damper

The motion equations of the Maxwell model could be written using the so-called relative internal variable $q_{r,i}$ (compare Fig. 1b). The above-mentioned damper motion equations are:

$$u_i = c_{d,i} (D_t^{\alpha_i} q_{d,i} - D_t^{\alpha_i} q_{r,i}) , \quad u_i = k_{d,i} q_{r,i} . \quad (2)$$

The damper of which the behaviour is described by the Kelvin model or the Maxwell model will be shortly referred to as the Kelvin damper or the Maxwell damper, respectively.

More information concerning the fractional rheological models can be found in [5, 7]. The equation of motion of the classical Kelvin and Maxwell models could be obtained after introducing $\alpha = 1$ into Equations (1) and (2).

2.2 The equations of motion of structures expressed in physical coordinates

The frame with VE dampers is treated as the elastic linear system and their model is the shear frame shown in Fig. 2a. The equation of motion of such a structure can be written as follows:

$$\mathbf{M}_s \ddot{\mathbf{q}}_s(t) + \mathbf{C}_s \dot{\mathbf{q}}_s(t) + \mathbf{K}_s \mathbf{q}_s(t) = \mathbf{s}(t) + \mathbf{p}(t) , \quad (3)$$

where the symbols \mathbf{M}_s , \mathbf{C}_s and \mathbf{K}_s denote the mass, the damping and the stiffness ($n \times n$) matrices, respectively. Moreover, $\mathbf{q}_s(t) = \text{col}(q_{s,1}, \dots, q_{s,n})$ and $\mathbf{p}(t) = \text{col}(p_1, \dots, p_n)$ denote the vector of displacements of the structure and the vector of excitation forces, respectively. The $\mathbf{s}(t) = \text{col}(s_1, s_2, \dots, s_n)$ vector is the ($n \times 1$) vector of interaction forces between the frame and dampers (compare Fig. 2b).

First of all, the structure with one damper, denoted as the damper number i , which is mounted between two successive storeys j and $j+1$ (shown in Fig. 2a), is considered. If the Kelvin damper is considered, the force interaction vector $\mathbf{s}(t)$ could be written as

$$\mathbf{s}(t) \equiv \mathbf{s}_i(t) = \text{col}(0, \dots, s_j = u_i, s_{j+1} = -u_i, \dots, 0) = \mathbf{e}_i u_i(t) , \quad (4)$$

where $\mathbf{e}_i = \text{col}(0, \dots, e_j = 1, e_{j+1} = -1, \dots, 0)$ is the allocation vector of the i -th damper.

Taking into account that the relative damper displacement, written in terms of structure displacements, is

$$q_{d,i}(t) = q_{s,j+1}(t) - q_{s,j}(t) = -\mathbf{e}_i^T \mathbf{q}_s(t) , \quad (5)$$

the damper force and the vector of interactive forces could be written as follows:

$$u_i(t) = -k_{d,i} \mathbf{e}_i^T \mathbf{q}_s(t) - c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) , \quad (6)$$

$$\mathbf{s}_i(t) = -\mathbf{e}_i k_{d,i} \mathbf{e}_i^T \mathbf{q}_s(t) - \mathbf{e}_i c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) . \quad (7)$$

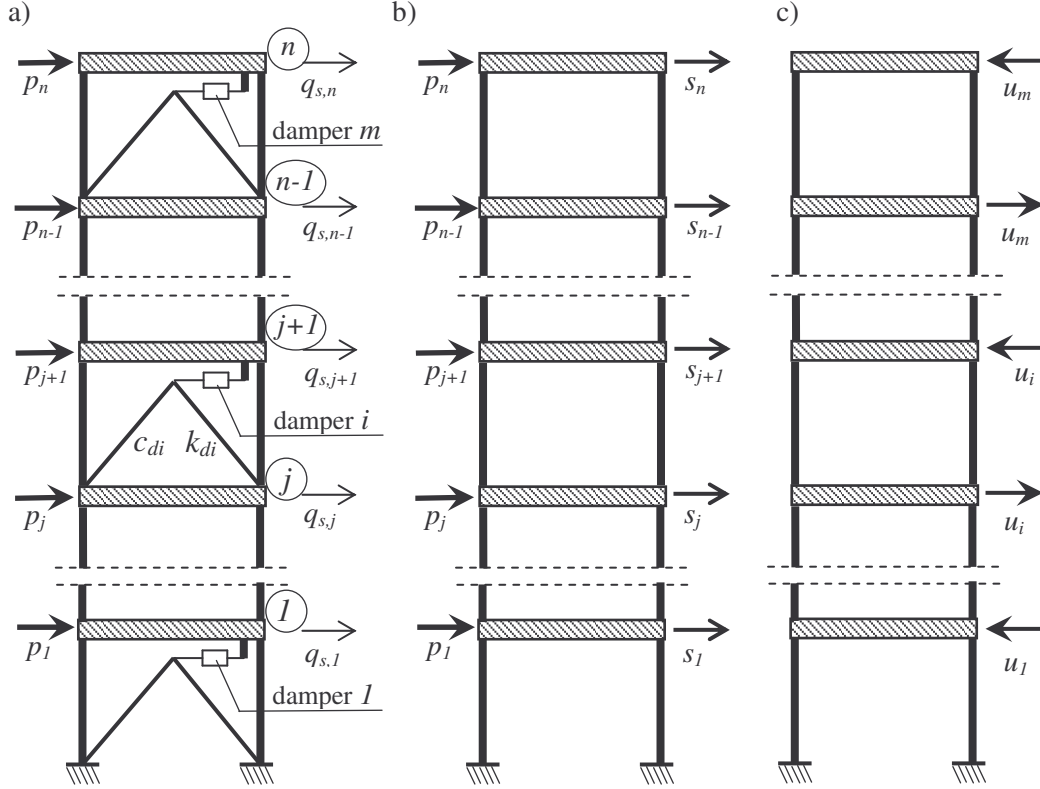


Figure 2: Diagram of a frame with VE dampers

If we have a structure with m dampers, the vector of interactive forces is given by:

$$\mathbf{s}(t) = \sum_{i=1}^m \mathbf{s}_i(t) = -\sum_{i=1}^m \mathbf{e}_i k_{d,i} \mathbf{e}_i^T \mathbf{q}_s(t) - \sum_{i=1}^m \mathbf{e}_i c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) , \quad (8)$$

and the equation of motion (3) could be rewritten in the form:

$$\mathbf{M}_s D_t^2 \mathbf{q}_s(t) + \mathbf{C}_s D_t^1 \mathbf{q}_s(t) + \sum_{i=1}^m \mathbf{e}_i c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) + (\mathbf{K}_s + \mathbf{K}_d) \mathbf{q}_s(t) = \mathbf{p}(t) , \quad (9)$$

where, in order to be consistent with the notation, a symbol such as $D_t^1(\bullet)$ is introduced to denote the first derivative with respect to time, and the matrix of dampers stiffness is

$$\mathbf{K}_d = \sum_{i=1}^m \mathbf{e}_i k_{d,i} \mathbf{e}_i^T . \quad (10)$$

Equation (9) is the matrix fractional differential equation which describes the dynamic behaviour of the considered frame with the Kelvin dampers. In this approach each damper can have its own values of parameters, different from others. The equation (9) is much simplified when all fractional parameters are equal, i.e., $\alpha_i = \alpha = const.$ In this case we can write:

$$\mathbf{M}_s D_t^2 \mathbf{q}_s(t) + \mathbf{C}_s D_t^1 \mathbf{q}_s(t) + \mathbf{C}_d D_t^\alpha \mathbf{q}_s(t) + (\mathbf{K}_s + \mathbf{K}_d) \mathbf{q}_s(t) = \mathbf{p}(t) , \quad (11)$$

where the damping matrix of the dampers is defined as

$$\mathbf{C}_d = \sum_{i=1}^m \mathbf{e}_i c_{d,i} \mathbf{e}_i^T . \quad (12)$$

Now the structure with the Maxwell dampers will be considered. Equations (2), containing the internal relative damper displacement $q_{r,i}(t)$ are used to describe the Maxwell damper behaviour. The vector of interactive forces $\mathbf{s}(t)$ is treated as a sum of two vectors, i.e., $\mathbf{s}(t) = \mathbf{s}_1(t) + \mathbf{s}_2(t)$. The $\mathbf{s}_1(t)$ vector contains interactive forces which are reactions of the elastic part of the Maxwell dampers to the frame, while the $\mathbf{s}_2(t)$ vector contains the interactive forces which are reactions of the dashpot part of the dampers.

If a structure with only one damper, denoted as the damper number i , mounted between two successive storeys j and $j+1$ is considered, then the vectors $\mathbf{s}_1(t)$ and $\mathbf{s}_2(t)$ could be written in the following form:

$$\mathbf{s}_1(t) \equiv \mathbf{s}_{1i}(t) = col(0, \dots, s_j = u_i, \dots, 0) = \tilde{\mathbf{e}}_i u_i(t) , \quad (13)$$

$$\mathbf{s}_2(t) \equiv \mathbf{s}_{2i}(t) = col(0, \dots, s_{j+1} = -u_i, \dots, 0) = \hat{\mathbf{e}}_i u_i(t) , \quad (14)$$

where $\tilde{\mathbf{e}}_i = col(0, \dots, \tilde{e}_j = 1, \tilde{e}_{j+1} = 0, \dots, 0)$, $\hat{\mathbf{e}}_i = col(0, \dots, \hat{e}_j = 0, \hat{e}_{j+1} = -1, \dots, 0)$.

In terms of displacements, the damping force $u_i(t)$ of the Maxwell damper could be shown in two equivalent forms:

$$u_i(t) = k_{d,i} (q_{r,i}(t) - q_{s,j}(t)) = k_{d,i} q_{r,i}(t) - k_{d,i} \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) , \quad (15)$$

$$u_i(t) = c_{d,i} (D_t^{\alpha_i} q_{s,j+1}(t) - D_t^{\alpha_i} q_{r,i}(t)) = -c_{d,i} D_t^{\alpha_i} q_{r,i}(t) - c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) , \quad (16)$$

and the interaction force vectors $\mathbf{s}_{1i}(t)$ and $\mathbf{s}_{2i}(t)$ are given by

$$\mathbf{s}_{1i}(t) = \tilde{\mathbf{e}}_i k_{d,i} q_{r,i}(t) - \tilde{\mathbf{e}}_i k_{d,i} \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) = \tilde{\mathbf{e}}_i k_{d,i} \mathbf{h}_i^T \mathbf{q}_r(t) - \tilde{\mathbf{e}}_i k_{d,i} \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) , \quad (17)$$

$$\begin{aligned} \mathbf{s}_{2i}(t) &= -\hat{\mathbf{e}}_i c_{d,i} D_t^{\alpha_i} q_{r,i}(t) - \hat{\mathbf{e}}_i c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) \\ &= -\hat{\mathbf{e}}_i c_{d,i} \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) - \hat{\mathbf{e}}_i c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) , \end{aligned} \quad (18)$$

where the vector of internal variables $\mathbf{q}_r(t) = col(q_{r,1}(t), \dots, q_{r,i}(t), \dots, q_{r,m}(t))$ and the vector $\mathbf{h}_i = col(0, \dots, h_i = 1, \dots, 0)$ have the dimension $(m \times 1)$.

When m dampers are present in the frame then the interaction force vectors are:

$$\mathbf{s}_1(t) = \sum_{i=1}^m \tilde{\mathbf{e}}_i k_{d,i} \mathbf{h}_i^T \mathbf{q}_r(t) - \sum_{i=1}^m \tilde{\mathbf{e}}_i k_{d,i} \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) = \mathbf{K}_{sr}^d \mathbf{q}_r(t) - \mathbf{K}_{ss}^d \mathbf{q}_s(t) , \quad (19)$$

$$\mathbf{s}_2(t) = - \sum_{i=1}^m \hat{\mathbf{e}}_i c_{d,i} \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) - \sum_{i=1}^m \hat{\mathbf{e}}_i c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) , \quad (20)$$

where

$$\mathbf{K}_{sr}^d = \sum_{i=1}^m \tilde{\mathbf{e}}_i k_{d,i} \mathbf{h}_i^T , \quad \mathbf{K}_{ss}^d = \sum_{i=1}^m \tilde{\mathbf{e}}_i k_{d,i} \tilde{\mathbf{e}}_i^T . \quad (21)$$

The dimensions of matrices \mathbf{K}_{sr}^d and \mathbf{K}_{ss}^d are $(n \times m)$ and $(n \times n)$, respectively.

Taking into account that $\mathbf{s}(t) = \mathbf{s}_1(t) + \mathbf{s}_2(t)$ and introducing Equations (19) and (20) into (3) we obtain the following motion equation of frame with the Maxwell dampers

$$\begin{aligned} \mathbf{M}_s D_t^2 \mathbf{q}_s(t) + \mathbf{C}_s D_t^1 \mathbf{q}_s(t) + \sum_{i=1}^m \hat{\mathbf{e}}_i c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) + (\mathbf{K}_s + \mathbf{K}_{ss}^d) \mathbf{q}_s(t) \\ + \sum_{i=1}^m \hat{\mathbf{e}}_i c_{d,i} \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) - \mathbf{K}_{sr}^d \mathbf{q}_r(t) = \mathbf{p}(t) . \end{aligned} \quad (22)$$

Equation (22) represents a set of n equations with $n+m$ unknowns which are elements of vectors $\mathbf{q}_s(t)$ and $\mathbf{q}_r(t)$. Additional m equations in the following form:

$$-c_{d,i} D_t^{\alpha_i} q_{s,j+1}(t) + c_{d,i} D_t^{\alpha_i} q_{r,i}(t) - k_{d,i} q_{s,j}(t) + k_{d,i} q_{r,i}(t) = 0 , \quad (23)$$

where $i=1,2,\dots,m$, are obtained from equilibrium condition of the internal node of Maxwell model of dampers.

In the matrix notation we can rewrite Equation (23), for $i=1,2,\dots,m$, in the form:

$$c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) + c_{d,i} \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) - k_{d,i} \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) + k_{d,i} \mathbf{h}_i^T \mathbf{q}_r(t) = 0 . \quad (24)$$

The final form of Equation (24) is obtained by pre-multiplying Equation (23) by \mathbf{h}_i and summing up all equations with respect to i . As the result, we have:

$$\begin{aligned} \sum_{i=1}^m \mathbf{h}_i c_{d,i} \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) + \sum_{i=1}^m \mathbf{h}_i c_{d,i} \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) \\ - \sum_{i=1}^m \mathbf{h}_i k_{d,i} \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) + \sum_{i=1}^m \mathbf{h}_i k_{d,i} \mathbf{h}_i^T \mathbf{q}_r(t) = \mathbf{0} . \end{aligned} \quad (25)$$

Equations (22) and (25) constitute a set of equations from which the dynamic response of structure with Maxwell dampers can be determined. It is a set of fractional differential equations. In this formulation each damper can have its own values of parameters different from others.

Equations (22) and (25) is much simplified when all fractional parameters are equal, i.e., $\alpha_i = \alpha = const$. In such case we can write:

$$\mathbf{M}_s D_t^2 \mathbf{q}_s(t) + \mathbf{C}_s D_t^1 \mathbf{q}_s(t) + \mathbf{C}_{ss}^d D_t^\alpha \mathbf{q}_s(t) + (\mathbf{K}_s + \mathbf{K}_{ss}^d) \mathbf{q}_s(t)$$

$$+ \mathbf{C}_{sr}^d D_t^\alpha \mathbf{q}_r(t) - \mathbf{K}_{sr}^d \mathbf{q}_r(t) = \mathbf{p}(t) , \quad (26)$$

$$\mathbf{C}_{rs}^d D_t^\alpha \mathbf{q}_s(t) + \mathbf{C}_{rr}^d D_t^\alpha \mathbf{q}_r(t) - \mathbf{K}_{rs}^d \mathbf{q}_s(t) + \mathbf{K}_{rr}^d \mathbf{q}_r(t) = \mathbf{0} , \quad (27)$$

where

$$\mathbf{C}_{ss}^d = \sum_{i=1}^m \hat{\mathbf{e}}_i c_{d,i} \hat{\mathbf{e}}_i^T , \quad \mathbf{C}_{sr}^d = \sum_{i=1}^m \hat{\mathbf{e}}_i c_{d,i} \mathbf{h}_i^T , \quad \mathbf{C}_{rs}^d = \sum_{i=1}^m \mathbf{h}_i c_{d,i} \hat{\mathbf{e}}_i^T = (\mathbf{C}_{sr}^d)^T , \quad (28)$$

$$\mathbf{C}_{rr}^d = \sum_{i=1}^m \mathbf{h}_i c_{d,i} \mathbf{h}_i^T , \quad \mathbf{K}_{rs}^d = \sum_{i=1}^m \mathbf{h}_i k_{d,i} \tilde{\mathbf{e}}_i^T = (\mathbf{K}_{sr}^d)^T , \quad \mathbf{K}_{rr}^d = \sum_{i=1}^m \mathbf{h}_i k_{d,i} \mathbf{h}_i^T . \quad (29)$$

2.3 The equations of motion of structures expressed in the state space

In many cases it is very convenient to use the equation of motion expressed in the state space. When the Kelvin model is used to describe dampers behaviour then the vector of state variables and the vectors of their derivatives could be defined as $\mathbf{z}(t) = \text{col}(\mathbf{q}_s, D_t^1 \mathbf{q}_s)$, $D_t^1 \mathbf{z}(t) = \text{col}(D_t^1 \mathbf{q}_s, D_t^2 \mathbf{q}_s)$, $D_t^{\alpha_i} \mathbf{z}(t) = \text{col}(D_t^{\alpha_i} \mathbf{q}_s, D_t^{\alpha_i+1} \mathbf{q}_s)$.

Moreover, when the following additional matrix equation

$$\mathbf{M}_s D_t^1 \mathbf{q}_s(t) - \mathbf{M}_s D_t^1 \mathbf{q}_s(t) = \mathbf{0} , \quad (30)$$

is added to the motion equation (9) the set of equations (9) and (30) could be rewritten using the state variables defined above. The resulting matrix equation is in the form:

$$\mathbf{A} D_t^1 \mathbf{z}(t) + \sum_{i=1}^m \mathbf{A}_i D_t^{\alpha_i} \mathbf{z}(t) + \mathbf{B} \mathbf{z}(t) = \tilde{\mathbf{p}}(t) , \quad (31)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{C}_s & \mathbf{M}_s \\ \mathbf{M}_s & \mathbf{0} \end{bmatrix} , \quad \mathbf{A}_i = \begin{bmatrix} \mathbf{e}_i c_{d,i} \mathbf{e}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} , \quad \mathbf{B} = \begin{bmatrix} (\mathbf{K}_s + \mathbf{K}_d) & \mathbf{0} \\ \mathbf{0} & -\mathbf{M}_s \end{bmatrix} , \quad \tilde{\mathbf{p}}(t) = \begin{Bmatrix} \mathbf{p}(t) \\ \mathbf{0} \end{Bmatrix} . \quad (32)$$

The motion equations in the state space can also be derived for frames with Maxwell dampers. In this case, the vector of state variables and vectors of state variables derivatives are defined as:

$$\mathbf{z}(t) = \text{col}(\mathbf{q}_r(t), \mathbf{q}_s(t), D_t^1 \mathbf{q}_s(t)) , \quad D_t^1 \mathbf{z}(t) = \text{col}(D_t^1 \mathbf{q}_r(t), D_t^1 \mathbf{q}_s(t), D_t^2 \mathbf{q}_s(t)) , \\ D_t^{\alpha_i} \mathbf{z}(t) = \text{col}(D_t^{\alpha_i} \mathbf{q}_r(t), D_t^{\alpha_i} \mathbf{q}_s(t), D_t^{\alpha_i+1} \mathbf{q}_s(t)) . \quad (33)$$

Now Equations (25), (22) and (30) can be rewritten in the form:

$$\mathbf{A} D_t^1 \mathbf{z}(t) + \sum_{i=1}^m \mathbf{A}_i D_t^{\alpha_i} \mathbf{z}(t) + \mathbf{B} \mathbf{z}(t) = \tilde{\mathbf{p}}(t) , \quad (34)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_s & \mathbf{M}_s \\ \mathbf{0} & \mathbf{M}_s & \mathbf{0} \end{bmatrix} , \quad \mathbf{A}_i = \begin{bmatrix} \mathbf{h}_i c_{d,i} \mathbf{h}_i^T & \mathbf{h}_i c_{d,i} \hat{\mathbf{e}}_i^T & \mathbf{0} \\ \hat{\mathbf{e}}_i c_{d,i} \mathbf{h}_i^T & \hat{\mathbf{e}}_i c_{d,i} \hat{\mathbf{e}}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} , \quad (35)$$

$$\mathbf{B} = \begin{bmatrix} \sum_{i=1}^m \mathbf{h}_i k_{d,i} \mathbf{h}_i^T & -\sum_{i=1}^m \mathbf{h}_i k_{d,i} \tilde{\mathbf{e}}_i^T & \mathbf{0} \\ -\sum_{i=1}^m \tilde{\mathbf{e}}_i k_{d,i} \mathbf{h}_i^T & \mathbf{K}_s + \sum_{i=1}^m \tilde{\mathbf{e}}_i k_{d,i} \tilde{\mathbf{e}}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{M}_s \end{bmatrix}, \quad \tilde{\mathbf{p}}(t) = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p}(t) \\ \mathbf{0} \end{Bmatrix}. \quad (36)$$

The presented above in the state space formulation is new. In comparison with previous ones, given, for example, in [8, 9], the proposed approach did not require matrices with huge dimensions. It is the main advantage of the proposed formula.

3 The eigenvalue problem

Applying the Laplace transform, and taking into account that (see [6]):

$$\mathcal{L}[\mathbf{z}(t)] = \mathbf{Z}, \quad \mathcal{L}[D_t^{\alpha_i} \mathbf{z}(t)] = s^{\alpha_i} \mathbf{Z}, \quad \mathcal{L}[D_t^1 \mathbf{z}(t)] = s \mathbf{Z}, \quad (37)$$

the equation of motion (31) or (34) can be written as:

$$\left(s \mathbf{A} + \sum_{i=1}^m s^{\alpha_i} \mathbf{A}_i + \mathbf{B} \right) \mathbf{Z} = \mathbf{0}. \quad (38)$$

Equation (38) constitutes the nonlinear eigenproblem, which can be solved using the continuation method. Description of the continuation method can be found in [10]. In this paper, the main parameter of the continuation method is a chosen fractional parameter α (say α_1). The first point on the $s(\alpha)$ curve and the $\mathbf{Z}(\alpha)$ curve is obtained for $\alpha = 1$. It can be determined after solving the following eigenvalue problem:

$$\left[s \left(\mathbf{A} + \sum_{i=1}^m \mathbf{A}_i \right) + \mathbf{B} \right] \mathbf{Z} = \mathbf{0}. \quad (39)$$

The typical calculation was made for a two-storey frame with the Maxwell VE damper mounted on the second floor. The following data were chosen: the mass of the first and second floors are $m_1 = 21.6 Mg$ and $m_2 = 17.28 Mg$, respectively; the height and rigidity of columns are $3.0 m$ and $EI = 11685.0 kNm^2$, the span and rigidity of the beam are $6.0 m$ and $EI = 47416.0 kNm^2$, respectively. The dampers data are: $k_{d,1} = k_{11}$ where k_{11} is the element of \mathbf{K}_s matrix, $c_d = 376.456 kNs/m$, $\tau_d = c_d / k_{d,1} = 0.02$.

The dynamic properties of the considered frame were calculated. The dependence of the first natural frequency of vibration and the dependence of the non-dimensional damping ratio versus the fractional parameter α are shown on Fig. 3 and 4, respectively. The natural frequency ω_i and the non-dimensional damping ratio γ_i are defined as:

$$\omega_i^2 = \mu_i^2 + \eta_i^2, \quad \gamma_i = -\mu_i / \omega_i, \quad \mu_i = \text{Re}(s_i), \quad \eta_i = \text{Im}(s_i). \quad (40)$$

It is easy to observe that both quantities, i.e. the natural frequency and the non-dimensional damping ratio increase when the fractional parameter increases.

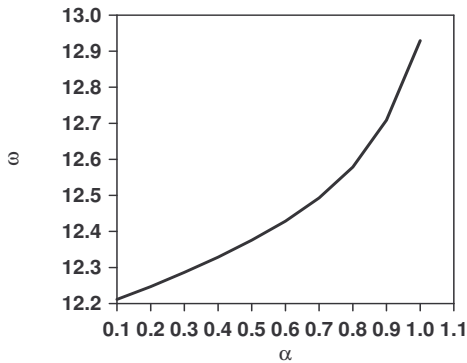


Figure 3: First natural frequency ω_1 versus fractional parameter α

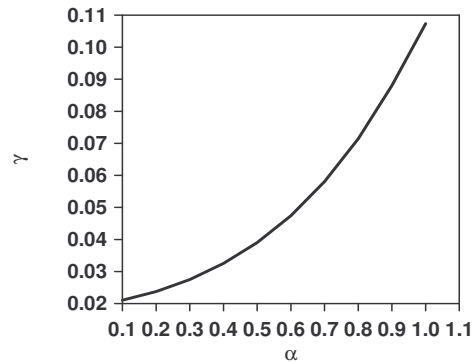


Figure 4: Non-dimensional damping ratio γ_1 versus fractional parameter α

4 Concluding remarks

In the paper, the motion equations of planar frames with VE dampers are derived. Two fractional, three-parameter rheological models are used to describe the dynamic behaviour of the considered systems. Moreover, the results of calculation of dynamic properties of typical frame with VE damper are presented.

Acknowledgments

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