

A Simple Method for Identification of the Parameters of the Fractional Kelvin-Voight Model

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A major difficulty connected with fractional rheological models is the estimation of the model parameters of experimental data. A new method for identification of the parameters of the fractional Kelvin-Voight model is presented. The procedure has been validated using artificial data and then used in estimating parameters from experimental data.

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1 Introduction

Fractional mechanical models are becoming more and more popular. The reason for their popularity is the ability of describing correctly the behaviour of viscoelastic materials using a small number of parameters [1,2]. A single equation is enough to describe the VE damper dynamics. This is an essential advantage of the fractional models. An important difficulty, connected with the fractional models, is the estimation of model parameters of the experimental data. In the past, many different methods were tested for identification of model parameters [3, 4, 5]. The process of parameter identification is an inverse problem which can be ill conditioned [5, 6]. It is the aim of this paper to present a new method for identification of the parameters of the fractional Kelvin-Voight model. The procedure has been tested using artificial and real experimental data.

2 A brief description of fractional rheological models

The motion equation of the fractional Kelvin-Voight model is in the following form:

$$u(t) = kq(t) + k\tau^\alpha D_t^\alpha q(t) \tag{1}$$

where: $\tilde{\tau} \equiv \tau^\alpha = \tilde{c}^\alpha / k = c / k$, $0 < \alpha \leq 1$ and $D_t^\alpha q(t)$ is the fractional derivative of order α with respect to time t .

The steady state solution to the equation (1) is in the form: $q(t) = q_c \cos \lambda t + q_s \sin \lambda t$, $u(t) = u_c \cos \lambda t + u_s \sin \lambda t$, where q_c , q_s , u_c and u_s are interrelated in the following way $u_c = \phi_1 q_c + \phi_2 q_s$, $u_s = -\phi_2 q_c + \phi_1 q_s$ and $\phi_1 = k[1 + (\tau\lambda)^\alpha \cos(\alpha\pi/2)]$, $\phi_2 = k(\tau\lambda)^\alpha \sin(\alpha\pi/2)$. The behaviour of the fractional Maxwell model is described by the following equation:

$$u(t) + \tau^\alpha D_t^\alpha u(t) = k\tau^\alpha D_t^\alpha q(t) \tag{2}$$

The steady state solution is in the same form as in the fractional Kelvin-Voight model although now the q_c, q_s, u_c, u_s parameters are interconnected in the following manner: $q_c = \phi_1 u_c - \phi_2 u_s$, $q_s = \phi_2 u_c + \phi_1 u_s$ and

$$\phi_1 = \frac{(\tau\lambda)^\alpha + \cos(\alpha\pi/2)}{k(\tau\lambda)^\alpha}, \quad \phi_2 = \frac{\sin(\alpha\pi/2)}{k(\tau\lambda)^\alpha}.$$

3 Parameter identification procedure based on the hysteresis loop equation

The equation of hysteresis loop of the fractional Kelvin-Voight model is given as follows

$$\left\{ \frac{u(t) - k[1 + (\tau\lambda)^\alpha \cos(\alpha\pi/2)]q(t)}{k(\tau\lambda)^\alpha q_0 \sin(\alpha\pi/2)} \right\}^2 + \left[\frac{q(t)}{q_0} \right]^2 = 1 \tag{3}$$

From Equation (3), and on the assumption that for the given frequency of excitation and $t=t_1$, we have $q(t_1)=q_0 > 0$, $u(t_1)=u_1 > 0$ and for $t=t_2$ when $q(t_2)=0$, $u(t_2)=u_2 > 0$ the following is obtained:

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$$k[1 + (\tau\lambda)^\alpha \cos(\alpha\pi/2)] = k + c\lambda^\alpha \cos(\alpha\pi/2) = u_1/q_0, \quad c\lambda^\alpha \sin(\alpha\pi/2) = u_2/q_0 \tag{4}$$

For a given λ , relationships (4) are a set of two nonlinear equations with three unknowns: k, c, α or k, τ, α . The parameters of u_1, u_2 and q_0 have clear physical meanings and can be easily obtained from experimental data. During experiments, the damper is several times harmonically excited with different frequencies λ_i . The steady state response of the damper is measured. The experimental damper displacements $q_{ei}(t)$ and the damper forces $u_{ei}(t)$ are known for each λ_i . Now the equations (4) can be rewritten for $i = 1, 2, \dots, n$ in the form:

$$r_i = k + c\lambda_i^\alpha \cos(\alpha\pi/2) - u_{1i}/q_{0i} = 0, \quad s_i = c\lambda_i^\alpha \sin(\alpha\pi/2) - u_{2i}/q_{0i} = 0. \tag{5}$$

The above equations constitute a set of over-determined nonlinear equations with respect to k, c and α . The pseudo-solution to (5) is obtained by minimizing the following functional:

$$\tilde{J}_{KV}(k, c, \alpha) = \sum_{i=1}^n (r_i^2 + s_i^2). \tag{6}$$

If the parameter α is known, then stationary conditions give us the following set of equations, which are linear with respect to k and c

$$kn + c \sum_{i=1}^n \lambda_i^\alpha \cos \frac{\alpha\pi}{2} = \sum_{i=1}^n \frac{u_{1i}}{q_{0i}}, \quad k \sum_{i=1}^n \lambda_i^\alpha \cos \frac{\alpha\pi}{2} + c \sum_{i=1}^n \lambda_i^{2\alpha} = \sum_{i=1}^n \lambda_i^\alpha \left(\frac{u_{1i}}{q_{0i}} \cos \frac{\alpha\pi}{2} + \frac{u_{2i}}{q_{0i}} \sin \frac{\alpha\pi}{2} \right) \tag{7}$$

The correct value of α is obtained using the method of systematic searching. The set of values of α , denoted as α_j ($j = 1, 2, \dots, m$) is chosen from a given range of α . For each α_j the corresponding values of k and c are determined from (7) and the value of functional (6) is calculated. The values of α_j, k_j and c_j for which the functional (6) has a minimal value are the searched parameters of the fractional Kelvin-Voight damper model. The same algorithm can be applied to the fractional Maxwell model and a very similar procedure of identification can be developed. The results of calculation are presented below

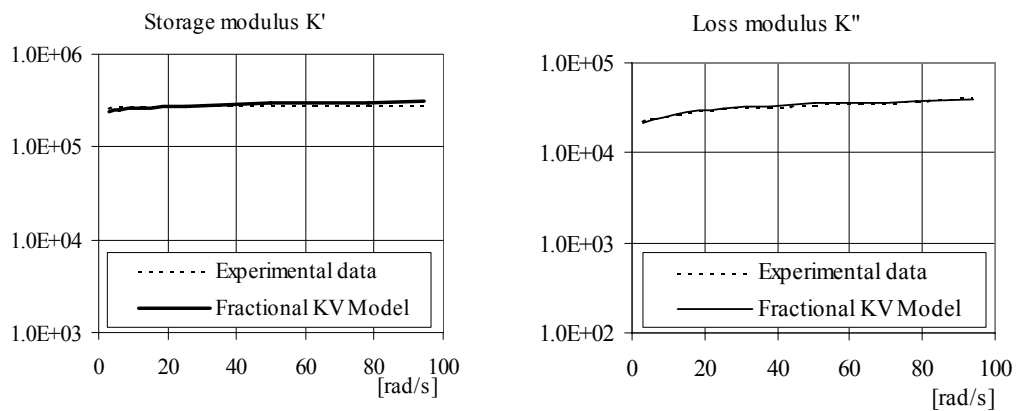


Fig. 1 Storage and loss modulus

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