

Optimal active and semi-active control of building structures using the acceleration feedback and non-standard performance index

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Abstract

The paper describes application of the acceleration-based feedback controllers to reduce vibrations of large building structures. The acceleration feedback is introduced in a direct way using the properly defined quadratic performance index. The static output feedback is used to calculate the control forces. The Kleinman method is applied to solve the algebraic Riccati equation. Both active and semi-active control systems are considered. The vibrations of building structures excited by strong winds are considered in particular. Results of calculations are also briefly discussed.

Keywords: active and semi-active control, acceleration feedback, non-standard performance index, static-output feedback

1. Introduction

The most widely used sensors to measure the response of structures such as buildings are accelerometers. Direct measurement of absolute accelerations is an inexpensive and reliable method of measuring. This fact is not directly taken into account in the controllers design procedures because usually only the influence of state-space (i.e. displacements and velocities) is considered. In this case, when accelerations at some points of structures are measured the velocities and displacements can be determined by integrating data from accelerometers. Papers [2-4,6,10,12] describe implementations of the acceleration-based feedback controllers, mainly in an indirect way [2-4,12]. In papers [10,12] the acceleration feedback is introduced in a direct way together with the instantaneous control method. Paper [6] is also dedicated to developing the acceleration-based controller in the direct way. Moreover, the non-standard performance index is introduced and the method is used to the active control of the first mode of beam loaded by the harmonic force. The reported control effects are promising.

The previously mentioned method (described in [6]) is rather new and the performance of such control system used to reduce vibrations of large structures like buildings or towers has not been considered as yet. The paper is dedicated to clarifying the question. In particular, the possibility of reducing building accelerations is of interest. Moreover, the acceleration feedback controller mentioned above is used to develop the semi-active control system with the viscous dampers. The static output feedback is used to calculate the control forces directly on the basing of the measured velocities and acceleration only. In this point, the proposed method differs from the one proposed in [6].

The linear theory is used to describe the behaviour of building structures treated as elastic systems. Vibrations are excited by strong winds. The dynamic part of wind forces is considered as the stochastic and ergodic process, which has zero mean value. It is assumed that the power spectral density

functions of fluctuations of wind velocities are known. The problem is solved in a time domain.

2. Formulation

The equation of motion of building structures considered as the discrete linear dynamic systems can be written in the form:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{D}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \tilde{\mathbf{B}}\mathbf{u}(t) + \tilde{\mathbf{H}}\mathbf{f}(t), \quad (1)$$

where $\mathbf{M}, \mathbf{D}, \mathbf{K}$ and $\tilde{\mathbf{B}}, \tilde{\mathbf{H}}$ are the mass, damping, stiffness and allocation matrices, respectively. Symbols $\mathbf{q}(t), \mathbf{u}(t), \mathbf{f}(t)$ denote the vector of nodal displacements, the vector of control forces and the vector of excitation forces, respectively.

In the context of structural control, the state space vector $\mathbf{z}(t) = \text{col}(\mathbf{q}(t), \dot{\mathbf{q}})$ is introduced and the equation of motion is usually written in the following form:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{H}\mathbf{f}(t). \quad (2)$$

The matrices \mathbf{A}, \mathbf{B} and \mathbf{H} are defined by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \quad (3)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\tilde{\mathbf{B}} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\tilde{\mathbf{H}} \end{bmatrix}. \quad (4)$$

However, in the considered method, the motion of structures equation is rewritten in a different way:

$$\mathbf{z}(t) = \mathbf{G}\dot{\mathbf{z}}(t) + \mathbf{H}\mathbf{u}(t) + \mathbf{N}\mathbf{f}(t), \quad (5)$$

where

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$$\mathbf{G} = \mathbf{A}^{-1} = \begin{bmatrix} -\mathbf{K}^{-1}\mathbf{D} & -\mathbf{K}^{-1}\mathbf{M} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad (6)$$

$$\mathbf{H} = -\mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} \mathbf{K}^{-1}\tilde{\mathbf{B}} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{N} = -\mathbf{A}^{-1}\mathbf{H} = \begin{bmatrix} \mathbf{K}^{-1}\tilde{\mathbf{H}} \\ \mathbf{0} \end{bmatrix}. \quad (7)$$

Equation (5) could be understood as the reciprocal one in comparison with Eqn (2). Stability conditions for both the standard state space and the reciprocal state description are the same because the eigenvalues of the \mathbf{G} matrix are inverses of the eigenvalues of the \mathbf{A} matrix.

The non-standard performance index, which directly takes into account the influence of structure velocities and accelerations, is chosen in the following form:

$$J = \frac{1}{2} \int_0^{\infty} \left(\dot{\mathbf{z}}^T(t) \mathbf{Q} \dot{\mathbf{z}}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) \right) dt. \quad (8)$$

The control vector $\mathbf{u}(t)$ is chosen in such a way that the performance index (8) is minimised, subject to the constraining Eqn (5). The matrices \mathbf{Q} and \mathbf{R} are referred to as weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables $\dot{\mathbf{q}}(t)$, $\ddot{\mathbf{q}}(t)$ and to the control forces in the minimisation procedure. The performance index is non-standard because the first derivative vector of state space is present instead of the vector of state space.

To solve the above mentioned problem the Lagrangian L is first formed using a time-varying Lagrange multiplier vector $\dot{\lambda}(t)$

$$L = \frac{1}{2} \int_0^{\infty} \left[\dot{\mathbf{z}}^T(t) \mathbf{Q} \dot{\mathbf{z}}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) + \dot{\lambda}^T(t) (\mathbf{G} \dot{\mathbf{z}}(t) + \mathbf{H} \mathbf{u}(t) + \mathbf{N} \mathbf{f}(t) - \mathbf{z}(t)) \right] dt. \quad (9)$$

Please note that, in fact, the first derivative of the Lagrange multipliers appears in the functional (9).

The necessary condition of the optimal control is that the first variation of L is equal to zero (i.e. $\delta L = 0$), which leads us to the following equation:

$$\int_0^{\infty} \delta \dot{\mathbf{z}}^T \frac{\partial H}{\partial \dot{\mathbf{z}}} dt + \int_0^{\infty} \delta \mathbf{u}^T \frac{\partial H}{\partial \mathbf{u}} dt + \int_0^{\infty} \delta \dot{\lambda}^T \frac{\partial H}{\partial \dot{\lambda}} dt = 0, \quad (10)$$

where the Hamiltonian H is defined by

$$2H = \dot{\mathbf{z}}^T(t) \mathbf{Q} \dot{\mathbf{z}}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) + \dot{\lambda}^T(t) (\mathbf{G} \dot{\mathbf{z}}(t) + \mathbf{H} \mathbf{u}(t) + \mathbf{N} \mathbf{f}(t) - \mathbf{z}(t)). \quad (11)$$

Because variations $\delta \dot{\mathbf{z}}$, $\delta \mathbf{u}$ and $\delta \dot{\lambda}$ are independent, each integral in Eqn (10) must be equal to zero, separately.

The third integral in Eqn (10) can be rewritten in the form:

$$\int_0^{\infty} \delta \dot{\lambda}^T (\mathbf{G} \dot{\mathbf{z}}(t) + \mathbf{H} \mathbf{u}(t) + \mathbf{N} \mathbf{f}(t) - \mathbf{z}(t)) dt = 0, \quad (12)$$

from which the constraining Eqn (5) is obtained.

The second integral leads us to the following condition:

$$\mathbf{u}(t) = -\mathbf{R}^{-1} \mathbf{H}^T \dot{\lambda}(t), \quad (13)$$

which means that the control forces are proportional to the Lagrange multipliers.

In a similar way the first integral in (10) could be transformed to the following form:

$$\int_0^{\infty} \delta \dot{\mathbf{z}}^T (\mathbf{Q} \dot{\mathbf{z}}(t) + \mathbf{G}^T \dot{\lambda}) dt - \delta \dot{\lambda}^T \int_0^{\infty} \dot{\lambda}^T \mathbf{z}(t) dt. \quad (14)$$

After integrating by parts the second component of Eqn (14), the following equation

$$\mathbf{Q} \dot{\mathbf{z}}(t) + \mathbf{G}^T \dot{\lambda}(t) + \lambda(t) = \mathbf{0}, \quad (15)$$

could be obtained.

When the velocities and accelerations of structure regulate the control forces one has:

$$\dot{\lambda}(t) = \mathbf{P}(t) \dot{\mathbf{z}}(t), \quad (16)$$

where $\mathbf{P}(t)$ is the unknown matrix. Moreover, the initial condition $\lambda(0) = \mathbf{0}$ is attached to Eqn (16).

It is assumed that the structures under consideration are at rest for $t \leq 0$ (i.e. $\mathbf{z}(0) = \mathbf{0}$). Integrating Eqn (16) with respect to time and taking into account the above-mentioned assumptions, the relation (17) is obtained

$$\lambda(t) = \mathbf{P}(t) \mathbf{z}(t) - \int_0^t \dot{\mathbf{P}}(\tau) \mathbf{z}(\tau) d\tau. \quad (17)$$

After introducing Eqns (16), (17) and (5) into Eqn (15), the following is obtained

$$\left(\mathbf{G}^T \mathbf{P}(t) + \mathbf{P}(t) \mathbf{G} - \mathbf{P}(t) \mathbf{H} \mathbf{R}^{-1} \mathbf{H}^T \mathbf{P}(t) + \mathbf{Q} \right) \dot{\mathbf{z}}(t) + \mathbf{P}(t) \mathbf{N} \mathbf{f}(t) - \int_0^t \dot{\mathbf{P}}(\tau) \mathbf{z}(\tau) d\tau = \mathbf{0}. \quad (18)$$

The following assumptions, which significantly simplify our considerations, are made:

- the influences of excitation forces in Eqn (18) are neglected, which means that the obtained solutions to the optimisation problem considered will only be sub-optimal,
- the first derivatives of the $\mathbf{P}(t)$ matrix is almost equal to zero during the time of regulation, which means that

$$\int_0^t \dot{\mathbf{P}}(\tau) \mathbf{z}(\tau) d\tau \approx \mathbf{0}, \quad (19)$$

and the matrix $\mathbf{P}(t) = \mathbf{P} = const.$ is not the matrix of the functions of time.

Please note that the above assumptions are typical in the field of structural control (see, for example [8,11]).

Under these conditions, the algebraic Riccati equation in the form of relation (20) follows from Eqn (18)

$$\mathbf{G}^T \mathbf{P} + \mathbf{P} \mathbf{G} - \mathbf{P} \mathbf{H} \mathbf{R}^{-1} \mathbf{H}^T \mathbf{P} + \mathbf{Q} = \mathbf{0}. \quad (20)$$

The considerations above are based on the assumption that all the components of the vector $\mathbf{z}(t)$ are measured. Actually, the number of measurement sensors distributed in the structure is limited, so the dynamic state of the system is measured at the selected points only. Therefore, the following equation

$$\dot{\mathbf{y}}(t) = \mathbf{C}\dot{\mathbf{z}}(t) , \quad (21)$$

should be added to the equations expressing the problem. The symbols $\dot{\mathbf{y}}(t)$ and \mathbf{C} in the equation denote the vector of the measured dynamic parameters of the structure (velocity and acceleration at the selected points, respectively) and the sensor allocation matrix.

Equation (16) should now be rewritten as follows:

$$\dot{\lambda}\mathbf{y}(t) = \mathbf{P}\dot{\mathbf{y}}(t) = \tilde{\mathbf{P}}\dot{\mathbf{z}}(t) , \quad (22)$$

where

$$\tilde{\mathbf{P}} = \mathbf{P}\mathbf{C} . \quad (23)$$

The unknown matrix $\tilde{\mathbf{P}}$ is found from the following Riccati equation:

$$\mathbf{G}^T\tilde{\mathbf{P}} + \tilde{\mathbf{P}}\mathbf{G} - \tilde{\mathbf{P}}\mathbf{H}\mathbf{R}^{-1}\mathbf{H}^T\tilde{\mathbf{P}} + \mathbf{Q} = \mathbf{0} . \quad (24)$$

Equation (23) remains to be solved: it is an overdeterminate equation and is solved by the least-square minimisation method. The objective function is introduced in the following form:

$$E_i = (\mathbf{p}_i\mathbf{C} - \tilde{\mathbf{p}}_i)^T (\mathbf{p}_i\mathbf{C} - \tilde{\mathbf{p}}_i) , \quad (25)$$

where the symbols \mathbf{p}_i , $\tilde{\mathbf{p}}_i$ denote the i -th rows of the matrix \mathbf{P} , $\tilde{\mathbf{P}}$ written as the row vectors.

The objective function E_i has its minimum when:

$$\mathbf{p}_i = \tilde{\mathbf{p}}_i\mathbf{C}^T(\mathbf{C}\mathbf{C}^T)^{-1} , \quad (26)$$

and the matrix \mathbf{P} is obtained by calculating the rows of the matrix one by one, using the formula (26).

The active control forces vector is calculated from the formula:

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{H}^T\mathbf{P}\dot{\mathbf{y}}(t) , \quad (27)$$

3. Solution to the Riccati equation

Let us give the Riccati Eqns (20) and (24) a coherent form:

$$\mathbf{G}^T\mathbf{P} + \mathbf{P}\mathbf{G} - \mathbf{P}\mathbf{D}\mathbf{P} + \mathbf{Q} = \mathbf{0} , \quad (28)$$

where $\mathbf{D} = \mathbf{H}\mathbf{R}^{-1}\mathbf{H}^T$ and \mathbf{P} denotes the matrix $\tilde{\mathbf{P}}$ in solving Eqn (24).

Equation (28) is solved using the Kleinman method [5], based on iterations. The steps of the calculation algorithm are described below:

- Step 1
Assume the initial approximation of the matrix \mathbf{P} . The approximation is denoted by the symbol \mathbf{P}_0 . An approximation should be selected for which the matrix

$\mathbf{G} - \mathbf{H}\mathbf{R}^{-1}\mathbf{H}^T\mathbf{P}$ is a negative definite. The calculations described in this paper were made assuming that $\mathbf{P}_0 = \mathbf{0}$.

- Step 2
Knowing the k -th approximation of the matrix \mathbf{P} (i.e. the matrix \mathbf{P}_k), solve the Lyapunov equation below with respect to \mathbf{P}_{k+1} ,

$$\mathbf{P}_{k+1}(\mathbf{G} - \mathbf{D}\mathbf{P}_k) + (\mathbf{G} - \mathbf{D}\mathbf{P}_k)^T\mathbf{P}_{k+1} + \mathbf{P}_k\mathbf{D}\mathbf{P}_k + \mathbf{Q} = \mathbf{0} . \quad (29)$$

- Step 3
Verify the following convergence conditions of the iteration process

$$\|\mathbf{P}_{k+1} - \mathbf{P}_k\| \leq \varepsilon_1 \|\mathbf{P}_{k+1}\| ; \quad \|\mathbf{R}_{k+1}\| \leq \varepsilon_2 \|\mathbf{Q}\| , \quad (30)$$

where the symbols ε_1 and ε_2 denote the assumed calculation accuracies, moreover

$$\mathbf{R}_{k+1} = \mathbf{G}^T\mathbf{P}_{k+1} + \mathbf{P}_{k+1}\mathbf{G} - \mathbf{P}_{k+1}\mathbf{D}\mathbf{P}_{k+1} + \mathbf{Q} = \mathbf{0} . \quad (31)$$

If the iteration conditions are satisfied, then the recent obtained approximation of the matrix \mathbf{P} (namely, the matrix \mathbf{P}_{k+1}) is considered to be the solution to the Riccati equation. Otherwise, go back to Step 2 to perform another iteration.

The Bartles-Stewart method as described in [1] was used to solve the Lyapunov equation (29).

4. The semi-active control of vibration

The possibility of using the subject method to designing semi-active vibration controllers is also considered in this paper. It is assumed that the semi-active control force actuators are of the viscous type. Therefore, the semi-active force regulation, excited in the actuator bearing the number “ i ” is calculated from the formula

$$u_i(t) = c_i(t)\dot{x}_i(t) , \quad (32)$$

where $x_i(t)$ denotes displacement of the actuator piston relative to its casing, and $c_i(t)$ denotes a time-varying damping factor.

The design of the damper enables the damping factor $c_i(t)$ to be changed in the range (c_{\min} , c_{\max}), i.e.

$$0 \leq c_{\min} \leq c_i(t) \leq c_{\max} . \quad (33)$$

The relative piston displacements $x_i(t)$ may easily be expressed by displacement of the points of the structure the actuator is connected with.

The process to design a semi-active control system comprises two steps. In the first step, the active control system is designed as described above. The active control system has the same configuration as the semi-active system (that is, the sensors and actuators are distributed at the same points) except that it uses active type actuators. Specifically, after finding the matrix \mathbf{P} from the Riccati equation, the control force vector is calculated from the formula

$$\tilde{\mathbf{u}}(t) = -\mathbf{R}^{-1}\mathbf{H}^T\mathbf{P}\dot{\mathbf{y}}(t) , \quad (34)$$

where the symbol $\tilde{\mathbf{u}}(t)$ denotes the vector of the desirable control forces.

Please note that the desirable control forces may not always be excited in the semi-active actuator due to the limited range of potential variation of the damping factor $c_i(t)$.

In the second step of designing the semi-active control system, the limitations expressed by Eqn (32) and inequality (33) are taken into account. The desirable damping factor $\tilde{c}_i(t)$ is calculated from the formula

$$\tilde{c}_i(t) = \frac{\tilde{u}_i(t)}{\dot{x}_i(t)} . \quad (35)$$

If inequality (33) is satisfied by the factor $\tilde{c}_i(t)$, then $c_i(t) = \tilde{c}_i(t)$. Otherwise, the factor $c_i(t)$ assumes the value c_{\min} or c_{\max} which can be expressed as follows:

$$c_i(t) = \begin{cases} c_{\min} & \text{if } c_i(t) < c_{\min} \\ \tilde{c}_i(t) & \text{if } c_{\min} \leq c_i(t) \leq c_{\max} \\ c_{\max} & \text{if } \tilde{c}_i(t) > c_{\max} \end{cases} , \quad (36)$$

and the semi-active control force may be calculated from the formula (32).

5. Solution to the motion equation

The solution to the motion equation (2) may be written as follows (see [8,11]):

$$\mathbf{z}(t) = \exp(\mathbf{A}t)\mathbf{z}_0 + \int_0^t \exp[\mathbf{A}(t-\tau)] [\mathbf{B}\mathbf{u}(\tau) + \mathbf{H}\mathbf{f}(\tau)] d\tau , \quad (37)$$

where \mathbf{z}_0 denotes the state vector at the initial instant.

The integral in the formula (37) is calculated numerically. It is assumed that in a small range of time (t_n, t_{n+1} , $t_{n+1} = t_n + h$), where h is the time step, the excitation forces and the control forces are constant, that is:

$$\mathbf{f}(\tau) = \mathbf{f}(t_n) = \mathbf{f}_n = \text{const.} \quad (38)$$

$$\mathbf{u}(\tau) = \mathbf{u}(t_n) = \mathbf{u}_n = \text{const.} \quad (39)$$

If the instant t_n is considered to be the initial instant, then the state of the system at the instant t_{n+1} may be determined from the formula ($\mathbf{z}(t_{n+1}) = \mathbf{z}_{n+1}$)

$$\mathbf{z}_{n+1} = \Phi(h)\mathbf{z}_n + \Gamma(h)(\mathbf{B}\mathbf{u}_n + \mathbf{H}\mathbf{f}_n) , \quad (40)$$

where (see [8])

$$\Phi(h) = \sum_{i=0}^{\infty} \frac{1}{i!} \mathbf{A}^i h^i , \quad (41)$$

$$\Phi(h) = \sum_{i=1}^{\infty} \frac{1}{(i+1)!} \mathbf{A}^i h^i . \quad (42)$$

The recurrence formula (40) is used to determine the state of the system in the consecutive instants of time.

6. Results of example calculations

Calculations were made for a 20-storey building, exposed to forces excited by the wind. The calculation model of a building is the so-called shear frame, described, *inter alia*, in [9]. The weight of the building is concentrated at the floor level, and the frame girders are infinitely rigid, compared with the columns.

The following characteristics of the building were assumed for the calculations:

- storey height $H = 4.0$ m ;
- column rigidity $EJ = 1.638 \cdot 10^8$ kNm² ;
- dimensionless damping factor $\gamma = 0.05$,
- storey weight $M = 27000$ kg .

The first natural frequency of building is 3.6536 rad/s.

The wind is considered to be the stationary random process with zero mean value. The random characteristics of the wind velocity fluctuations are described by the spectral density function, proposed by Kaimal. Typical time variances of the wind velocity fluctuations $w_j(t)$ were generated as described in [7].

The dynamic forces excited by the wind are calculated from the formula

$$f_j(t) = \rho C_d A_j U_j w_j(t) , \quad (43)$$

where the symbols ρ , C_d , A_j and U_j denote the air density, drag factor, exposure area related to the point "j", and the wind mean velocity at that very point. The forces excited by the wind operate at the floor level. The time variances of the excitation force exerted onto the 20th floor storey are shown in Fig. 1.

The control system comprises 8 actuators distributed on the storeys 1-8. Calculations were made based on the assumption that all the components of the vector $\mathbf{z}(t)$ are measured.

Moreover, it was assumed that $\mathbf{Q} = 4\mathbf{I}$, $\mathbf{R} = \varepsilon\mathbf{I}$, $\varepsilon = 10^{-10}$ where \mathbf{I} is the unit matrix.

The calculation results are shown in Figs 2–5. Figure 2 shows the time variances of floor displacement on the 20th storey of the building. The thick line shows the displacement of the structure with the active control system installed in it. The thin line shows the displacement of the structure without any control system. It can be seen that the active control system may significantly reduce vibration amplitudes.

Figures 3 and 4 show a similar comparison for the time variances of the velocity and acceleration of the floor at the 20th level. Figure 5 illustrates the time variance of the control force, excited by the actuator installed on the 1st floor.

It can be seen that the accelerations are also significantly reduced. It is very important because the occupants' comfort is a prime aim in a case of buildings exposed to strong wind excitations. Moreover, the largest peaks of displacements, velocities and accelerations are reduced in greater degree than small peaks.

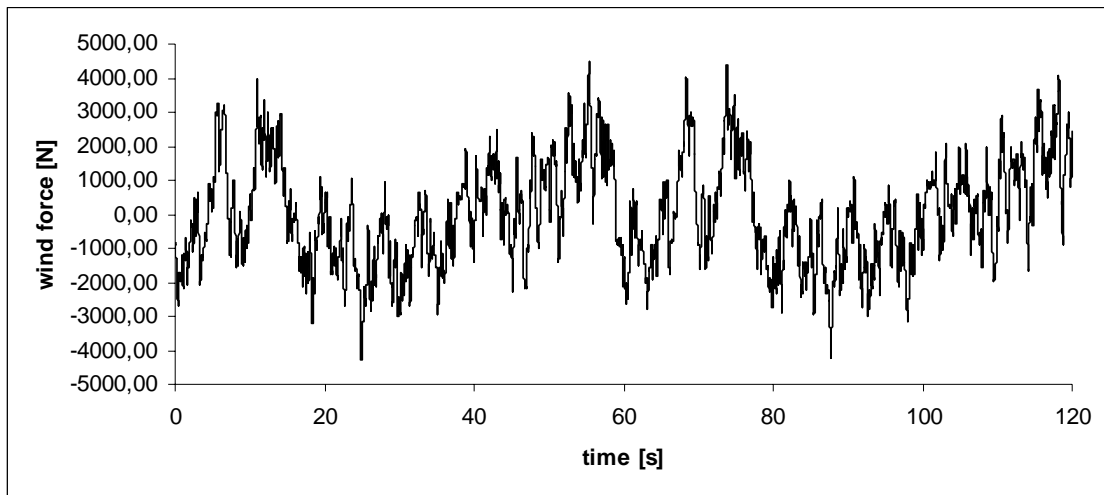


Fig.1 Variances of wind force acting on twentieth floor

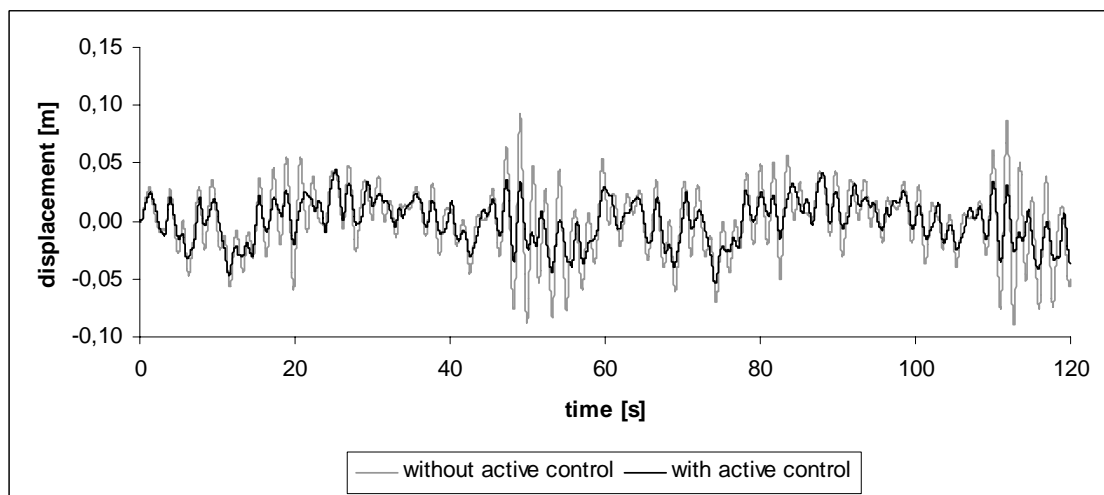


Fig. 2 Comparison of horizontal displacement of twentieth floor

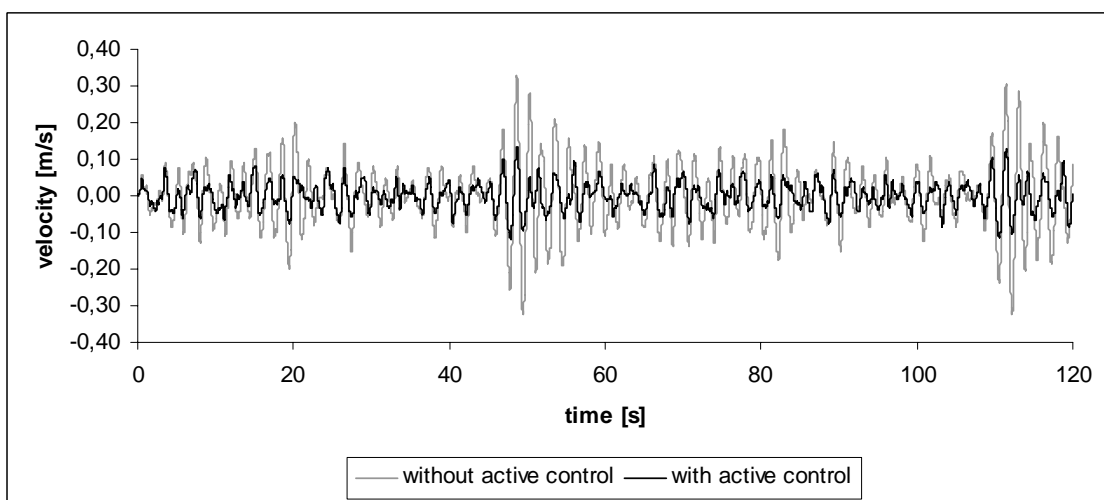


Fig. 3 Comparison of velocity of twentieth floor

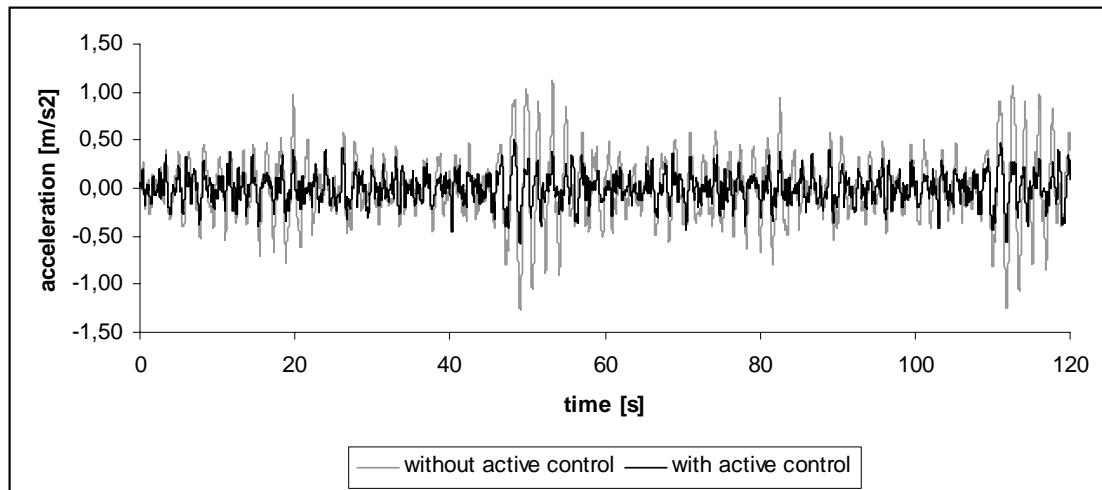


Fig. 4 Comparison of acceleration of twentieth floor

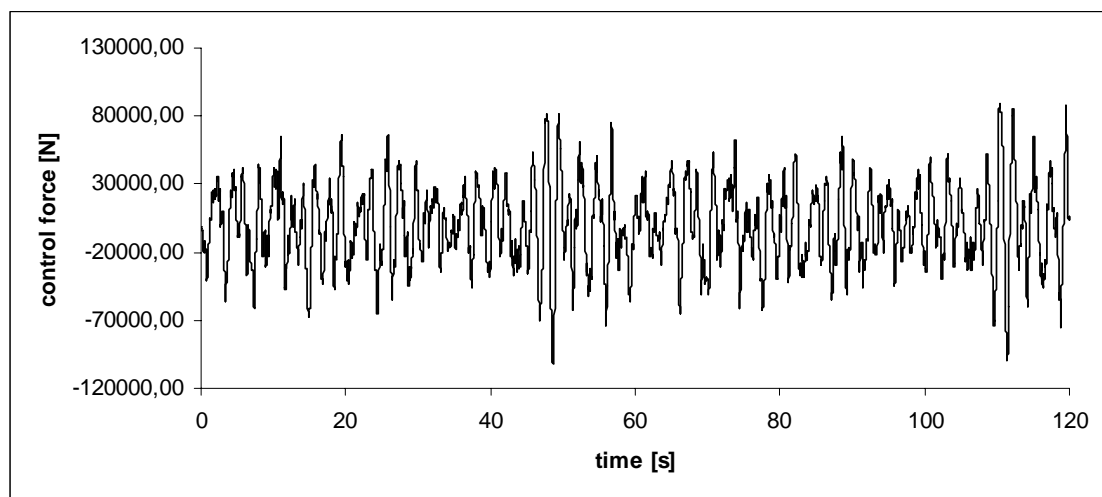


Fig. 5 Variance of the active force excited by the actuator installed on the first floor

The Kleinman method is very effective if the Riccati equation for small buildings is solved. Some problems with the iteration convergence occur if the tall buildings are analysed. This problem needs additional studies.

7. Final remarks

In this paper, the possibility of reducing of civil structures vibrations using the acceleration-based feedback controller is investigated. The non-standard performance index is used and the acceleration feedback is introduced in a direct way. The behaviour of structures with and without control system is simulated numerically. The preliminary results of calculation are promising because the control system can suppress accelerations significantly.

References

- [1] Bartels, R. and Stewart, G., Solution of the matrix equation $AX+XB=C$; Algorithm 432, *Communications of ACM*, 15, pp.820-826, 1972
- [2] Chung, L.L., Wu, L.Y. and Jin, T.G., Acceleration feedback control of seismic structures, *Engineering Structures*, 20, pp.62-74, 1998.
- [3] Dyke, S.J., Spencer, B.F., Quast, P., Sain, M.K., Kaspari, D.C. and Soong, T.T., Acceleration feedback control of MDOF structures, *Journal of Engineering Mechanics*, 122, pp. 907-917, 1996.
- [4] Jabbari, F., Schmitendorf, W.E. and Yang J.N., H-infinity control for seismic-excited buildings with acceleration feedback, *Journal of Engineering Mechanics*, 121, pp.994-1002, 1995.
- [5] Kleinman, D.L., On an iterative technique for Riccati equation computations, *IEEE Transactions on Automatic Control*, AC-13, pp.114-115, 1968.

- [6] Kwak, S.K., Washington, G. and Yedavalli R.K., Acceleration-based vibration control of distributed parameter systems using the “reciprocal state-space framework”, *Journal of Sound and Vibration*, 251, pp.543-557, 2002.
- [7] Lewandowski, R.: Application of semi-empirical model to analysis of vortex-excited vibrations of beams near synchronisation region, in *Computational Civil and Structural Engineering* (eds. G.De Roeck and B.H.V. Topping), Civil-Comp Press, Edinburgh, pp.133-141, 2000.
- [8] Meirovitch, L., *Dynamics and control of structures*, John Wiley, New York, 1990.
- [9] Paz, M.: *Structural dynamics: Theory and computation*, Van Nostrand Reinhold Company, New York, 1985.
- [10]Rofooei, F.R. and Tadjbakhsh, I.G., Optimal control of structures with acceleration, velocity and displacement feedback, *Journal of Engineering Mechanics*, 119, pp.1993-2010, 1993.
- [11]Soong, T., *Active structural control*, Longman Scientific & Technical, New York, 1990.
- [12]Yang, J.N., Li, Z., and Liu, S.C., Instantaneous optimal control with velocity and acceleration feedbacks, *Journal of Probabilistic Engineering Mechanics*, 16, pp.204-211, 1991.