Implementation of a hyperelastic model for arterial layers considering damage and distributed collagen fiber orientations

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Abstract. The continuous progress in medical technology requires more accurate and advanced computational tools for improving medical treatments. Mechanically dominated clinical interventions can be improved by case studies, where an appropriate mechanical description of biological tissues plays a crucial role. The main goal of this contribution is to present the implementation of a constitutive equation for arteries considering discontinuous damage and distributed collagen fiber orientations. This type of model has a wide practical application and gives a good prediction. The material law can particularly be used for the simulation of balloon angioplasty where the applied loads are far beyond the physiological domain so that tissue damage occurs. In particular we present the implementation of the constitutive model by Holzapfel et al. [1], and the generalized version by Gasser et al. [2] and Weisbecker et al. [3], which includes fiber dispersion and damage, respectively.

Keywords: computational mechanics, constitutive modeling, anisotropy, arterial wall mechanics

1. Introduction

Progress in medical technology is of fundamental importance for our society. The fast-developing health service needs new and more innovative tools for improving the treatment quality, which calls for new challenges for researchers and engineers in the area of science, engineering and related applications.

Several medical treatments are mechanically based, thus, in several cases, related modeling requires continuum mechanics. Numerical simulations of interventions such as balloon angioplasty may be helpful in supporting physicians in their decision making and may provide them with useful parameters in order to improve interventions. Thereby, constitutive relations of biological materials play an essential role.

This contribution presents an implementation of a constitutive relation for the layered structure of the arterial wall, crucial for its biomechanical analysis in health and disease. With respect to the arterial wall structure under supra-physiological loading, the constitutive law may be seen as an isotropic matrix material reinforced by two families of fibers which is combined with damage [1–3]. As it is shown in Fig. 1, the arterial wall consists of three layers: intima, media, adventitia. It is important to note that human arteries with non-atherosclerotic intimal thickening need to be modeled as a three-layer thick-walled tube, where each of the layer consists of a complex collagen fabric [4].

2. Model including damage and collagen dispersion

The implemented strain-energy function has the form

\[ \Psi(C, A_1, A_2) = U(J) + \Psi_d(C) + \eta_f \sum_{i=1,2} \Psi_i(C, A_1, A_2), \]  

where \( U(J) \) is a penalty function, \( \Psi_d(C) \) is an isotropic deviatoric contribution due to the ground matrix, \( \Psi_i(C, A_1, A_2) \) is an anisotropic deviatoric contribution due to the embedded families of fibers, \( \eta_f \) is the damage variable, \( C \) denotes the right Cauchy-Green tensor, \( J = (\text{det}C)^{\frac{1}{2}} > 0 \) is the volume ratio, \( C = J^{-2/3} \) is the related volume-preserving tensor and \( A_1 \) and \( A_2 \) are structural tensors.

\[ U(J) = \frac{K}{2} (J - 1)^2, \quad \Psi_d(C) = \frac{c}{2} (\bar{I}_1 - 3), \]  

where \( K \) is the bulk modulus, \( c \) is the shear modulus, \( \bar{I}_1 \) is the first invariant of the modified right Cauchy-Green tensor \( C \). The energy stored in the fibers is provided by

\[ \Psi_i(C, A_1, A_2) = \frac{k_i}{2k_0} \left[ \exp(k_2 E_i^2) - 1 \right], \quad i = 1, 2, \]  

where \( k_i \) is a stress-like parameter, \( k_2 \) is a dimensionless parameter, and \( E_i \) is the strain in the direction of the mean orientation, say \( a_{i0} \), of the \( i \)-th fiber family. The orientations \( a_{i0} \) of the fibers and the related dispersion \( \kappa \) of the two fiber families are expressed by the tensors \( A_1 \), \( A_2 \) characterizing the structure of the tissue.

Damage of the material is included by

\[ \eta_f = 1 - \frac{1}{r_f} \text{erf} \left( \frac{1}{m_f} (\Psi^\text{max} - \Psi^0) \right), \]

where \( r_f \) and \( m_f \) are damage parameters, \( \Psi^\text{max} \) is the variable of strain-energy history and \( \Psi^0 \) is the deviatoric strain energy of the undamaged material.
3. Implementation

The set of parameters in this specific anisotropic damage model for arterial tissues can be summarized as: $K, c, k_1, k_2, a_{0i}$ and the damage parameters $r_i, m_i$. The presented material model was implemented in the commercial finite element software Abaqus Standard (Dassault Systèmes Simulia Corp.) as a UMAT subroutine for continuum stress/displacement, with three-dimensional eight-noded linear elements.

4. Model verification

We verify here two cases, which we label as A and B, and which we refer to as ‘qualitative’ and ‘quantitative’, respectively. In the first analysis, i.e. case A, the elastic material model is tested without the inclusion of damage, what basically boils down to the Holzapfel-Gasser-Ogden (HGO) model implemented in Abaqus [1, 2]. In the second analysis, i.e. case B, we verify the damage model [3].

In the simulation A, a simple tension of an iliac adventitial strip in the circumferential direction is considered. Geometry, boundary and loading conditions as well as the material parameters for the arterial strip are taken from [2]. The results of the simulation A are illustrated in Fig. 2, where the distributions of the maximal principal Cauchy stress (see Fig. 2a), and the displacement in the axial direction (see Fig. 2b), are presented. In Fig. 2a) two plots are shown. The upper plot shows the results obtained from the constitutive model we have implemented, while the plot underneath illustrates the distribution obtained from the default HGO model available in Abaqus Standard. The two plots in Fig. 2b) are generated in an analogous way.

In the simulation B, an ideal cylindrical geometry of an artery with three layers is analyzed. The inner surface of the tube is first loaded by an internal pressure of 68 mmHg (0.064 kPa), and then the pressure is decreased to 34 mmHg (0.037 kPa). The results of the simulation B are presented in Table 1. Note that we did not use realistic material parameters for this specific example: in vivo there would be no damage at a pressure level of 68 mmHg. Simulation B is just a benchmark test for evaluating the implementation of damage. The column ‘Abaqus impl.’ refer to the results of the implemented material model, while the column ‘Reference (FEAP)’ refer to the corresponding values obtained from simulations performed with the Finite Element Analysis Program FEAP. Note that the qualitative (simulation A) and the quantitative (simulation B) comparisons of both stress and displacement give very good agreement.

5. Summary

The implemented constitutive model, provided that appropriate parameters are given, shows a good prediction of the mechanical response of arterial tissues, confirmed by examples from the literature, see e.g. [1–3]. The constitutive model has several useful applications in arterial wall mechanics, with a reasonable number of parameters which can be physically interpreted and determined by experimental tests.

The inclusion of damage allows to simulate the Mullins effect, which is a typical behavior of soft tissues due to the straightening of the collagen fibers during loading. Correct implementation of the constitutive model was shown by two examples.

| Table 1: Numerical simulation of artery inflation: I – pressure 34 mmHg (then loading to 68 mmHg), II – pressure 34 mmHg (unloading, damage accumulated due to pressure reduction from 68 mmHg). |

<table>
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<th>Abaqus impl.</th>
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References


