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NON-LINEAR VIBRATION OF BEAMS WITH GAPS AT SUPPORTS

The problem of non-linear, periodic vibration of beams with gaps at supports is considered. The steady state responses of structures excited by harmonic forces are of particular interest. The periodic solutions are described using single harmonic function of time. The unknown coefficients of harmonic function are determined from the non-linear amplitude equations. The matrix amplitude equation is derived with the help of the Galerkin method. The unilateral constrains are taken into account in a course of determination of amplitude equation coefficients. The amplitude equation is treated as the equation with parameter and the frequency of excitation is chosen as the main parameter. The incremental-iterative procedure is used to solve the amplitude equation and to determine the response curves. Results of example calculations are also presented and briefly discussed.

Key words: non-linear vibration, steady state response, unilateral constrains, beams

1. INTRODUCTION

In this paper we consider the problem of non-linear, steady state vibration of beams with clearances at supports. In particular, we analyse the beams excited by harmonic forces. These types of structures have a piecewise linear characteristic. The behaviour of such structures could be strongly non-linear.

The static problems of the so-called slackened frame structures (i.e. structures with gaps at structural joints and supports) have been analysed by Gawęcki *et al.* (see [1,2]) where an advanced computational model is introduced.

The particular problem considered in this paper belongs to a wider class of dynamics of systems with non-smooth characteristics. The dynamic problems of such systems, treated as systems with many degrees of freedom, are rarely considered [3,4]. Unfortunately, the beam structures are not investigated in detail. Only one-degree of freedom systems with piecewise linear stiffness are extensively studied in many papers (see, for example [5,6]).

The paper is organised as follows. The assumptions and equations of motion are discussed in Section 2. In Section 3 the matrix amplitude equations are derived with the use of the Galerkin method. The method of determination of response curves is described in Section 4. The results of example calculations are presented and discussed in Section 5.

2. ASSUMPIONS AND EQUATIONS OF MOTION

Consider the elastic beam structure excited by harmonic forces. The gaps exist between the beam and some beam supports. However, it is assumed that there are supports without gaps and the beam on these supports can be understood as a geometrically immovable structure. The beam with all gaps open will be called the basic structure. Two kinds of gaps, the linear and angular ones, are taken into account in particular. The friction forces at supports are neglected. Moreover, the mass of the is lumped at some points. It is also assumed that there are no masses at points where the gaps can occur. Displacements of beams and gaps are small enough so the linear theory of kinematics can be used. An example of the considered system is shown in Fig.1.

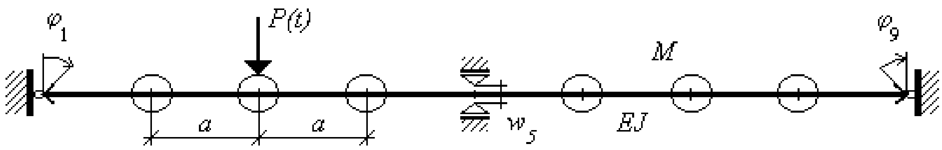

Fig.1 The beam with gaps at supports

The unilateral conditions are written in the following form [1]:

$$\mathbf{g}(t) = \mathbf{N}^T \mathbf{w}(t) - \mathbf{w}_o \leq \mathbf{0} \tag{2.1}$$

$$sign\mathbf{r}(t) = sign\mathbf{w}(t) \tag{2.2}$$

$$\mathbf{r}^T(t)\mathbf{g}(t) = 0 \tag{2.3}$$

where $\mathbf{r}(t), \mathbf{w}(t), \mathbf{w}_o, \mathbf{N}$ are, respectively, the vector of support reactions, the vector of beam displacements at supports, the vector of limits gaps at supports and the matrix of compatibility. The inequality (1) can be also written in the form $\mathbf{w}_o^- \leq \mathbf{w}(t) \leq \mathbf{w}_o^+$, where $\mathbf{w}_o^-, \mathbf{w}_o^+$ are the vectors of lower and upper limits of gaps.

The finite element method is used to model the beam in a usual way. Taking into account the above assumptions, we can write the equation of motion in the form:

$$\widetilde{\mathbf{M}}\ddot{\widetilde{\mathbf{v}}}(t) + \widetilde{\mathbf{C}}\dot{\widetilde{\mathbf{v}}}(t) + \widetilde{\mathbf{K}}\widetilde{\mathbf{v}}(t) - \widetilde{\mathbf{p}}(t) - \widetilde{\mathbf{r}}(t) = \mathbf{0} \tag{2.4}$$

where symbols $\widetilde{\mathbf{M}}, \widetilde{\mathbf{C}}, \widetilde{\mathbf{K}}, \widetilde{\mathbf{v}}(t), \widetilde{\mathbf{p}}(t), \widetilde{\mathbf{r}}(t)$ denote the mass and damping matrices, the stiffness matrix of the basic structure, the vector of nodal displacements of beam, the vector of nodal excitation forces and the vector of reactions at supports with gaps, respectively. Dots indicate differentiation with respect to time. In general, at time t, the vector $\widetilde{\mathbf{v}}(t) = col(\mathbf{v}_d(t), \mathbf{v}_l(t), \mathbf{v}_a(t))$ contains three types of displacements, i.e. the displacements $\mathbf{v}_d(t)$ that cannot be in contact with support, the displacements $\mathbf{v}_l(t)$, which can be in contact, but they are not at the present time, and the displacements $\mathbf{v}_a(t)$, which are currently in contact with supports. It is obvious that dimensions of vectors $\mathbf{v}_l(t)$ and $\mathbf{v}_a(t)$ change in time but in all cases $\mathbf{w}(t) = col(\mathbf{v}_l(t), \mathbf{v}_a(t))$.

Now, taking into account these assumptions, the above equation could be also written in the following partitioned form:

$$\mathbf{M}\ddot{\mathbf{v}}_d + \mathbf{C}\dot{\mathbf{v}}_d + \mathbf{K}_{dd}\mathbf{v}_d + \mathbf{K}_{dl}\mathbf{v}_l + \mathbf{K}_{da}\mathbf{v}_a - \mathbf{p}(t) = \mathbf{0} \tag{2.5}$$

$$\mathbf{K}_{ld}\mathbf{v}_d + \mathbf{K}_{ll}\mathbf{v}_l + \mathbf{K}_{la}\mathbf{v}_a = \mathbf{0} \tag{2.6}$$

$$\mathbf{K}_{ad}\mathbf{v}_d + \mathbf{K}_{al}\mathbf{v}_l + \mathbf{K}_{aa}\mathbf{v}_a - \mathbf{r}_a(t) = \mathbf{0} \tag{2.7}$$

For the given vectors $\mathbf{v}_d(t)$, $\mathbf{v}_l(t)$ and $\mathbf{v}_a(t) = \mathbf{v}_{ao}$, where $\mathbf{v}_{ao}$ is the vector of currently closed gaps, the vector of reactions $\mathbf{r}(t) = col(\mathbf{0}, \mathbf{r}_a(t))$ could be determined using Eqn (2.7).

Using Eqn (2.6), we obtain

$$\mathbf{v}_l = -\mathbf{K}_{ll}^{-1}\mathbf{K}_{ld}\mathbf{v}_d - \mathbf{K}_{ll}^{-1}\mathbf{K}_{la}\mathbf{v}_{ao} \tag{2.8}$$

and Eqn (2.5) can be rewritten in the following form:

$$\mathbf{z}(t) \equiv \mathbf{M}\ddot{\mathbf{v}}_d + \mathbf{C}\dot{\mathbf{v}}_d + \mathbf{K}(t)\mathbf{v}_d - \mathbf{p}(t) + \mathbf{f}_o(t) = \mathbf{0} \tag{2.9}$$

where

$$\mathbf{K}(t) = \mathbf{K}_{dd} - \mathbf{K}_{dl}\mathbf{K}_{ll}^{-1}\mathbf{K}_{ld} \tag{2.10}$$

$$\mathbf{f}_o(t) = \left(\mathbf{K}_{da} - \mathbf{K}_{dl}\mathbf{K}_{ll}^{-1}\mathbf{K}_{la}\right)\mathbf{v}_{ao} \tag{2.11}$$

The motion equation (2.9) describes oscillations of beams in terms of nodal displacements, which cannot be in contact with supports. The elements of the stiffness matrix $\mathbf{K}(t)$ are functions of time because dimensions of matrices $\mathbf{K}_{dl}$, $\mathbf{K}_{ll}$ and $\mathbf{K}_{ld}$ change in time. The non-linearity also comes into our problem through the vector $\mathbf{f}_o(t)$.

In this formulation the damping matrix $\mathbf{C}$ could be non-proportional but, in this paper, we assume that

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{2.12}$$

where $\mathbf{K}$ is the stiffness matrix of the basic structure.

3. STEADY STATE SOLUTIONS AND AMPLITUDE EQUATIONS

It is assumed that the vector of excitation forces can be described by:

$$\mathbf{p}(t) = \mathbf{p}_c \cos\lambda t + \mathbf{p}_s \sin\lambda t \tag{3.1}$$

where λ and $T = 2\pi/\lambda$ are the frequency and the period of excitation, respectively and $\mathbf{p}_c$, $\mathbf{p}_s$ are vectors of amplitudes of excitation forces.

The steady state vibration of beams is assumed to be described by the simple harmonic function, i.e.

$$\mathbf{v}_d(t) = \mathbf{a}_c \cos\lambda t + \mathbf{a}_s \sin\lambda t \tag{3.2}$$

where $\mathbf{a}_c$ and $\mathbf{a}_s$ are unknown vectors of amplitudes of vibration. Notice that the above form of solution is valid only for displacements which cannot be in contact with supports.

Sometimes, in non-linear systems, higher harmonics have a significant part in steady state responses of systems. For example, the complex behaviour of a system with piecewise-non-linear stiffness is reported in [6]. This problem needs additional investigations and, at this moment, is out of scope of this paper. However, it will be shown later that the response of a particular beam with a gap at support is periodic and that the influence of higher harmonics is small.

The assumed solution of motion equations is not exact and after introducing Eqn (3.2) into (2.9) we obtain a vector of residuals denoted by $\mathbf{z}(t)$.

The equilibrium conditions are fulfilled in a weak form and the Galerkin method is used to derive the amplitude equations. The Galerkin conditions are:

$$\frac{1}{2T}\int_0^T \mathbf{z}(t)\cos\lambda t dt = \mathbf{0}\ , \qquad \frac{1}{2T}\int_0^T \mathbf{z}(t)\sin\lambda t dt = \mathbf{0} \tag{3.3}$$

from which we obtain the following coupled, non-linear amplitude equations:

$$\begin{aligned}&\left(\mathbf{K}_{cc} - \lambda^2\mathbf{M}\right)\mathbf{a}_c + \left(\mathbf{K}_{cs} + \lambda\mathbf{C}\right)\mathbf{a}_s + \mathbf{f_c} = \mathbf{p}_c\\ &\left(\mathbf{K}_{sc} - \lambda\mathbf{C}\right)\mathbf{a}_c + \left(\mathbf{K}_{ss} - \lambda^2\mathbf{M}\right)\mathbf{a}_s + \mathbf{f_s} = \mathbf{p}_s\end{aligned} \tag{3.4}$$

The matrices $\mathbf{K}_{cc}$, $\mathbf{K}_{cs}$, $\mathbf{K}_{sc}$, $\mathbf{K}_{ss}$ and vectors $\mathbf{f_c}$, $\mathbf{f_s}$ depend on unilateral conditions, which are active during the period of oscillations. These matrices and vectors are defined as follows:

$$\mathbf{K}_{cc} = \frac{1}{2T}\int_0^T \mathbf{K}(t)\cos^2\lambda t dt\ , \qquad \mathbf{K}_{ss} = \frac{1}{2T}\int_0^T \mathbf{K}(t)\sin^2\lambda t dt \tag{3.5}$$

$$\mathbf{K}_{cs} = \mathbf{K}_{sc} = \frac{1}{2T}\int_0^T \mathbf{K}(t)\cos\lambda t\sin\lambda t dt \tag{3.6}$$

$$\mathbf{f}_{co} = \frac{1}{2T}\int_0^T \mathbf{f}_o(t)\cos\lambda t dt\ , \qquad \mathbf{f}_{so} = \frac{1}{2T}\int_0^T \mathbf{f}_o(t)\sin\lambda t dt \tag{3.7}$$

Taking into account the fact that the elements of matrix $\mathbf{K}(t)$ and vector $\mathbf{f}_o(t)$ are constant in intervals of time where the number of closed gaps is constant, we can rewrite Eqs (3.5) – (3.7) in the following form:

$$\mathbf{K}_{cc} = \frac{1}{2T}\sum_i \mathbf{K}_i \int_{t_{il}}^{t_{ip}} \cos^2 \lambda t dt\,, \qquad \mathbf{K}_{ss} = \frac{1}{2T}\sum_i \mathbf{K}_i \int_{t_{il}}^{t_{ip}} sin^2 \lambda t dt \tag{3.8}$$

$$\mathbf{K}_{cs} = \mathbf{K}_{sc} = \frac{1}{2T}\sum_i \mathbf{K}_i \int_{t_{il}}^{t_{ip}} cos\,\lambda t\, sin\,\lambda t dt \tag{3.9}$$

$$\mathbf{f}_c = \frac{1}{2T}\sum_i \mathbf{f}_{oi} \int_{t_{il}}^{t_{ip}} cos\,\lambda t dt\,, \qquad \mathbf{f}_c = \frac{1}{2T}\sum_i \mathbf{f}_{oi} \int_{t_{il}}^{t_{ip}} sin\,\lambda t dt \tag{3.10}$$

where $\mathbf{K}_i$ and $\mathbf{f}_{oi}$ are the matrix $\mathbf{K}(t)$ and the vector $\mathbf{f}_o(t)$, respectively, determined in the interval "i". Summation in (3.8) – (3.10) is over all intervals. It is easy to check that

$$\int_{t_{il}}^{t_{ip}} cos^2\,\lambda t dt = \frac{t_{ip}-t_{il}}{2} + \frac{1}{4\lambda}\left(sin\,2\lambda t_{ip} - sin\,2\lambda t_{il}\right) \tag{3.11}$$

$$\int_{t_{il}}^{t_{ip}} sin^2\,\lambda t dt = \frac{t_{ip}-t_{il}}{2} - \frac{1}{4\lambda}\left(sin\,2\lambda t_{ip} - sin\,2\lambda t_{il}\right) \tag{3.12}$$

$$\int_{t_{il}}^{t_{ip}} sin\,\lambda t\, cos\,\lambda t dt = \frac{1}{2\lambda}\left(sin^2\,\lambda t_{ip} - sin^2\,\lambda t_{il}\right) \tag{3.13}$$

$$\int_{t_{il}}^{t_{ip}} cos\,\lambda t dt = \frac{1}{\lambda}\left(sin\,\lambda t_{ip} - sin\,\lambda t_{il}\right)\,, \qquad \int_{t_{il}}^{t_{ip}} sin\,\lambda t dt = \frac{1}{\lambda}\left(-cos\,\lambda t_{ip} + cos\,\lambda t_{il}\right) \tag{3.14}$$

The limits of intervals t_{il} and t_{ip}, appearing in the above integrals, must be determined numerically. The following procedure is used. For a given time t, we can calculate the vector $\mathbf{v}_d(t)$, using Eqn (3.2). The dimensions of vectors $\mathbf{v}_l(t)$ and $\mathbf{v}_{ao}$ must be determined in an iterative way. Having $\mathbf{v}_d(t)$, the vector $\mathbf{w}(t)$ can be calculated from

$$\mathbf{w}(t) = -\mathbf{K}_{ll}^{-1}\mathbf{K}_{ld}\mathbf{v}_d(t) \tag{3.15}$$

Equation (3.15) follows from (2.8) if we assume that all gaps are open, which means that $\mathbf{w}(t) \equiv \mathbf{v}_l(t)$ and dimension of the vector $\mathbf{v}_a(t) = \mathbf{v}_{ao}$ is equal to zero.

The second possible choice is to take the vector $\mathbf{v}_a(t)$ from the previous time and calculate the vector $\mathbf{v}_l(t)$ from Eqn (2.8). Now, we can verify the unilateral conditions (2.1) and we can find out which gaps are closed. The nodal displacements corresponding to closed gaps are equal to lower or upper limits of gaps and create the next approximation of vector $\mathbf{v}_a(t) = \mathbf{v}_{ao}$. This ends the first iteration. In the second iteration, having vectors $\mathbf{v}_d(t)$, $\mathbf{v}_{ao}$ and their respective dimensions, we can build new matrices $\mathbf{K}_{ll}$, $\mathbf{K}_{ld}$, $\mathbf{K}_{la}$ and calculate the new approximation of vector $\mathbf{v}_l(t)$, using Eqn (2.8). If all elements of $\mathbf{v}_l(t)$ fulfil the unilateral condition (2.1) the iteration process is completed. If not, the next iteration is performed. In our calculations, only two iterations are needed to obtain the correct vector $\mathbf{w}(t) = col(\mathbf{v}_l(t), \mathbf{v}_a(t))$ fulfilling the unilateral conditions.

If for two successive time instances t_n and t_{n+1} different gaps are closed, there exist a time instance t_i in which at least one gap closes or opens. This particular time can be determined using the method of bisection. The time t_i must be precisely determined because it has significant influence on calculation results.

In this way, the unilateral conditions are taken into account during the process of calculation of matrices $\mathbf{K}_{cc}$, $\mathbf{K}_{cs}$, $\mathbf{K}_{sc}$, $\mathbf{K}_{ss}$ and vectors $\mathbf{f_c}$, $\mathbf{f_s}$.

The second possible approach to calculate the above mentioned quantities is simply to use the trapezoidal rule to calculate the integrals appearing in Eqs (3.5) – (3.7). The numerical formulas are very simple and they are not given here due to limited space. In this case, the time instances where the stiffness of the system changes must also be precisely determined.

For the given frequency of excitation λ, the amplitude equations (2.16) are solved with respect to $\mathbf{a}_c$ and $\mathbf{a}_s$. The problem is non-linear because of the unilateral conditions (2.1) - (2.3). For this reason, the iterative procedure must be used. This procedure is described in the next section.

4. DETERMINATION OF RESPONSE CURVES

In many cases, the response curves must be determined in order to show the dynamic properties of the system under consideration. The response curve is obtained if the amplitude equations are solved for a set of values of excitation frequency, taken from a prescribed range $(\lambda_a, \lambda_\mathrm{b})$. The incremental – iterative method is used to determine the response curves efficiently.

The solutions of amplitude equations are represented by a sequence of excitation frequencies ${}^m\lambda$ and the amplitude vectors ${}^m\mathbf{a}_c$ and ${}^m\mathbf{a}_s$ for *m=1,2,....*. For any incremental step, the vectors ${}^m\mathbf{a}_c$ and ${}^m\mathbf{a}_s$ and ${}^m\lambda$ of the preceding step *m* are assumed to be given. At the beginning of the incremental procedure we can take λ_a far from the resonance regions. In non-resonance regions usually all unilateral conditions are non-active and the problem is linear.

For given ${}^m\lambda$, ${}^m\mathbf{a}_c$ and ${}^m\mathbf{a}_s$, the frequency of excitation is increased by $\Delta\lambda$, i.e. ${}^{m+1}\lambda = {}^m\lambda + \Delta\lambda$ and the amplitude equations are solved in an iterative way, taking the vectors ${}^m\mathbf{a}_c$ and ${}^m\mathbf{a}_s$ as the first approximation of vectors $\mathbf{a}_c$ and $\mathbf{a}_s$ in the iteration procedure. This completes the incremental step.

It is assumed that in a typical iteration we know an approximate solution denoted by $\mathbf{a}_c^i$ and $\mathbf{a}_s^i$. Now it is possible to calculate the matrices $\mathbf{K}_{cc}$, $\mathbf{K}_{cs}$, $\mathbf{K}_{sc}$, $\mathbf{K}_{ss}$ and vectors $\mathbf{f_c}$, $\mathbf{f_s}$, using the procedure described in the previous section. The next approximation of the solution of amplitude equations $\mathbf{a}_c^{i+1}$ and $\mathbf{a}_s^{i+1}$ is determined from Eqs (2.16). The iterations are repeated until the following conditions are fulfilled:

$$\left\| \mathbf{a}_c^{i+1} - \mathbf{a}_c^i \right\| \le \varepsilon_1 \left\| \mathbf{a}_c^{i+1} \right\|, \qquad \left\| \mathbf{a}_s^{i+1} - \mathbf{a}_s^i \right\| \le \varepsilon_1 \left\| \mathbf{a}_s^{i+1} \right\| \tag{4.1}$$

where ε_1 is the assumed accuracy of calculations.

Using the above procedure, only the stable parts of the response curve can be determined. A more advanced procedure, similar to the continuation method described in [7], must be used if the whole response curve is needed.

3. RESULTS OF EXAMPLE CALCULATIONS

The response curve of the simply supported beam, shown in Fig.2, has been calculated and is presented in Fig.3. The data are as follows: the mass $M = 200.0\ \mathrm{kg}$, half of the beam length $a = 2.0\ \mathrm{m}$, the damping factor $c = 150.0\ \mathrm{Ns/m}$, the beam rigidity $EJ = 350550.0\ \mathrm{Nm}^2$ the amplitude of excitation force $P = 250.0\ \mathrm{N}$ and the accuracy of calculation $\varepsilon_1 = 0.01$. The beam has a rotational clearance at left support. The upper and lower bounds of rotation at support are $\varphi_{10}^{+} = 0.005\ \mathrm{rad}$ and $\varphi_{10}^{-} = -0.005\ \mathrm{rad}$, respectively.

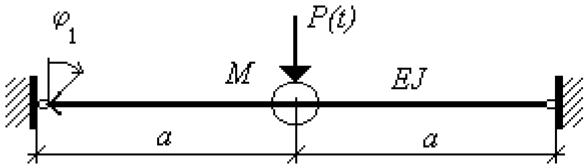

Fig.2 The simply supported beam with rotational gap

The results obtained by the method presented in this paper are shown as the thick line in Fig.3 and denoted as Curve 1. In [8], for this particular case we obtain the steady state solutions by means of the harmonic balance method and the continuation method. The three harmonics are taken into account in the Fourier series describing the steady state solution of motion equations and it was found that the influence of higher harmonics is very small. The results are shown as the thin curve denoted as Curve 2. Moreover, the steady responses are obtained by means of the time integration method. The well-known Newmark method is used and the results are depicted in Fig. 3 by small crosses. It is obvious that there is strong agreement between results obtained with the use of all method. It is also demonstrated in [8] that it is possible to determine the whole response curve by the continuation method. The obtained results clearly indicate that the considered beam is a strongly non-linear system. In Fig. 3 two additional curves, which represent the linear solutions, denoted as Curves 3 and 4, are shown for comparison. Curve 3 and 4 are the response curves of the simply supported beam and of the fixed-simply supported beam without gaps, respectively. The peak of non-linear response curve is lower then the peak of response curve for simply supported beam but higher than the response curve peak for fixed-simply supported beam. We also observe from Fig.3 that the non-linear response curve tends to be vertical in the upper part and that the main resonance region is between the natural frequencies of the simply supported beam and the fixed-simply supported beam, respectively.

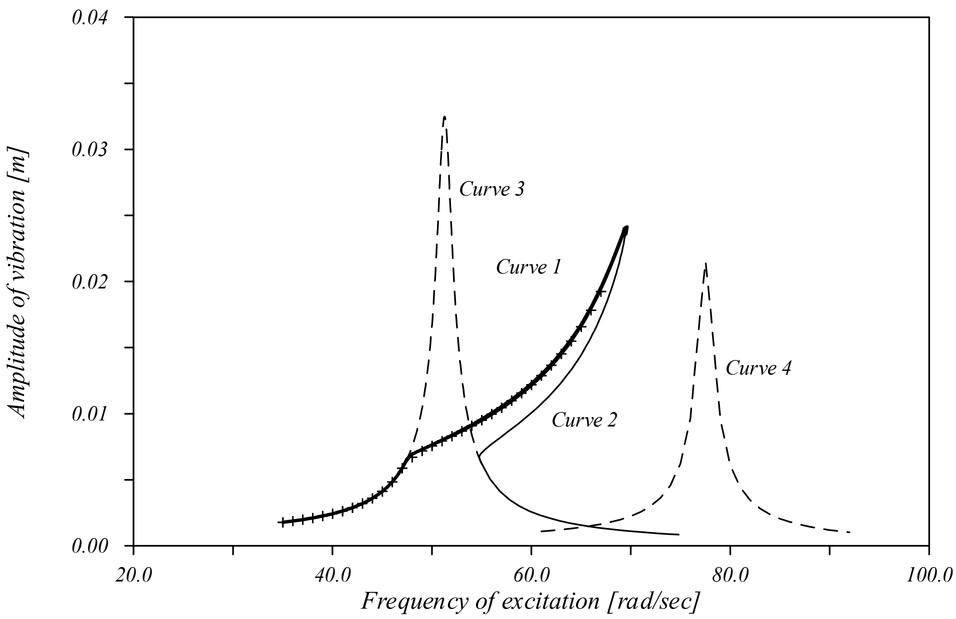

Fig.3 Response curve of simply supported beam with rotational gap at support

The second example concerns the beam shown in Fig.1. The beam is divided into 8 equal finite elements of the length $a = 0.6$ m each; the granulated masses are also identical and $M = 10.0$ kg . The beam rigidity is $EJ = 2{,}000{,}000.0$ Nm^2, the non-dimensional modal damping factors of the first and second linear modes of vibration are $\gamma_1 = \gamma_2 = 0.008$. There are three gaps in this case, two rotational gaps at left and right support and one gap in the middle of the beam. The limits of gaps are: $\varphi_{1o}^{+} = 0.04$ rad , $\varphi_{1o}^{-} = -0.04$ rad , $\varphi_{9o}^{+} = 0.002$ rad , $\varphi_{9o}^{-} = -0.002$ rad , $w_{5o}^{+} = 0.02$ m and $w_{5o}^{-} = -0.02$ m, respectively. The excitation force acts on the second mass and $p_c = 100.0$ N while $p_s = 0.0$ N . Moreover, $\varepsilon_1 = 0.01$. Now the system with many degrees of freedom is analysed.

The response curve is shown in Fig.4 and denoted as Curve 1. This picture shows the amplitude of vibration of the second mass versus the frequency of excitation. In this case, the gap at the right support is closed during a part of period of vibration. The curves denoted as Curves 2 and 3 are the response curves for the simply supported beam and the simply supported-fixed beam, respectively. The results are similar to those presented in Fig.3.

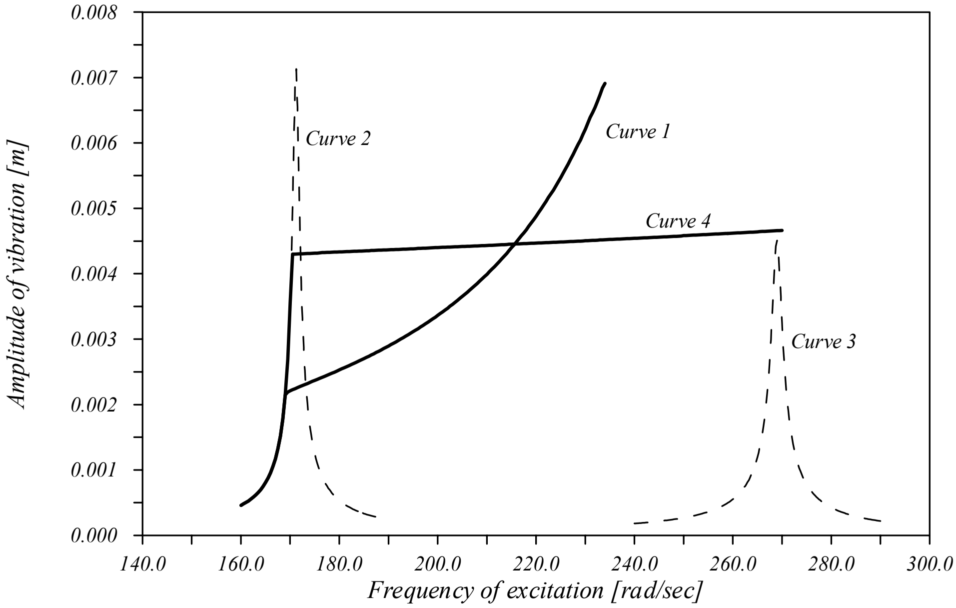

Fig.4 Example 2 - Response curves

The dynamic behaviour of this beam changes significantly if the gap in the middle of the beam can be closed. The response curve for the considered beam with different gaps at supports is shown in Fig.4 and denoted as Curve 4. Now the limits of gaps are: $\varphi_{1o}^{+} = 0.04$ rad, $\varphi_{1o}^{-} = -0.04$ rad, $\varphi_{9o}^{+} = 0.04$ rad, $\varphi_{9o}^{-} = -0.04$ rad, $w_{5o}^{+} = 0.006$ m and $w_{5o}^{-} = -0.006$ m. We see that in this case the response curve is almost horizontal and does not grow as the previously discussed curves. The reason is that in this case the first natural frequency of a two-span, simply supported beam is far from the considered range of excitation frequency. The non-linear part of the response curve of the beam with gaps is similar to the response curve of the above mentioned two span beam in non-resonance region but with much greater amplitudes of vibration.

3. CONCLUDING REMARKS

A method of analysis of steady state vibration of beams with gaps at supports is proposed in the paper. The Galerkin method is used to derive the amplitude equations. The incremental-iterative procedure is used to determine the response curves. Results of example calculations are also presented and briefly discussed. These results show that the dynamic behaviour of the considered beams can be strongly non-linear and depend very much on reciprocal proportions of limits values of gaps. Only beams with symmetric limits of gaps are considered.

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NIELINIOWE DRGANIA BELEK Z LUZAMI NA PODPORACH

Streszczenie

W pracy rozważa się problem okresowych, nieliniowych drgań belek z luzami na podporach. W szczególności analizuje się drgania ustalone wzbudzane siłami harmonicznymi. Względem czasu rozwiązanie ustalone jest opisywane za funkcji harmonicznej. Nieznane współczynniki tej funkcji są wyznaczane z nieliniowego równania amplitud. Macierzowe równanie amplitud otrzymuje się za pomocą metody Galerkina. Warunki jednostronności więzów podporowych są uwzględniane w trakcie obliczania współczynników równania amplitud. Równanie amplitud jest traktowane jako równanie z parametrem, a częstość sił wymuszających jest uważana za główny parametr. Do wyznaczania krzywych rezonansowych stosuje się procedurę przyrostowo-iteracyjną. Podano i omówiono rezultaty przykładowych obliczeń.

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