AN APPLICATION OF THE HARTLEN-CURRIE MODEL AND THE FINITE ELEMENT METHOD TO TRANSIENT AND STEADY STATE ANALYSIS OF VORTEX-INDUCED VIBRATION OF BEAMS

Roman Lewandowski

Poznan University of Technology, Institute of Structural Engineering,
ul. Piotrowo 5, 60-965 Poznan, Poland

SUMMARY: In this paper, the strip and finite element methods together with the Hartlen-Currie type vortex-shedding model are employed to study the vortex-induced vibration of beams in a synchronisation region. Both the transient and steady state behaviour of beams are analysed. The time integration method and the harmonic balance method are used to solve the equations of motion. The Newmark method is used to determine the transient and steady state behaviour of beams. The harmonic balance method is used to determine the periodic responses. The amplitudes of vortex-induced vibration are determined by solving the so-called amplitude equation with a help of continuation method. The mean wind velocity is chosen as the main parameter. The results obtained by both methods are compared.

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1. INTRODUCTION

Slender engineering structures are to be sensitive to wind induced vibrations such as vortex-shedding galloping and torsional flutter. In civil engineering application, we are often mainly interested in the maximum vortex-induced response of structure. For this reason, the empirical models providing a reasonable approximation of aeroelastic response are useful. Among models of this kind the one proposed by Hartlen and Currie [1] is in quite good agreement with the experimental data. In paper [1] the elastically supported, rigid cylinder in air flow is considered. Cylinder motion is restricted to pure translation in the direction perpendicular to the flow direction and cylinder axis. The equations of motion for this model can be written in the following form:

$$\ddot{w} + 2x\dot{w} + w = a w_s^2 c_L \; , \qquad \ddot{c}_L - a w_s \dot{c}_L + g\dot{c}_L^3 / w_s + w_s^2 c_L = b\dot{w} \; , \qquad (1)$$

where w, c_L, x, w_s, a, g, a, b are the cylinder displacement, the „hidden" aerodynamic variable interpreted as the lift coefficient, the damping factor for cylinder, the shedding frequency which is proportional to the wind mean velocity U, and some aerodynamic constants, respectively. The aerodynamic constants are determined experimentally. Equation 1.2 is the non-linear, van der Pol differential equation. More detailed description of the mentioned model is given in [1]. In next section, the Hartlen - Currie model will be extended for beams treated as systems with multi degrees of freedom.

2. EQUATIONS OF MOTION

The considered system (the beam and flow field) is divided into finite elements (beam) and strips (flow field). Each strip is parallel to the direction of undisturbed flow and has a width equal to the finite element length. The strips are also perpendicular to the finite elements. The main assumption is that flows in strips are mutually independent which means that the aerodynamic forces are induced only by flow in the associated strip. The distribution of lift coefficient along the strip width is assumed in the following form:

$$c_L(x,t) = \mathbf{N}_L^T(x)\mathbf{c}_e(t) \ , \qquad (2)$$

where $\mathbf{N}_L(x) = col(N_1(x), N_2(x))$, $N_1(x) = 1-\eta$, $N_2(x) = \eta$, $\eta = x/l$, $\mathbf{c}_e = col(c_a, c_b)$ are the vector of shape functions and the vector of nodal parameters for strip, respectively.

The cross-wind transverse displacements w of the typical two node beam finite element with two degrees of freedom per node are described by using the Hermitan polynomials shape functions i.e.

$$w(x,t) = \mathbf{N}_b^T(x)\mathbf{w}_e(t) \ , \qquad (3)$$

where $\mathbf{N}_b(x) = col(N_3(x), N_4(x), N_5(x), N_6(x))$ and $\mathbf{w}_e(t) = col(w_a, \varphi_a, w_b, \varphi_b)$ are the vector of beam shape functions and the vector of nodal parameters, respectively.

The kinetic and potential energy of finite element can be written in the form

$$T_b^e = \tfrac{1}{2}\dot{\mathbf{w}}_e^T\mathbf{M}_b^e\dot{\mathbf{w}}_e \ , \qquad W_b^e = \tfrac{1}{2}\mathbf{w}_e^T\mathbf{K}_b^e\mathbf{w}_e \ , \qquad (4)$$

where $\mathbf{M}_b^e$ and $\mathbf{K}_b^e$ are the mass and stiffness matrices, respectively.

The virtual work of non-conservative forces consists of damping term and the external excitation term. The damping term is represented by

$$\delta L_{bd}^e = \delta\mathbf{w}_e^T\mathbf{D}_b^e\dot{\mathbf{w}}_e \ , \qquad (5)$$

where the damping matrix is given by $\mathbf{D}_b^e = \kappa_1\mathbf{M}_b^e + \kappa_2\mathbf{K}_b^e$ (κ_1, κ_2 are some parameters).

The aerodynamic external forces acting on beam are assumed to be similar in a mathematical form depicted in the Hartlen - Currie model so the virtual work of these forces can be written as

$$\int_{t_1}^{t_2}\delta L_{ba}^e dt = \int_{t_1}^{t_2}\int_0^l \delta w(x)\tfrac{1}{2}\rho U^2(x)d(x)c_L(x,t)dxdt = \int_{t_1}^{t_2} \omega_s^2\delta\mathbf{w}_e^T\mathbf{S}_L^e\mathbf{c}_e(t)dt \ , \qquad (6)$$

where symbols ρ, $U(x)$, $d(x)$, denote the air density, the mean velocity of wind and the cross-section characteristic dimension. The matrix $\mathbf{S}_L^e$ is defined by

$$\mathbf{S}_L^e = \frac{D^3 \rho_e^2 d_e}{8\pi^2 S^2}\int_0^l \mathbf{N}_b(x)\mathbf{N}_L^T(x)dx \ , \qquad (7)$$

where $\omega_s = 2\pi SU/D$, D is the reference cross-section dimension, S is the Strouhal number, d_e is the non-dimensional cross-section characteristic dimension and l is the length of element.

Equation (1.2) in the Hartlen-Currie model describes the motion of some artificial variable which characterises the flow action in a globally way. This equation take into account only the primary characteristic deduced from experiments. However, many details connected with flow are omitted. From the mathematical point of view Eqn (1.2) could be understood as the motion equation of fictitious mechanical oscillator with non-linear damping characteristic. In order to make possible the weak formulation for whole considered system the 'kinetic and potential energy' and the 'virtual work of non-conservative forces' for the ficitous oscillators are also introduced. Of course, these quantities must be considered as some kind of functional which lead us to the counterparts of Eqn (1.2) in case of system with many degrees of freedom. The 'kinetic and potential energy' for lift coefficient is defined as follows:

$$T_L^e = \tfrac{1}{2}\dot{\mathbf{c}}_e^T\mathbf{M}_L^e\dot{\mathbf{c}}_e \ , \qquad W_L^e = \tfrac{1}{2}\omega_s^2\mathbf{c}_e^T\mathbf{K}_L^e\mathbf{c}_e \ , \qquad (8)$$

where

$$\mathbf{M}_L^e = \frac{1}{l}\int_0^l \mathbf{N}_L(x)\mathbf{N}_L^T(x)dx \ , \quad \mathbf{K}_L^e = \frac{\rho_e^2}{l d_e^2}\int_0^l \mathbf{N}_L(x)\mathbf{N}_L^T(x)dx \ . \qquad (9)$$

The ‘virtual work of damping forces’ for ‘hidden variable’ is defined by:

$$\int_{t_1}^{t_2}\int_0^l dc_L(x)(-a w_s \dot{c}_L(x,t) + g\dot{c}_L^3(x,t)/w_s)dxdt = \int_{t_1}^{t_2} d\mathbf{c}_e^T\left[-w_s\mathbf{D}_L^e + w_s^{-1}\mathbf{D}_{NL}^e(\dot{\mathbf{c}}_e,\dot{\mathbf{c}}_e)\right]\dot{\mathbf{c}}_e(t)dt \ , \qquad (10)$$

where

$$\mathbf{D}_L^e = \frac{a_e p_e}{l d_e}\int_0^l \mathbf{N}_L(x)\mathbf{N}_L^T(x)dx\,, \quad \mathbf{D}_{NL}^e(\dot{\mathbf{c}}_e,\dot{\mathbf{c}}_e) = \frac{g_e d_e}{l p_e}\int_0^l \mathbf{N}_L^T(x)\dot{\mathbf{c}}_e(t)\dot{\mathbf{c}}_e^T(t)\mathbf{N}_L(x)\mathbf{N}_L(x)\mathbf{N}_L^T(x)dx\,. \quad (11)$$

The non-linear damping matrix $\mathbf{D}_{NL}^e(\dot{\mathbf{c}}_e,\dot{\mathbf{c}}_e)$ is the quadratic function of velocity of nodal parameters. In above relations the symbols a_e and g_e denote the constants which must be determined experimentally and they can be different for each strip. The ‘virtual work of external forces’ is defined by:

$$\int_{t_1}^{t_2} dL_{Le}dt = \int_{t_1}^{t_2}\int_0^l dc(x)b\dot{w}(x,t)dxdt = \int_{t_1}^{t_2} d\mathbf{c}_L^T\mathbf{S}_b^e\dot{\mathbf{w}}_e(t)dt \ , \qquad (12)$$

where

$$\mathbf{S}_b^e = \frac{b_e}{l}\int_0^l \mathbf{N}_L(x)\mathbf{N}_b^T(x)dx \ . \qquad (13)$$

The equations of motion are derived on a basis of Hamilton principle, which states that

$$\int_{t_1}^{t_2}\left[d(T-W) + dL\right]dt = 0 \ , \qquad (14)$$

where d is the variational operator and T, W and dL denote the total kinetic and potential energy of system and the virtual work of non-conservative forces, respectively. Using the Hamilton principle we can derive the following equations of motion for the typical beam element and strip, respectively:

$$\mathbf{R}_b^e = \mathbf{M}_b^e\ddot{\mathbf{w}}_e(t) + \mathbf{D}_b^e\dot{\mathbf{w}}_e(t) + \mathbf{K}_b^e\mathbf{w}_e(t) - w_s^2\mathbf{S}_L^e\mathbf{c}_e(t) \ , \qquad (15)$$

$$\mathbf{R}_L^e = \mathbf{M}_L^e\ddot{\mathbf{c}}_e(t) - w_s\mathbf{D}_L^e\dot{\mathbf{c}}_e(t) + w_s^{-1}\mathbf{D}_{NL}^e(\dot{\mathbf{c}}_e(t),\dot{\mathbf{c}}_e(t))\dot{\mathbf{c}}_e(t) + w_s^2\mathbf{K}_L^e\mathbf{c}_e(t) - \mathbf{S}_b^e\dot{\mathbf{w}}_e(t) \ . \qquad (16)$$

After assembling procedure the motion equations for the entire system can be written as:

$$\mathbf{R}_b(t) = \mathbf{M}_b\ddot{\mathbf{w}}(t) + \mathbf{D}_b\dot{\mathbf{w}}(t) + \mathbf{K}_b\mathbf{w}(t) - w_s^2\mathbf{S}_L\mathbf{c}(t) = \mathbf{0} \ , \qquad (17)$$

$$\mathbf{R}_L(t) = \mathbf{M}_L\ddot{\mathbf{c}}(t) - w_s\mathbf{D}_L\dot{\mathbf{c}}(t) + w_s^{-1}\mathbf{D}_{NL}(\dot{\mathbf{c}}(t),\dot{\mathbf{c}}(t))\dot{\mathbf{c}}(t) + w_s^2\mathbf{K}_L\mathbf{c}(t) - \mathbf{S}_b\dot{\mathbf{w}}(t) = \mathbf{0} \ , \qquad (18)$$

where $\mathbf{M}_b$, $\mathbf{M}_L$, $\mathbf{D}_b$, $\mathbf{D}_L$, $\mathbf{D}_{NL}(\dot{\mathbf{c}}(t),\dot{\mathbf{c}}(t))$, $\mathbf{K}_b$, $\mathbf{K}_L$, $\mathbf{S}_b$, $\mathbf{S}_L$, $\mathbf{w}(t)$, $\mathbf{c}(t)$ are the global counterparts of previously defined, on a level of element and strip, matrices and vectors. The residual vectors $\mathbf{R}_b(t)$ and $\mathbf{R}_L(t)$ vanish in an equilibrium state.

3. SOLUTION OF MOTION EQUATIONS BY THE NEWMARK METHODS

For convenience, Eqns (17) and (18) are first rewritten in the following more compact form:

$$\mathbf{R}(t) = \mathbf{M}\mathbf{a}(t) + \mathbf{D}(\mathbf{v}(t))\mathbf{v}(t) + \mathbf{K}\mathbf{d}(t) = \mathbf{0} \ ; \qquad (19)$$

where

$$\mathbf{R} = col(\mathbf{R}_b,\mathbf{R}_L) \ , \quad \mathbf{d}(t) = col(\mathbf{w}(t),\mathbf{c}(t)) \ , \quad \mathbf{v}(t) = \dot{\mathbf{d}}(t) \ , \quad \mathbf{a}(t) = \dot{\mathbf{v}}(t) \ ,$$

$$\mathbf{M} = \begin{bmatrix}\mathbf{M}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_L\end{bmatrix} , \quad \mathbf{K} = \begin{bmatrix}\mathbf{K}_b & -w_s^2\mathbf{S}_L\\ \mathbf{0} & w_s^2\mathbf{K}_L\end{bmatrix} , \quad \mathbf{D} = \begin{bmatrix}\mathbf{D}_b & \mathbf{0}\\ -\mathbf{S}_b & -w_s\mathbf{D}_L + w_s^{-1}\mathbf{D}_{NL}(\mathbf{v},\mathbf{v})\end{bmatrix} .$$

The following Newmark formulas:

$$\mathbf{d}_{n+1} = \mathbf{d}_n + t\mathbf{v}_n + 0.25t^2(\mathbf{a}_{n+1} + \mathbf{a}_n) \ , \qquad \mathbf{v}_{n+1} = \mathbf{v}_n + 0.5t(\mathbf{a}_{n+1} + \mathbf{a}_n) \ , \qquad (20)$$

give us the system state in time $t_{n+1} = t_n + t$, where t is the small time interval, if the state in t_n (i.e.

vectors $\mathbf{a}_n$, $\mathbf{v}_n$, $\mathbf{d}_n$) and the acceleration vector $\mathbf{a}_{n+1}$ are known. If the motion equation will be understood as the equilibrium one in time t_{n+1} , i.e.

$$\mathbf{R}_{n+1} = \mathbf{M}\mathbf{a}_{n+1} + \mathbf{D}(\mathbf{v}_{n+1})\mathbf{v}_{n+1} + \mathbf{K}\mathbf{d}_{n+1} = \mathbf{0} \ , \qquad (21)$$

a solution of Eqns. (20)-(21) give us the system state in time t_{n+1}. Eqn. (21) is non-linear one and the Newton method is needed to solve it. Starting with the given initial conditions, the solution of motion equations can be determined by applying recurrently the above method for number of t intervals.

4. HARMONIC BALANCE METHOD - DERIVATION OF AMPLITUDE EQUATIONS

The steady state, periodic response of system can be described in a first approximation by:

$$\mathbf{w}(t) = \mathbf{w}_c \cos wt + \mathbf{w}_s \sin wt \ , \qquad \mathbf{w}_e(t) = \mathbf{w}_{ce} \cos wt + \mathbf{w}_{se} \sin wt \ , \qquad (22)$$

$$\mathbf{c}(t) = \mathbf{c}_c \cos wt + \mathbf{c}_s \sin wt \ , \qquad \mathbf{c}_e(t) = \mathbf{c}_{ce} \cos wt + \mathbf{c}_{se} \sin wt \ ,$$

(23)

where $\mathbf{w}_c$, $\mathbf{w}_s$, $\mathbf{c}_c$, $\mathbf{c}_s$, $\mathbf{w}_{ce}$, $\mathbf{w}_{se}$, $\mathbf{c}_{ce}$, $\mathbf{c}_{se}$ are the unknown vectors of harmonic amplitudes of nodal parameters of beam and strips on a level of entire system and the finite element and strip, respectively. Also the frequency of oscillation w is the unknown quantity. In this work, the solution with only one harmonic is taken into account because the results of experiments show that it is sufficiently accurate in the synchronisation region. The in time Galerkin procedure is used to derive the amplitude equations. These equations follow from the Galerkin conditions which state that:

$$\frac{2}{T}\int_0^T \mathbf{R}_b(t)\cos wt dt = \mathbf{0} \ , \ \frac{2}{T}\int_0^T \mathbf{R}_b(t)\sin wt dt = \mathbf{0}, \ \frac{2}{T}\int_0^T \mathbf{R}_L(t)\cos wt dt = \mathbf{0} \ , \ \frac{2}{T}\int_0^T \mathbf{R}_L(t)\sin wt dt = \mathbf{0}, \qquad (24)$$

where $T = 2p / w$ denotes the period of limit cycle. The residual vectors $\mathbf{R}_b(t)$, $\mathbf{R}_L(t)$ appearing in Eqn (25) are determined by introducing the assumed solution of motion equations into Eqs (17) and (18). After calculation of resulting integrals, from the Galerkin conditions one obtains the following set of non-linear algebraic equations:

$$(\mathbf{K}_b - w^2\mathbf{M}_b)\mathbf{w}_c + w\mathbf{D}_b\mathbf{w}_s - w_s^2\mathbf{S}_L\mathbf{c}_c = \mathbf{0} \ , \qquad (25)$$

$$-w\mathbf{D}_b\mathbf{w}_c + (\mathbf{K}_b - w^2\mathbf{M}_b)\mathbf{w}_s - w_s^2\mathbf{S}_L\mathbf{c}_s = \mathbf{0} \ , \qquad (26)$$

$$(w_s^2\mathbf{K}_L - w^2\mathbf{M}_L)\mathbf{c}_c - ww_s\mathbf{D}_L\mathbf{c}_s + \tfrac{3}{4}w^3w_s^{-1}\left[\mathbf{D}_{NL}(\mathbf{c}_c,\mathbf{c}_c) + \mathbf{D}_{NL}(\mathbf{c}_s,\mathbf{c}_s)\right]\mathbf{c}_s - w\mathbf{S}_b\mathbf{w}_s = \mathbf{0} \ , \qquad (27)$$

$$ww_s\mathbf{D}_L\mathbf{c}_c - \tfrac{3}{4}w^3w_s^{-1}\left[\mathbf{D}_{NL}(\mathbf{c}_c,\mathbf{c}_c) + \mathbf{D}_{NL}(\mathbf{c}_s,\mathbf{c}_s)\right]\mathbf{c}_c + (w_s^2\mathbf{K}_L - w^2\mathbf{M}_L)\mathbf{c}_s + w\mathbf{S}_b\mathbf{w}_c = \mathbf{0} \ . \qquad (28)$$

The elements of non-linear matrix $\mathbf{D}_{NL}(\mathbf{c}_c,\mathbf{c}_s)$ are defined on a strip level as follow:

$$d_{11} = b\left[12c_{ca}c_{sa} + 3(c_{ca}c_{sb} + c_{cb}c_{sa}) + 2c_{cb}c_{sb}\right] \ , \qquad d_{22} = b\left[2c_{ca}c_{sa} + 3(c_{ca}c_{sb} + c_{cb}c_{sa}) + 12c_{cb}c_{sb}\right] \ ,$$

$$d_{12} = d_{21} = b\left[3c_{ca}c_{sa} + 2(c_{ca}c_{sb} + c_{cb}c_{sa}) + 3c_{cb}c_{sb}\right] \ , \qquad b = g_e d_e / (60 p_e) \ .$$

Using the following notation:

$$\tilde{\mathbf{K}}(w, w_s, \tilde{\mathbf{c}}) = \begin{bmatrix} (\tilde{\mathbf{K}}_b - w^2\tilde{\mathbf{M}}_b + w\tilde{\mathbf{D}}_b) & -w_s^2\tilde{\mathbf{S}}_L \\ -w\tilde{\mathbf{S}}_b & (w_s^2\tilde{\mathbf{K}}_L - w^2\tilde{\mathbf{M}}_L - ww_s\tilde{\mathbf{D}}_L + \tfrac{3}{4}w^3w_s^{-1}\tilde{\mathbf{D}}_{NL}(\tilde{\mathbf{c}},\tilde{\mathbf{c}})) \end{bmatrix} ,$$

$$\tilde{\mathbf{M}}_b = \begin{bmatrix} \mathbf{M}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_b \end{bmatrix} , \quad \tilde{\mathbf{K}}_b = \begin{bmatrix} \mathbf{K}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_b \end{bmatrix} , \quad \tilde{\mathbf{D}}_b = \begin{bmatrix} \mathbf{0} & \mathbf{D}_b \\ -\mathbf{D}_b & \mathbf{0} \end{bmatrix} , \quad \tilde{\mathbf{S}}_L = \begin{bmatrix} \mathbf{S}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_L \end{bmatrix} , \ \tilde{\mathbf{a}} = col(\tilde{\mathbf{w}}, \tilde{\mathbf{c}})$$

$$\tilde{\mathbf{M}}_L = \begin{bmatrix} \mathbf{M}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_L \end{bmatrix} , \quad \tilde{\mathbf{K}}_L = \begin{bmatrix} \mathbf{K}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_L \end{bmatrix} , \quad \tilde{\mathbf{D}}_L = \begin{bmatrix} \mathbf{0} & \mathbf{D}_L \\ -\mathbf{D}_L & \mathbf{0} \end{bmatrix} , \quad \tilde{\mathbf{S}}_b = \begin{bmatrix} \mathbf{0} & \mathbf{S}_b \\ -\mathbf{S}_b & \mathbf{0} \end{bmatrix} , \ \tilde{\mathbf{w}} = col(\mathbf{w}_c, \mathbf{w}_s)$$

$$\tilde{\mathbf{D}}_{NL} = \begin{bmatrix} \mathbf{0} & \mathbf{D}_{NL}(\mathbf{c}_c, \mathbf{c}_c) + \mathbf{D}_{NL}(\mathbf{c}_s, \mathbf{c}_s) \\ -\mathbf{D}_{NL}(\mathbf{c}_c, \mathbf{c}_c) - \mathbf{D}_{NL}(\mathbf{c}_s, \mathbf{c}_s) & \mathbf{0} \end{bmatrix}, \qquad \tilde{\mathbf{c}} = col(\mathbf{c}_c, \mathbf{c}_s) \quad .$$

the mentioned amplitude equations can be rewritten in the following compact form:

$$\tilde{\mathbf{G}}(w, w_s, \mathbf{a}) = \tilde{\mathbf{K}}(w, w_s, \mathbf{c})\tilde{\mathbf{a}} = \mathbf{0} \ . \tag{29}$$

5. CONTINUATION PROCEDURE

Equation (29) is solved for different w_s (or U due to relation $w_s = 2pSU / D$) by using the continuation method. The wind velocity is chosen as a main parameter. Obviously, $\tilde{\mathbf{a}} = \mathbf{0}$ is the solution of amplitude equation. Moreover, it is easy to verify that if w and $\tilde{\mathbf{a}} \neq \mathbf{0}$ are, for the particular U, the solution of amplitude equation then w, and $-\tilde{\mathbf{a}}$ are also the solution. It is follow from facts that the amplitude equation is homogenous and the non-linear matrices of type $\mathbf{D}_{NL}(\mathbf{c}_c, \mathbf{c}_s)$ are the quadratic functions of amplitudes of lift coefficients $\mathbf{c}_c$ and $\mathbf{c}_s$. The non-trivial solution of amplitude equation is represented by a sequence of vortex-shedding frequency, frequency of periodic response and the amplitude vectors, i.e. ${}^m w_s$, ${}^m w$, ${}^m\tilde{\mathbf{a}}$ for *m=1,2,...* For any incremental step, the vector ${}^m\tilde{\mathbf{a}}$ and ${}^m w_s$, ${}^m w$ of the proceeding step is assumed to be given. The purpose of an incremental process is to find the following increments $\mathbf{D}w_s$, $\mathbf{D}w$, $\mathbf{D}\tilde{\mathbf{a}}$ which can be accumulated to yield:

$${}^{m+1}w_s = {}^m w_s + \mathbf{D}w_s , \qquad {}^{m+1}w = w + \mathbf{D}w, \qquad {}^{m+1}\tilde{\mathbf{a}} = {}^m\tilde{\mathbf{a}} + \mathbf{D}\tilde{\mathbf{a}} \ . \tag{30}$$

In the equation (29) there are *n+1* unknowns (i.e. w, $\tilde{\mathrm{w}}$ and $\tilde{\mathrm{c}}$). However, since the considered dynamic system is autonomous, one of the Fourier coefficients can be fixed (e.g. $w_{c(s),i} = 0$ or $c_{c(s),i} = 0$), which causes only a shift of response on the time axis. The above mentioned autonomous condition and the following constraint equation:

$$\mathbf{D}\tilde{\mathbf{w}}^T \mathbf{D}\tilde{\mathbf{w}} / m_b^2 + \mathbf{D}\tilde{\mathbf{c}}^T \mathbf{D}\tilde{\mathbf{c}} / m_L^2 = (\mathbf{D}s)^2 \ , \tag{31}$$

are added to the matrix amplitude equation. These set of non-linear equations can be solved only by an iterative procedure. A detailed description of continuation method can be found in [2].

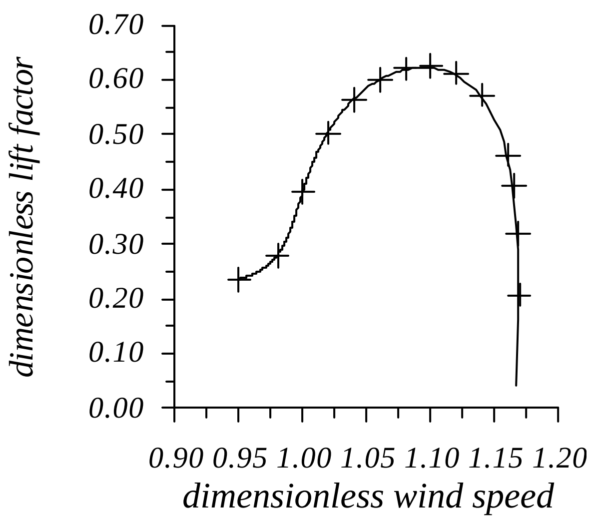

Fig.1 Rigid cylinder - the dimensionless amplitude vs. dimensionless wind speed

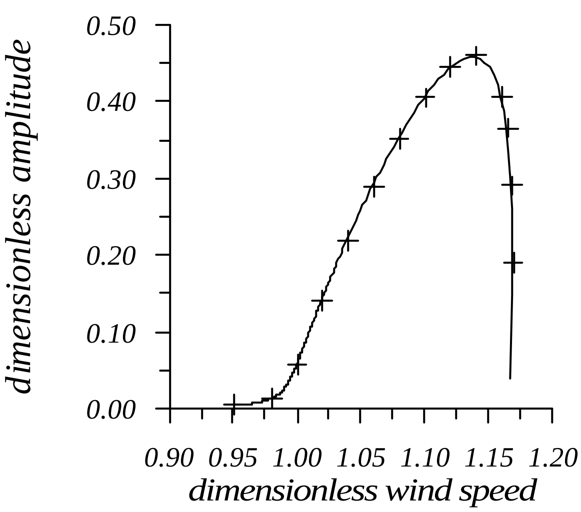

Fig.2 Rigid cylinder - the dimensionless lift factor vs. dimensionless wind speed

6. NUMERICAL RESULTS

As a first example the rigid cylinder elastically supported at both ends is considered. The results of calculation are shown in Figs 1 and 2. The results obtained by the harmonic balance method are shown as the solid lines whereas the ones obtained by the Newmark method is pictured by the small crosses. The perfect agreement between results of both methods is visible.
Moreover, the results for fixed-free beam is also presented. The following data are used: the beam length *L=32.0* m, the beam cross-section diameter *D=1.2* m, the bending rigidity $EJ = 2.0\ 10^9\ \mathrm{Nm}^2$ The beam is divided into 10 elements. The first natural frequency is $w_1 = 7.2387$ rad/s and the damping matrix is calculated in such a way that the modal damping ratios for first and second modes are 1.0 per cent. The air density $r = 1..2\ \mathrm{kg/m^3}$, the Strouhal number *S=0.20* and the aerodynamic constants are $a = 0.02$, *b=0.4*, $g = 2/3$. The wind speed is equal *U* in a range of first 8 elements from the top of beam and equal zero for others. The results of calculation is shown in Fig.3 where the ones obtained by the harmonic balance method are shown as the solid line whereas the ones obtained by the Newmark method is pictured by the small crosses. In this figure the amplitude of vibration of the top of beam versus shedding frequency is shown. A good agreement between results of both methods is obtained.

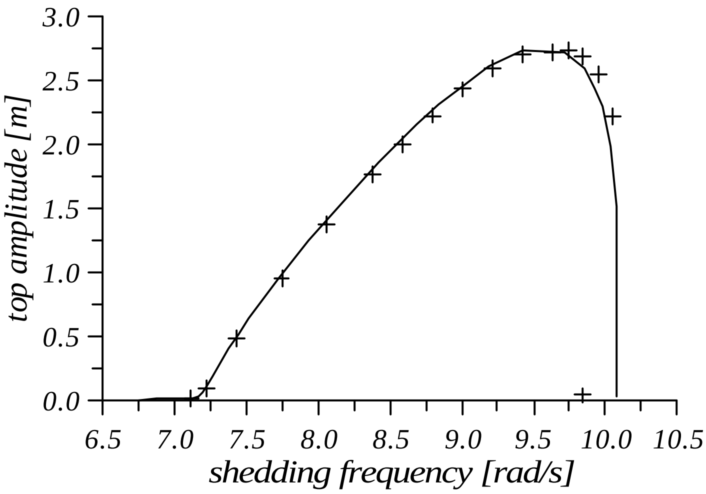

Fig.3 The fixed-free beam - top amplitude vs. shedding frequency

7. CONCLUDING REMARKS

In this paper, the computational methods for analysis of vortex-induced vibration of beams are presented. Both the transient and steady state solutions are determined by means the time integration. The semi-analytical method are employed to determine the steady state solution for set of values of mean wind velocity. The validity of the harmonic balance results are confirmed by means the time integration method

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