

Optimization of Viscoelastic Dampers as described by the Fractional Rheological Model

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Abstract

Frame structures with viscoelastic dampers (VE) mounted on them are considered. It is the aim of this paper to find the optimal damper placement and to determine the optimal parameters of the dampers. The dynamic behaviour of the VE dampers described by the fractional Kelvin rheological model is written in the form of fractional differential equation. The objective function, which we minimize, is the weighted sum of amplitudes of the transfer functions of interstorey drifts, evaluated at the fundamental, natural frequency of the frame with the dampers. The solution is obtained using the sequential optimization method and the particle swarm optimization method (PSO). The results obtained by both methods are compared.

Keywords: viscoelastic dampers, fractional rheological models, particle swarm optimization, optimal dampers location.

1 Introduction

In civil engineering VE dampers are successfully applied to reduce excessive vibrations of buildings caused by winds and earthquakes. It was found that incorporation of the VE dampers in a structure leads to a significant reduction of unwanted vibrations, see Soong [1]. A number of applications of VE dampers in civil engineering are listed in [2]. The dampers behaviour depends mainly on the rheological properties of the VE material the dampers are made of and some of their geometric parameters. In the past, several rheological models were proposed to describe the dynamic behaviour of VE materials and dampers. Both the classic and the so-called fractional-derivative models of dampers and VE materials are available. In the classic approach, the mechanical models consisting of springs and dashpots are used to describe the rheological properties of VE dampers [3]. A good description of the VE dampers requires mechanical models consisting of a set of appropriately connected springs and dashpots. In this approach, the dynamic

behaviour of a single damper is described by a set of differential equations. The rheological properties of VE dampers are also described using the fractional calculus and the fractional mechanical models. This approach has received considerable attention and has been used in modelling the rheological behaviour of VE materials [4] and dampers [5]. The fractional models have an ability to correctly describe the behaviour of VE materials and dampers using a small number of model parameters. A single equation is enough to describe the VE damper dynamics, which is an important advantage of the discussed model. However, in this case, the VE damper equation of motion is the fractional differential equation.

The dynamic analysis of frame or building structures with dampers is presented in many papers, where the Kelvin model [6, 7] is used to describe the dampers' dynamic behaviour. In papers [8, 9] the three-parameter fractional-derivative rheological model is used to model the dampers' behaviour.

In this paper, planar frame structures with the VE dampers mounted on them are considered. The VE dampers are modelled using the fractional rheological model. The three-parameter fractional rheological Kelvin model is considered. The structures are treated as linear elastic systems. The equations of motion of the whole system (the structure with dampers) are written in terms of both physical and state-space variables. The proposed approach in the state space formulation is new. This is the main advantage of the proposed formulation, which does not require matrices with huge dimensions. However, the resulting matrix equation of motion is a fractional differential equation.

It is aim of the paper to find the optimal placements of the dampers and to determine their optimal parameters. The objective function, which we minimize, is the weighted sum of amplitudes of the transfer functions of interstorey drifts, evaluated at the fundamental, natural frequency of the frame with the dampers. The optimality criterion is expressed by the vector consisting of the values of the above mentioned transfer functions of the interstorey drifts.

The solution to the considered optimization problem is arrived at using the sequential optimization method and the particle swarm optimization method (PSO), which is based on the study of social behaviour in a self-organized population system [10]. Numerical tests carried out for a multi-storey building structure modelled as a shear plane frame with VE dampers mounted on it show that the presented methods are simple and efficient.

2 Equations of motion

2.1 The rheological model of damper

In this paper, the fractional Kelvin model is used to represent the rheological properties of VE dampers. The considered model consists of a fractional dashpot with constants $c_{d,i}$, α_i ($0 < \alpha_i \leq 1$), in parallel with a spring of stiffness $k_{d,i}$, see Figure 1. The governing equation of a typical damper i could be written in the form:

$$u_i = k_{d,i}q_{d,i} + c_{d,i}D_t^{\alpha_i}q_{d,i} \quad (1)$$

where u_i is the damper force and $q_{d,i}$ is the relative damper displacement.

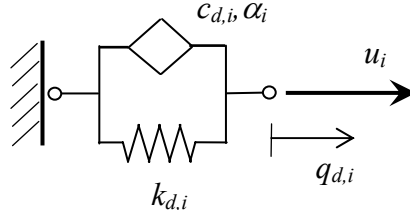


Figure 1: Rheological model of damper

Moreover, $D_t^{\alpha_i}(\bullet)$ denotes the Riemann-Liouville fractional derivative of the order α_i with respect to time, t , (see [11] for details). More information concerning the fractional rheological models can be found in [5]. The equation of motion of the classic Kelvin model could be obtained after introducing $\alpha_i = 1$ into Equation (1).

2.2 Equations of motion as expressed in physical coordinates

The frame with VE dampers is treated as the elastic linear system and their model could be the shear frame shown in Figure 2a.

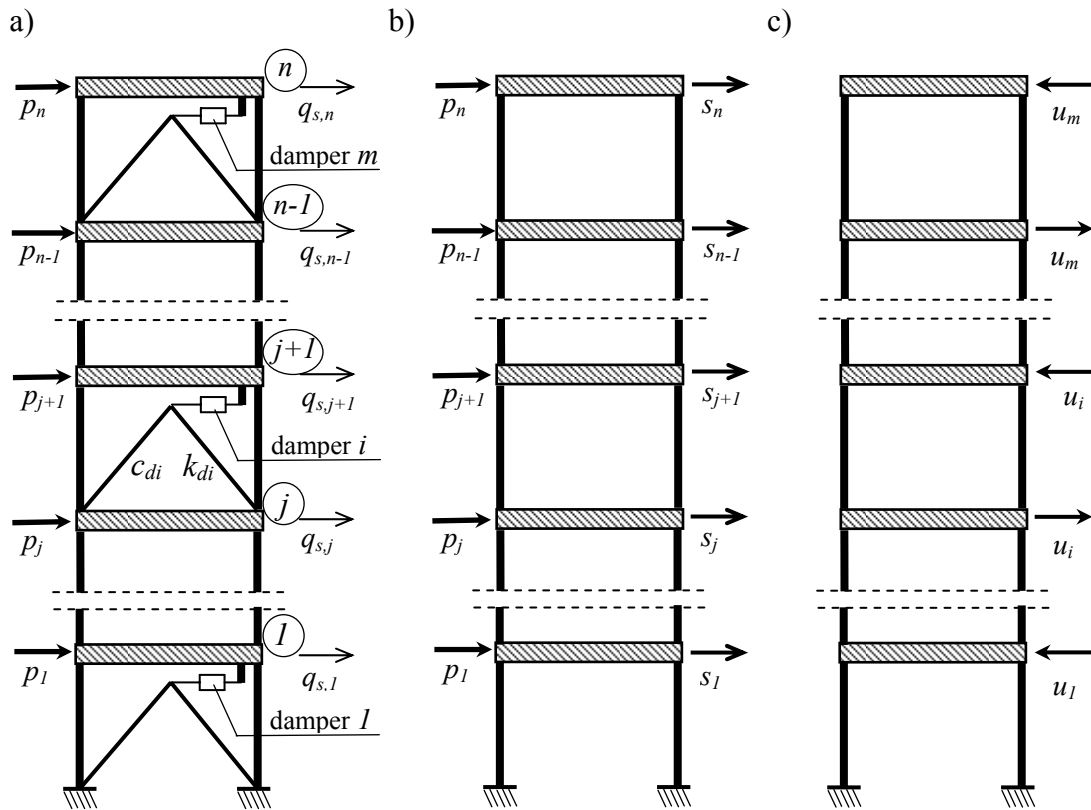


Figure 2: Diagram of frame with VE dampers

The mass of the system is lumped at the level of storeys. The frame can also be modelled as the structure with flexible beams. In this case we assume that beams and columns are axially inextensible. Moreover, static condensation could be used to eliminate the rotational nodal parameters from the equations of motion. Finally, the equation of motion of such a structure can be written as follows:

$$\mathbf{M}_s \ddot{\mathbf{q}}_s(t) + \mathbf{C}_s \dot{\mathbf{q}}_s(t) + \mathbf{K}_s \mathbf{q}_s(t) = \mathbf{s}(t) + \mathbf{p}(t) \quad (2)$$

where the symbols \mathbf{M}_s , \mathbf{C}_s and \mathbf{K}_s denote the mass, the damping and the stiffness ($n \times n$) matrices, respectively. Moreover, $\mathbf{q}_s(t) = \text{col}(q_{s,1}, \dots, q_{s,j}, \dots, q_{s,n})$ and $\mathbf{p}(t) = \text{col}(p_1, \dots, p_j, \dots, p_n)$ denote the vector of displacements of the structure and the vector of excitation forces, respectively. The $\mathbf{s}(t) = \text{col}(s_1, s_2, \dots, s_n)$ vector is the ($n \times 1$) vector of interaction forces between the frame and dampers (see Figure 2b).

First of all, the structure with one damper, denoted as the damper number i , which is mounted between two successive storeys j and $j+1$ (shown in Figure 2a), is considered. The force interaction vector $\mathbf{s}(t)$ could be written as:

$$\mathbf{s}(t) \equiv \mathbf{s}_i(t) = \text{col}(0, \dots, s_j = u_i, s_{j+1} = -u_i, \dots, 0) = \mathbf{e}_i u_i(t) \quad (3)$$

where $\mathbf{e}_i = \text{col}(0, \dots, e_j = 1, e_{j+1} = -1, \dots, 0)$ is the i -th damper allocation vector of dimension ($n \times 1$), $u_i(t)$ is the damper force given in (1). We assume the brace systems used for connecting dampers with the successive storeys are rigid.

Taking into account that the relative damper displacement, written in terms of structure displacements, is

$$q_{d,i}(t) = q_{s,j+1}(t) - q_{s,j}(t) = -\mathbf{e}_i^T \mathbf{q}_s(t), \quad (4)$$

the damper force and the vector of interactive forces could be written as follows:

$$u_i(t) = -k_{d,i} \mathbf{e}_i^T \mathbf{q}_s(t) - c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t), \quad (5)$$

$$\mathbf{s}_i(t) = -\mathbf{e}_i k_{d,i} \mathbf{e}_i^T \mathbf{q}_s(t) - \mathbf{e}_i c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t). \quad (6)$$

If we have a structure with m dampers, the vector of interactive forces is given by:

$$\mathbf{s}(t) = \sum_{i=1}^m \mathbf{s}_i(t) = -\sum_{i=1}^m \mathbf{e}_i k_{d,i} \mathbf{e}_i^T \mathbf{q}_s(t) - \sum_{i=1}^m \mathbf{e}_i c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t), \quad (7)$$

and the equation of motion (2) could be rewritten in the form:

$$\mathbf{M}_s D_t^2 \mathbf{q}_s(t) + \mathbf{C}_s D_t^1 \mathbf{q}_s(t) + \sum_{i=1}^m \mathbf{e}_i c_{d,i} \mathbf{e}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) + (\mathbf{K}_s + \mathbf{K}_d) \mathbf{q}_s(t) = \mathbf{p}(t), \quad (8)$$

where, in order to be consistent with the notation, a symbol such as $D_t^1(\bullet)$ is introduced to denote the first derivative with respect to time and the matrix of dampers stiffness is

$$\mathbf{K}_d = \sum_{i=1}^m \mathbf{e}_i k_{d,i} \mathbf{e}_i^T. \quad (9)$$

Equation (8) is the matrix fractional differential equation which describes the dynamic behaviour of the considered frame with the Kelvin dampers. In this approach each damper can have its own values of parameters, different from others.

2.3 The state-space formulation of equations of motion

In many cases it is very convenient to use the equation of motion expressed in the state space. The vector of state variables $\mathbf{z}(t) = \text{col}(\mathbf{q}_s(t), D_t^1 \mathbf{q}_s(t))$ is associated with the vectors of their derivatives defined as:

$$D_t^1 \mathbf{z}(t) = \text{col}(D_t^1 \mathbf{q}_s(t), D_t^2 \mathbf{q}_s(t)),$$

$$D_t^{\alpha_i} \mathbf{z}(t) = \text{col}(D_t^{\alpha_i} \mathbf{q}_s(t), D_t^{\alpha_i+1} \mathbf{q}_s(t)).$$

Moreover, when the following additional matrix equation:

$$\mathbf{M}_s D_t^1 \mathbf{q}_s(t) - \mathbf{M}_s D_t^1 \mathbf{q}_s(t) = \mathbf{0}, \quad (10)$$

is appended to the motion Equation (8) we get the set of Equations (8) and (10) which could be rewritten using the state variables defined above. The resulting matrix equation is in the form:

$$\mathbf{A} D_t^1 \mathbf{z}(t) + \sum_{i=1}^m \mathbf{A}_i D_t^{\alpha_i} \mathbf{z}(t) + \mathbf{B} \mathbf{z}(t) = \tilde{\mathbf{p}}(t), \quad (11)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{C}_s & \mathbf{M}_s \\ \mathbf{M}_s & \mathbf{0} \end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} \mathbf{e}_i c_{d,i} \mathbf{e}_i^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} (\mathbf{K}_s + \mathbf{K}_d) & \mathbf{0} \\ \mathbf{0} & -\mathbf{M}_s \end{bmatrix}, \quad \tilde{\mathbf{p}}(t) = \begin{Bmatrix} \mathbf{p}(t) \\ \mathbf{0} \end{Bmatrix} \quad (12)$$

The approach to the state space formulation as presented above is new. In comparison with previous ones, given, for example, in [8, 12], the proposed approach did not require matrices with huge dimensions. This is the main advantage of the proposed formulation. Moreover, all matrices appearing in Equation (12) are symmetrical.

3 Dynamic analysis

3.1 The dynamic characteristics of structures

In order to simplify the further derivations we assume that fractional parameters, α_i , are identical (i.e., $\alpha_i = \alpha = \text{const.}$) for all dampers. Applying the Laplace transform, taking into account that $\tilde{\mathbf{p}}(t) = \mathbf{0}$ and ([13]):

$$\mathcal{L}[\mathbf{z}(t)] = \mathbf{Z}, \quad \mathcal{L}[D_t^\alpha \mathbf{z}(t)] = s^\alpha \mathbf{Z}, \quad \mathcal{L}[D_t^1 \mathbf{z}(t)] = s\mathbf{Z}, \quad (13)$$

the equation of motion (11) can be written as:

$$(s\mathbf{A} + s^\alpha \tilde{\mathbf{A}}_1 + \mathbf{B})\mathbf{Z} = \mathbf{0}. \quad (14)$$

where $\tilde{\mathbf{A}}_1 = \sum_{i=1}^m \mathbf{A}_i$.

Equation (14) constitutes the nonlinear eigenproblem, which can be solved using the continuation method. Methods for solving the eigenproblem that occurs in the dynamic analysis of viscoelastic structures or systems, when the damping forces are modelled using the fractional derivative, are investigated in [15, 18]. The solution to the considered non-linear equation could be shown as a curve in the configuration space, i.e., the s, \mathbf{Z} space ([18]). The first point in this curve is obtained for $\alpha = 1$, in this case Equation (14) expresses the linear eigenvalue problem. Next, the solution to the eigenproblem (14) for the chosen value of $\alpha \in (0, 1)$ is investigated. The incremental-iteration method, presented in detail in [18], is used. Usually, one incremental step and three or four iterations are enough to reach the solution for the final value of the fractional parameter. The continuation method enables the eigenvalues, s_i , to be determined. In this work we propose to characterize the dynamic behavior of frame with viscoelastic dampers by the natural frequency ω_i and the non-dimensional damping parameter γ_i . Similarly to viscous damping, we define the above-mentioned properties as:

$$\omega_i^2 = \mu_i^2 + \eta_i^2, \quad \gamma_i = -\mu_i / \omega_i, \quad (15)$$

where $\mu_i = \text{Re}(s_i)$, $\eta_i = \text{Im}(s_i)$.

3.2 Frequency response functions

In this section we focus on steady state harmonic responses of structures governed by Equations (8). For the harmonic external forces described by:

$$\mathbf{p}(t) = \mathbf{P} \exp(i\lambda t), \quad (16)$$

where $i = \sqrt{-1}$, λ is the frequency of excitation, the displacement response of structure can be expressed as

$$\mathbf{q}_s(t) = \mathbf{Q}_s(\lambda) \exp(i\lambda t). \quad (17)$$

If we substitute relationships (16) and (17) into the motion Equations (8), written in terms of physical coordinates the following equation is obtained:

$$\mathbf{Q}_s(\lambda) = \tilde{\mathbf{H}}(\lambda) \tilde{\mathbf{P}}, \quad (18)$$

where:

$$\tilde{\mathbf{H}}(\lambda) = \left[-\lambda^2 \mathbf{M}_s + i\lambda \mathbf{C}_s + \sum_{j=1}^m (i\lambda)^{\alpha} \mathbf{e}_j c_{d,j} \mathbf{e}_j^T + \mathbf{K}_s + \mathbf{K}_d \right]^{-1}. \quad (19)$$

When the structure is subjected to a base acceleration $\ddot{u}_g(t)$, the excitation vector is written as $\mathbf{p}(t) = -\mathbf{M} \mathbf{r} \ddot{u}_g(t)$, where $\mathbf{r} = \text{col}\{1, 1, \dots, 1\}$. For the harmonic external forces, we have $\ddot{u}_g(t) = \ddot{U}_g \exp(i\lambda t)$. The displacement response of the structure is given by relationship (17) and $\mathbf{Q}_s(\lambda)$ is determined from:

$$\mathbf{Q}_s(\lambda) = \mathbf{H}(\lambda) \ddot{U}_g, \quad (20)$$

where the vector $\mathbf{H}(\lambda) = -\tilde{\mathbf{H}}(\lambda) \mathbf{M} \mathbf{r}$ will be called the vector of frequency transfer functions of displacements.

4 Optimization problem

It is the aim of this paper to find the optimal dampers placements and to determine the optimal parameters of the dampers k_{di} and c_{di} . The objective function, which we minimize, is the weighted sum of amplitudes of the transfer functions of interstorey drifts, $h_i(\lambda)$, evaluated at the fundamental, natural frequency ($\lambda = \omega_1$) of the frame with the dampers. The optimality criteria may be described as follows:

$$F = \mathbf{w}^T \mathbf{h}(\omega_1), \quad (21)$$

where, the vector $\mathbf{h}(\omega_1) = \text{col}(h_1(\omega_1), h_2(\omega_1), \dots, h_n(\omega_1))$ consists of the values of the above mentioned amplitudes of transfer functions of interstorey drifts, $\mathbf{w} = \text{col}(w_1, w_2, \dots, w_n)$ is the vector of weight coefficients, and n stands for the number of the structure degrees of freedom.

The considered optimization problem is subjected to some constraints. We assume that the sum of damping coefficients and the sum of stiffness parameters are

known and constant. Moreover, the values of the damping $c_{d,i}$ and the stiffness $k_{d,i}$ parameters of every damper must be non-negative. The above constraints are written as:

$$\sum_{i=1}^m c_{d,i} = C_d, \quad \sum_{i=1}^m k_{d,i} = K_d, \quad c_{d,i} \geq 0, \quad k_{d,i} \geq 0. \quad (22)$$

The vector $\mathbf{H}_d(\lambda)$ of the frequency transfer functions of interstorey drifts can be calculated from the following formula:

$$\mathbf{H}_d(\lambda) = \mathbf{TH}(\lambda), \quad (23)$$

where the transformation matrix is

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ -1 & 1 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & -1 & 1 \end{bmatrix}. \quad (24)$$

The solution is obtained using the sequential optimization method and the particle swarm optimization method (PSO). In the first case, for each possible location of one damper the values of the objective function are calculated. The right fixed location of the damper is the position for which the minimum value of the objective function is obtained. When the first damper location is determined the procedure is repeated until all locations for the dampers are found. However, there is no proof for the solution convergence but many examples show that this method is simple and efficient in many engineering applications (for example [16]).

The algorithm of PSO which is based on the study of social behaviour in a self-organized population system (i.e., ant colonies, fish schools), searches a space by adjusting the trajectories of so-called particles. In the considered optimization problem the vector of i -th particle position \mathbf{p}_i consists of damping coefficients of dampers currently mounted on structure, i.e., $\mathbf{p}_i = \text{col}(c_{d,1}^{(i)}, c_{d,2}^{(i)}, \dots, c_{d,n}^{(i)})$. The dimension of vector \mathbf{p}_i is equal to the number of building storeys. Moreover, to reduce the number of elements of the particle position vector and because of damper technological requirements the stiffness parameter of damper is calculated assuming that the ratio $c_{d,i} / k_{d,i}$ is given and will not change during iteration.

A population of particles is initialized with random positions and velocities ([17]). Taking into account the best positions of the particles at subsequent iteration $k+1$, the algorithm adjusts the behaviour of the particles by the following rules:

$$\mathbf{v}_i(k+1) = w(k)\mathbf{v}_i(k) + \frac{c_1}{\Delta t} \mathbf{R}_1(k)(\mathbf{b}_i(k) - \mathbf{p}_i(k)) + \frac{c_2}{\Delta t} \mathbf{R}_2(k)(\mathbf{g}_i(k) - \mathbf{p}_i(k)),$$

$$\mathbf{p}_i(k+1) = \mathbf{p}_i(k) + \mathbf{v}_i(k+1)\Delta t, \quad (25)$$

where $\Delta t = 1$, $\mathbf{p}_i(k)$ is the position of i -th particle at k -th iteration, $\mathbf{v}_i(k)$ is the corresponding velocity vector; $\mathbf{b}_i(k)$ and $\mathbf{g}_i(k)$ stand for the best position found by the particle i and the best position in the particle's neighbourhood achieved so far, respectively; $\mathbf{R}_1(k)$, $\mathbf{R}_2(k)$ are the diagonal matrices of independent random numbers uniformly distributed in the range $(0, 1)$; $w(k)$ is the inertia factor providing balance between exploration and exploitation, c_1 is the individuality constant, and c_2 is the sociality constant. To speed up convergence, the inertia weight was linearly reduced from w_{\max} to w_{\min} , i.e.:

$$w(k+1) = w_{\max} - \frac{(w_{\max} - w_{\min})}{k_{\max}}k, \quad (26)$$

where k_{\max} denotes the maximal number of iterations.

A new velocity, which moves the particle in the direction of a potentially better solution, is calculated based on its previous value, and the particle location at which the best fitness so far has been achieved.

5 Numerical tests

In the numerical examples we investigate a ten-storey building structure modelled as a shear plane frame with VE dampers mounted on it. The bending rigidity of columns varies in sequence, at each two storeys: $k_1 = k_2 = 68710.0 \text{ kN/m}$, $k_3 = k_4 = 54010.0 \text{ kN/m}$, $k_5 = k_6 = 42170.0 \text{ kN/m}$, $k_7 = k_8 = 28660.0 \text{ kN/m}$, $k_9 = k_{10} = 16450.0 \text{ kN/m}$, but the mass value is the same on each floor: $m_s = 2.07Mg$. The structure damping ratios correspond to the stiffness of storeys are: $c_1 = c_2 = 4.76 \text{ kNs/m}$, $c_3 = c_4 = 3.73 \text{ kNs/m}$, $c_5 = c_6 = 2.91 \text{ kNs/m}$, $c_7 = c_8 = 1.98 \text{ kNs/m}$, $c_9 = c_{10} = 1.44 \text{ kNs/m}$ (data taken from [16]).

Firstly, the calculations were carried out for frame without dampers (see Figure 3a), only the damping properties of structure were taken into account.

The solution to Equation (14), where $\tilde{\mathbf{A}}_1 = \mathbf{0}$, leads to the eigenvalues s_i which enable determination of the dynamic properties of the structure described by Equation (15). The results, the natural frequencies of structure and the values of non-dimensional damping factor are presented in Table 1.

Next, the authors investigated the structure with one damper mounted on each storey (see Figure 3b). The assumed value of the sum of the damping coefficients and the sum of the stiffness parameters are: $C_d = 500 \text{ kNs/m}$, and $K_d = 25000 \text{ kNm}^2$, respectively. If dampers are uniformly distributed over a

structure, the data for every single damper are: $k_d = 2500 \text{ kNm}^2$, $c_d = 50 \text{ kNs/m}$, $\tau_d = c_d / k_d = 0.02$. The values of fractional parameters for all dampers are identical, i.e., $\alpha = 0.6$. Using the suggested procedure we computed the dynamic properties of the considered system (see Table 1).

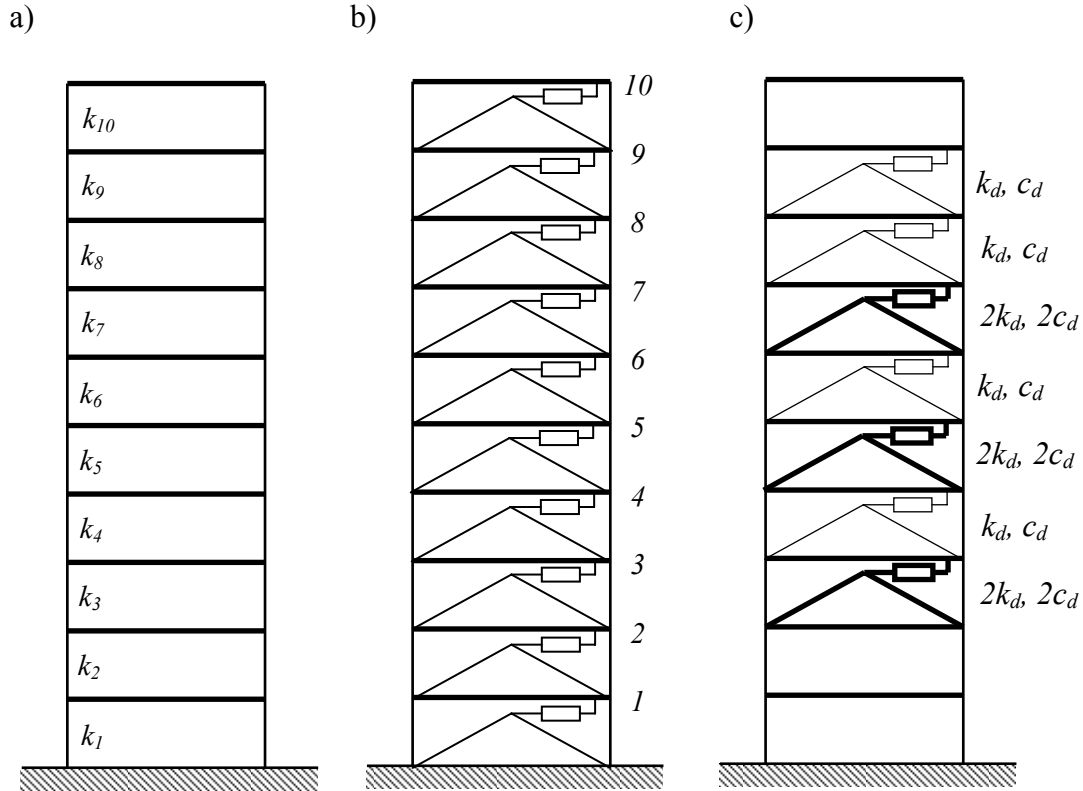


Figure 3: Diagram of a 10-storey frame with different distributions of dampers

A first solution to the optimization problem is obtained using the sequential optimization method. For each possible location of one damper the values of fundamental frequency are calculated (see Figure 4). Next the objective function is evaluated for the frame, taking into account every possible damper position. The results are presented in Figure 5.

The right fixed location of the first damper is at the seventh storey, for which the minimum value of the objective function is obtained. When the first damper's location is determined the procedure is repeated until all locations for the dampers are found. The optimal locations of successive dampers are found to be on the following storeys (see also Figure 3c): 7, 9, 5, 7, 3, 6, 5, 8, 4, 3. The dynamic properties of structures with optimally distributed dampers are shown in Table 1. It can be noticed that the non-dimensional damping ratio of the first mode of vibration is greater (by about 13%) than the same ratio for the structure with uniformly distributed dampers. Moreover, the damping ratios of the second, third, fourth and fifth modes of vibration are smaller. This redistribution of damping properties of the

structure is justified by the fact that the resonant pick at $\lambda = \omega_1$ is a few times as high as the resonant picks for higher modes of vibration.

Modal number	No dampers		Dampers distribution			
			uniform		optimal	
	ω_i	γ_i	ω_i	γ_i	ω_i	γ_i
1	22.690	0.0008	23.439	0,0038	23.568	0.0043
2	56.534	0.0022	59.493	0.0099	59.009	0.0085
3	91.909	0.0035	96.806	0.0137	95.568	0.0107
4	127.472	0.0047	134.031	0.0162	131.590	0.0118
5	151.769	0.0061	162.555	0.0231	159.097	0.0181
6	182.399	0.0066	190.891	0.0195	191.232	0.0200
7	208.638	0.0073	219.001	0.0219	222.836	0.0261
8	245.147	0.0085	254.727	0.0208	257.510	0.0237
9	281.524	0.0097	290.884	0.0209	292.657	0.0228
10	324.052	0.0112	333.127	0.0212	332.421	0.0214

Table 1: Natural frequencies ω_i and non-dimensional damping factors γ_i

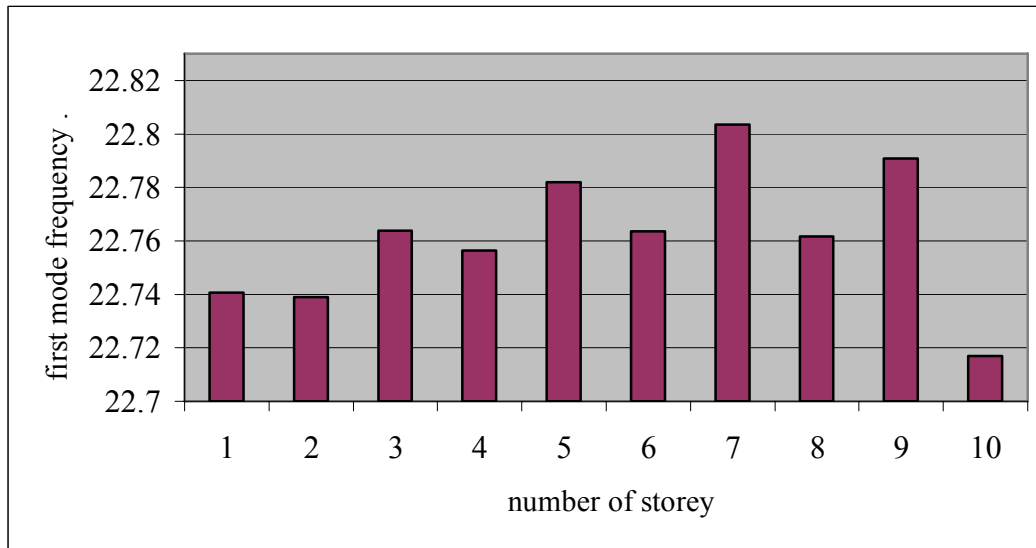


Figure 4: Fundamental frequency versus first damper position

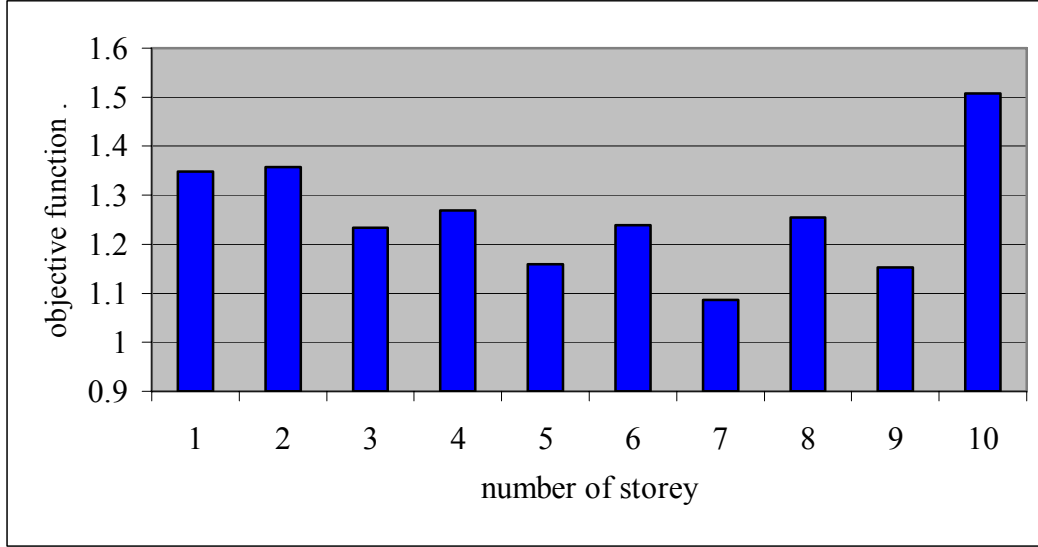


Figure 5: Objective function versus first damper position

In the second approach the PSO method is applied. In Equation (25) we define the values of coefficients $c_1 = c_2 = 2$ and the declining value of the inertia factor; started with $w = 0.9$, it decreased at each step of iteration by 0.005. A population of fifteen particles was initialized with random positions. The coordinates of every particle position describe the current distribution of damping properties on the frame. On every storey the value of the damping coefficient must be non-negative, and smaller than the assumed constant value:

$$0 \leq c_{d,i} \leq C_d, \quad (27)$$

where $C_d = 500.0 \text{ kNs/m}$ is the sum of damping coefficients of dampers over a structure. The stiffness parameters of dampers are calculated from the value of the $c_{d,i} / k_{d,i}$ ratio which is equal to 0.02 and constant for each damper.

Changes of the best value of the objective function during the iteration process are presented in Figure 6.

The solution to the optimization problem, i.e., the optimal distribution of VE dampers obtained with the help of both optimization methods is shown in Table 2.

The objective function, the weighted sum of amplitudes of the transfer functions of interstorey drifts is: $F_0 = 1.7053$, $F_U = 0.3286$, $F_S = 0.2972$ for the frame without dampers, for uniformly distributed dampers, and for the optimal solution obtained by the sequential and PSO methods, respectively.

It can be concluded that results obtained by both methods yield similar dampers distributions on the frame. The differences between optimal values of damping coefficients obtained as the result of optimization procedures are partially affected by an incremental way of distribution of damping coefficients in the sequential optimization method.

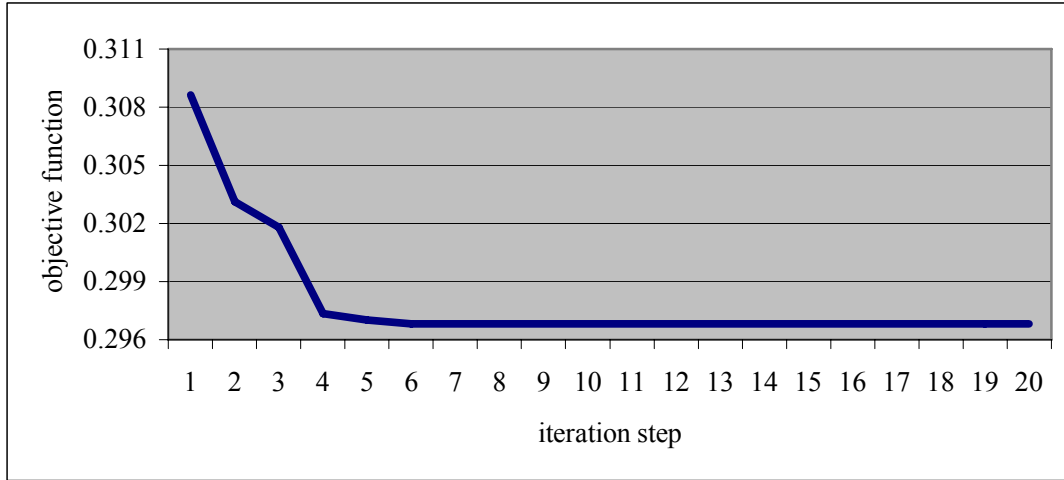


Figure 6: Convergence of objective function at PSO iteration

Number of storey	Damping coefficient	
	Sequential method	PSO method
1	0	0
2	0	0
3	100	87.57
4	50	47.68
5	100	106.18
6	50	56.68
7	100	120.77
8	50	26.68
9	50	54.45
10	0	0
Total	500	500.01

Table 2: Optimal distribution of VE dampers

6 Concluding remarks

In this paper, frame structures with viscoelastic dampers mounted on them are considered. Viscoelastic dampers are modelled using a three-parameter, fractional rheological Kelvin model which more precisely describes the VE damper properties. The resulting matrix equation of motion is the fractional differential equation. The

problem of optimal distribution of VE dampers modelled by the fractional rheological Kelvin model is solved for the first time. The considered optimization problem is solved by minimization of the objective function, which is the weighted sum of amplitudes of the transfer functions of the interstorey drifts. The sequential optimization method and the particle swarm optimization method are used to successfully solve the optimization problem. Examples of numerical calculations were shown. The presented results demonstrate the effectiveness and applicability of the proposed approach.

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