# The Design of an Active Seismic Control System for a Building Using the Particle Swarm Optimization

Adam Schmidt[1] and Roman Lewandowski[2]

[1] Poznan University of Technology, Institute of Control and Information Engineering
Adam.Schmidt@put.poznan.pl
[2] Poznan University of Technology, Institute of Structural Engineering
Roman.Lewandowski@put.poznan.pl

**Abstract.** Recently significant attention has been paid to the active reduction of vibrations in civil constructions. In this paper we present the synthesis of an active control system using the particle swarm optimization method. The controller design is analyzed as a building stories' displacement minimalization problem. The proposed fitness function is computationally efficient and incorporates the constraints on the system's stability and actuators' maximum output. The performance of the obtained controller was tested using historical earthquake records. The performed numerical simulations proved that the designed controller is capable of efficient vibrations reduction.

**Keywords:** active vibration reduction, particle swarm optimization, earthquake engineering.

## 1 Introduction

Recently it is common to design and construct lightweight and cost-efficient buildings. However, these light constructions are often more susceptible to vibrations caused either by human or by natural sources such as earthquakes. Therefore, it is important to design methods and technologies which would provide means to reduce the unwanted vibrations in buildings.

The active control system concept, which was proposed for the first time in a context of earthquake engineering by Yaoby Yao [1], is one of the possible solutions to that problem. It consists of a controller, sensors measuring the state of the building (displacements, velocities and accelerations) and actuators generating forces opposing to the forces induced by the environment. As the active control system introduces additional forces to the structure it is neccessary to make sure that it would not destabilize the building.

Various methods have been used to design the controller for the active control systems. These include linear quadratic regulators (LQR) [4][5], instantaneuos optimal control [6][7], linear quadratic Gaussian (LQG) [8][9][10], $H_2$ and $H_\infty$ [8][11][12][13], pole placement[14], modal control[15], sliding mode control[16], fuzzy control[17] or artificial neural networks[18][19].

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Over the last years the Particle Swarm Optimization has been succesfully used in the controller design. In [20] and [21] the PSO has been used in the optimization of the PI and PID controllers. Wang et al.[22] have applied the PSO to find the control system poles resulting in a robust control system. In [23] the PSO has been succesfully used to find the optimal feedback gain in the vehicle navigation system controller.

In this paper we present the design of the building active control system using the PSO. The constrained controller design is formulated as an optimization problem. The proposed fitness function minimizes the building's structure displacements and incorporates constraints on the system's stability and requirements concerning maximum forces generated by actuators. The effectiveness of the presented method is assessed on the model of a six-story building under different earthquakes.

## 2 Structure Model

The building is modeled as a shear planar frame with actuators installed between some of its stories. It is assumed that the braces supporting actuators are infinitely stiff. The motion equations of the system can be defined as:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = Eu(t) + f(t) \tag{1}$$

where $M$, $C$, $K$ and $E$ stand for the mass, damping, stiffness and location matrices. The $q(t)$ is a vector of the stories displacements relative to the ground, $u(t)$ is a vector of the forces generated by the actuators and the $f(t)$ is a vector of the forces induced by the earthquake.

Under the assumption that the mass of each floor is lumped the mass matrix $M$ of N-stories building takes a form of a diagonal matrix with the masses of the subsequent floors on the main diagonal:

$$M = diag(m_1, m_2, \dots m_N) \tag{2}$$

The stiffness matrix $K$ is defined as:

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -k_{N-1} & k_{N-1} + k_N & -k_N \\ 0 & 0 & \dots & 0 & -k_N & k_N \end{bmatrix} \tag{3}$$

where $k_i$ is the stiffness of the $i$-th story. In our study we assumed that $N = 6$, $m_i = 10000kg$ and $k_i = 2250000\frac{N}{m}$. The damping matrix is calculated as a linear combination of the mass and stiffness matrices:

$$C = \alpha M + \beta K \tag{4}$$

$$\alpha = \frac{2\omega_1\omega_2(\gamma_1\omega_2 - \gamma_2\omega_1)}{(\omega_2^2 - \omega_1^2)} \tag{5}$$

$$\beta = \frac{2(\gamma_1\omega_2 - \gamma_2\omega_1)}{(\omega_2^2 - \omega_1^2)} \tag{6}$$

where $\omega_1 = 3.6161\frac{rad}{s}$ and $\omega_2 = 10.6381\frac{rad}{s}$ are the structural modal frequencies of the first and the second mode of the uncontrolled system and $\gamma_1 = 0.02$ and $\gamma_2 = 0.02$ are the dimensionless modal damping ratios.

The location matrix $E$ defines the actuators positions in the building structure. We considered a building with two actuators placed below the first and the third stories. The resulting location matrix took form:

$$E = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

The vector of earthquake induced forces is calculated as:

$$f(t) = M \cdot 1_{N\times 1} a_{gr}(t) \tag{8}$$

where $a_{gr}(t)$ is the acceleration of the ground.

The forces generated by the actuators are calculated according to the structure displacements $q(t)$ and velocities $\dot{q}(t)$. It is assumed, that all displacements and velocities are measured, which means that:

$$u(t) = -G_1 q(t) - G_2 \dot{q}(t) \tag{9}$$

where $G_1$ and $G_2$ are the gain matrices of the control system feedback loop. The Eqn. 1 can be rewritten as:

$$M\ddot{q}(t) + (C + EG_2)\dot{q}(t) + (K + EG_1)q(t) = M \cdot 1_{N\times 1} a_{gr}(t) \tag{10}$$

The model can be presented in the following form of the state equations:

$$z(t) = \begin{bmatrix} q(t) \ \dot{q}(t) \end{bmatrix}^T \tag{11}$$

$$\dot{z}(t) = Az(t) + Ba_{gr}(t) \tag{12}$$

$$A = \begin{bmatrix} 0_{(N\times N)} & 1_{(N\times N)} \\ -M^{-1}(K + EG_1) & -M^{-1}(C + EG_2) \end{bmatrix} \tag{13}$$

$$B = \begin{bmatrix} 0_{1\times N} \ 1_{1\times N} \end{bmatrix}^T \tag{14}$$

## 3 Particle Swarm Optimization

### 3.1 Description

The Particle Swarm Optimization (PSO) introduced by Kennedy and Eberhart [2] is an population-based optimization technique inspired by social behaviour of animals e.g. birds flocking or fish schooling. Similarly to the genetic algorithm the populations consists of possible solutions (called particles) and the search for an optimal solutions is performed by updating subsequent generations. However, no evolutionary operators such as cross-over or mutation are used. Instead, each particle explores the problem space being drawn to the current optimal solutions.

The main difference between GA and PSO is the memory of particles. Each particle keeps a record of its best fitness achieved so far (along with the associated solution $Pb$) and the best fitness and corresponding solution achieved in the particle's neighborhood - $Lb$. It was shown that using the global neighborhood (all particles are fully aware of other particles' fitness) minimizes the median number of iterations needed to converge. On the other hand the neighborhood of size 2 gives the highest resistance to local minima.

At each iteration $i$ of the PSO the velocities of the particles are changed (accelerated) towards the $Pb$ and the $Lb$ and the particles are moved to new positions:

$$v_j(i) = w \cdot v_j(i-1) + c_1 \cdot rand \cdot (Pb_j - p_j(i)) + c_2 \cdot rand \cdot (Lb_j - p_j(i)) \quad (15)$$
$$p_j(i) = p_j(i-1) + v_j(i) \quad (16)$$

where $v_j$ and $p_j$ are the velocity and the position of the $j$-th particle, $w$ is the intertia factor providing balance between the exploration and the exploitation, $c_1$ is the individuality constant and $c_2$ is the sociality constant.

To avoid the "velocity explosion" the maximum velocity constraint $v_{max} = 100000$ is introduced. Whenever the velocity violates the $[-v_{max}, v_{max}]$ limits it is truncated to that range. Additionally, to speed up the convergence the inertia weight was linearly reduced from $w_{max} = 0.9$ to $w_{min} = 0.1$.

In our experiments we have used $n = 20$ particles, the maximum number of iterations $i_{max} = 200000$ and $c_1 = c_2 = 2$. The neighborhood of size 4 was used as a tradeoff between the fast convergence and the resistance to local minima. More information on the parameters selection of the algorithm and its variations can be found in [3].

### 3.2 Fitness Function and Covergence Criterion

The optimization goal was to find the gain matrices $G_1$ and $G_2$ that would minimize the displacements of building's stories under the earthquake. Additionally, the resulting model had to be stable and the generated forces had to be lower than an assumed value ($F_{max} = 100000N$). Those constraints were incorporated into the fitness function:

$$\mathrm{fit}(p) = \mathrm{fit}_{displacement}(p) + a \cdot \mathrm{fit}_{stability}(p) + b \cdot \mathrm{fit}_{forces}(p) \quad (17)$$

where $a$ and $b$ are constraints coefficients.

The displacements of the structure were analyzed in the frequency domain under the simplifying assumption that the ground acceleration is a sinusoidal signal. The following transfer function was defined:

$$H_{disp}(j\omega) = \frac{Q(j\omega)}{A_{gr}(j\omega)} = (-M\omega^2 + j(C + E \cdot G_2)\omega + K + E \cdot G_1)^{-1} M \cdot 1_{N\times 1} \quad (18)$$

where $Q(j\omega)$ and $A_{gr}$ are the Fourier transforms of the displacements and ground acceleration respectively.

The biggest (and thus the most dangerous for the structure) displacements are generated for the modal frequencies of the resulting system. Therefore, the following fitness function component was defined:

$$\text{fit}_{displacement}(p) = \sum_{i=1}^{N} 1_{1\times N}|H_{disp}(j\omega_i)| \tag{19}$$

where $\omega_i$ is the $i$-th modal frequency of the closed-loop system.

The resulting system would be stable if the real parts of all the system's poles were smaller than 0. The fitness function stability component was calculated as:

$$\text{fit}_{stability}(p) = \begin{cases} 1 + \max(\Re(e_i)) - \rho & \text{if } \max(\Re(e_i)) \geq \rho \\ 0 & \text{if } \max(\Re(e_i)) < \rho \end{cases} \tag{20}$$

where $e_i$ is the $i$-th eigenvalue of the state matrix $A$ and $\rho$ is the maximal allowed real part of the system's poles.

It was assumed that the actuators should not saturate until the ground acceleration amplitude reached a certain value ($A_{max} = 1\frac{m}{s^2}$) at any of the system's modal frequencies. The following transfer function was defined:

$$H_{force}(j\omega) = \frac{U(j\omega)}{A_{gr}(j\omega)} = (-G_1 - jG_2\omega)H_{disp}(j\omega) \tag{21}$$

The force component of the fitness function was calculated according to:

$$\text{fit}_{force}(p) = \begin{cases} \max\left(\frac{|H_{force}(j\omega)|A_{max}}{F_{max}}\right) & \text{if } \max\left(\frac{|H_{force}(j\omega)|A_{max}}{F_{max}}\right) > 1 \\ 0 & \text{if } \max\left(\frac{|H_{force}(j\omega)|A_{max}}{F_{max}}\right) \leq 1 \end{cases} \tag{22}$$

The $a$ parameter was set to 1000000 and the $b$ was set to 1000. This ensured that any solution resulting in an unstable system would have higher fitness function value than any of the stable ones and that solutions violating the maximum force limits would have worse fitness than those conforming to both constraints.

The proposed fitness function can be calculated without the time consuming simulations which is an important advantage in any iterative optimization algorithm.

The convergence of the PSO was assumed if the best fitness value in the population had not changed over $i_{conv} = 5000$ iterations and the best fitness value had been smaller than $\min(a, b)$ meaning that the solution conformed to both constraints.

## 4 Results

The optimization process was executed 100 times. On average the algorithm needed 20848 iteration to converge (median = 19856). The best result obtained

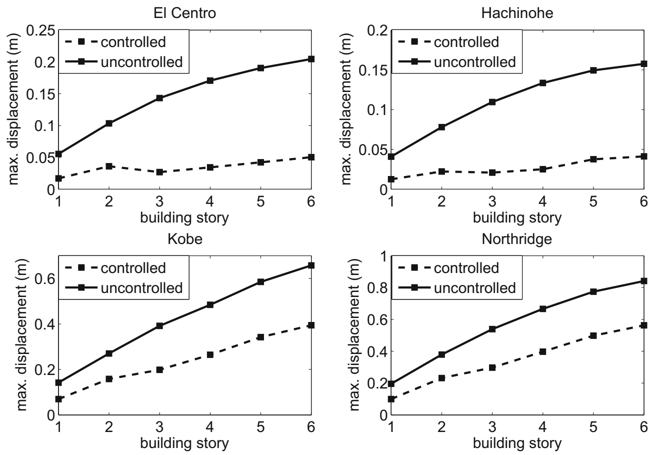


**Fig. 1.** The maximum displacement of building stories

**Table 1.** The normalized RMS of building displacement

| El Centro | Hachinohe | Kobe | Northridge |
|---|---|---|---|
| 0.1525 | 0.1703 | 0.353 | 0.4469 |

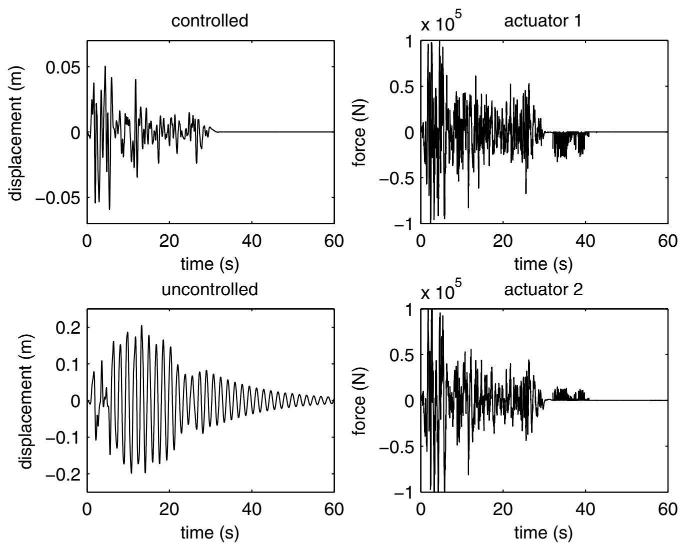


**Fig. 2.** The displacement of the top floor(left) and the forces generated by the actuators (right) under the El Centro earthquake

was equal to 0.1745, average fitness function of the converged solutions was equal to 0.3784 with the standard deviation of 0.0825 .

The performance of the obtained controller was tested in numerical simulations. Four different earthquake records were used: El Centro, Hachinohe, Kobe, Northridge. The peak ground accelerations of these earthquakes were: $0.3188g$, $0.2294g$, $0.8337g$, $0.8428g$ respectively. The maximum displacement of the building stories is shown in the Figure 1. The example of the top story displacement of both controlled and uncontrolled building as well as the forces generated by the actuators are shown in the Figure 2. Additionally, the normed RMS of structure displacement was calculated for all of the considered earthquakes (Table 1):

$$D_{RMS} = \frac{\max_i \sqrt{\frac{1}{T}\int_0^T q_i^c(t)^2 \mathrm{dt}}}{\max_i \sqrt{\frac{1}{T}\int_0^T q_i^{uc}(t)^2 \mathrm{dt}}} \tag{23}$$

where $i$ is the story number, $T$ is the duration time of the earthquake, $q_i^c$ is the displacement of the $i$-th story of the controlled building and $q_i^{uc}$ is the displacement of the $i$-th story of the uncontrolled building.

## 5 Conclusions

This paper presents a successfull attempt to design an active vibrations control system using the PSO technique. To the extent of the authors' knowledge this was the first attempt to design an active building controller by optimization of static, direct feedback gain matrices. A novel, computationally efficient fitness function minimizing stories displacements and incorporating the control system constraints was defined.

The obtained controller was tested under different historical earthquake loads. It achieved excellent results in the terms of the absolute stories displacements reduction and the normalized RMS of displacements.

The presented study shows that the PSO can be succesfully used to design active control systems. In our future work we will focus on modifying the fitness function to take into account the accelerations of building stories as well as on incorporating the number and positions of sensors and actuators into the optimization scheme. Moreover, the performance of different optimization methods such as simulated annealing or evolutionary algorithms will be tested.

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