Structural Glass Modelling - Application of Linear Elastic Fracture Mechanics and XFEM

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Abstract
This paper concerns the application of Finite Element Methods in Fracture Mechanics. The advantages, disadvantages and limitation of different computational methods are shown for numerical experiment of glass beam bending. Results obtained in laboratory tests, analytical method and by means of Linear Elastic Fracture Mechanics (LEFM) and eXtended Finite Element Method (XFEM) are presented, compared and discussed. The results presented in this paper are the first stage of a research leading to determine fracture criterion and proper computational simulation of behaviour of glass for both static and dynamic loadings.

Keywords: computational mechanics, fracture mechanics, LEFM, XFEM, glass

1. Introduction

The application of structural glass has significantly increased in recent decades. Glass is used mostly in facades of buildings, but modern architecture employs glass as a material of loads bearing members of constructions. It is used in staircases, gangways, floors and is subjected to loads of different kinds. The use of this very attractive for its esthetic value material, unfortunately may carry a risk. Damage of glass element comes suddenly and without any significant signs of overloading, such as large deformation, because of brittle nature of glass. Like all other brittle materials glass is very sensitive to any structural imperfection. Voids, surface scratches, cracks, etc. have significant influence on its strength. The resistance of glass elements, bend by forces acting in plane, depends largely on the condition of edges i.e. existence of stresses and strains near the crack tip. The fracture toughness of glass, measured on the basis of cracked beam, shows much scatter of values of damage force (the lowest value is 890N and the highest is 2210N).

The strength of 27 glass beams bending by concentrated force have been tested during authors’ research. All samples, after cutting to the geometry shown in Fig. 1, have every edge ground. None intended scratch of particular size has been made on the edges of the beams.

Results of bending tests are presented in Fig. 2. The plot load versus deflection confirms linear elasticity and brittleness of glass and shows much scatter of values of damage force (the lowest value is 890N and the highest is 2210N).

3. Analytical solution

Fracture Mechanics describes the mechanical behaviour of body containing a crack. One of the strength criterion in FM assumed that crack propagation starts when the stress intensity factor $K_I$ reaches a critical value $K_{IC}$, characteristic for material. Stress intensity factor describes the distribution of values of stresses and strains near the crack tip. The fracture toughness $K_{IC}$ in plane strain condition for Na$_2$O–CaO–SiO$_2$ glass gets the values from the range $K_{IC}=22.77–25.93$ [MPa√mm] [3]. For the beam shown in Fig. 1 the analytical equation of $K_I$ is available in the form [2]:

$$K_I = \frac{P l}{g \sqrt{h^3}} \left( 2.9 \sqrt{\frac{a}{h}} - 4.6 \left(\frac{a}{h}\right)^3 + 21.8 \left(\frac{a}{h}\right)^5 - 37.6 \left(\frac{a}{h}\right)^7 + 38.7 \left(\frac{a}{h}\right)^9 \right)$$

where $a$ is a crack in the middle of the beam span. Substituting geometry of samples used in laboratory test, values of damage force obtained in tests and fracture toughness of glass equal $K_{IC}=22.77$ [MPa√mm] to Eqn (1) one gets, that sizes of cracks, which caused damage during bending, are included in the range from 0.02mm to 0.136mm. The cracks of these sizes are a consequence of edge grinding process [1].
4. Numerical analysis

4.1. LEFM in 2D numerical analysis of three-point bending test

The geometry, boundary and loading conditions for the numerical analysis of three-point bending test are presented in Fig. 3. Because of symmetry only a half of beam has been considered. During the analysis crack length of value $a=0.136\,\text{mm}$ has been imposed in the middle of the beam span. The following values of parameters for glass, as isotropic and linear elastic material, have been chosen: Young’s modulus $E=70\,\text{GPa}$, and Poisson’s ratio $\nu=0.20$.

![Figure 3: Scheme of three-point bending beam: geometry, loads and boundary conditions for a half of the model.](image)

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Numerical analysis has been performed utilising ABAQUS, the finite element analysis software. During the analysis for applied force only mode I of loading - symmetrical opening of traction-free crack surfaces will appear, so only values of stress intensity factor $K_I$ has been calculated. 2D analysis has been done for plain strain state with CPE8R elements. The LEFM requires application of very dense mesh near crack tip (see Fig. 3). During analysis convergence of stress intensity factor $K_I$ has been examine as a function of mesh density. The results are presented in Fig. 4.

![Figure 4: Convergence of $K_I$ as a function of mesh density.](image)

Figure 4: Convergence of $K_I$ as a function of mesh density.

Obtained results confirm that application of finite elements of size 5mm and dense mesh near crack tip gives sufficiently accurate values of $K_I$.

4.2. LEFM in 3D numerical analysis of three-point bending test

The 3D analysis has been performed to verify accuracy of 2D model. The analysis was done by means of C3DSR finite elements with the sizes 5mm/5mm/1.25mm, where the last size is the thickness of the element perpendicular to the plane of the beam. The changes of values of stress intensity factor $K_I$ with the distance through thickness are shown in Fig. 5. The results show much differences in $K_I$ values on the edges and within the thickness of the specimen. This confirms the phenomenon of existing plain stress state (for which $K_I$ reaches always bigger values) on the edges and plain strain in the middle area of thick elements [2].

![Figure 5: Variation of $K_I$ for LEFM and XFEM analysis through the thickness of the beam g.](image)

Figure 5: Variation of $K_I$ for LEFM and XFEM analysis through the thickness of the beam g.

4.3. XFEM in numerical analysis of three-point bending test

The major disadvantage of LEFM application in numerical analysis of fracture mechanics is time consuming preparation of a proper mesh i.e. very dense near crack tip. This disadvantage is not present in XFEM. The shape and path of crack are independent of mesh, it is not required to define the position of imperfection and the crack propagation analysis is available.

XFEM analysis has been repeated for the same model as in the case of 3D LEFM analysis (but without increasing mesh density near crack tip). Likewise in other numerical analysis values of stress intensity factor $K_I$ have been obtained and shown in Fig. 5.

![Table 1: Values of stress intensity factor](image)

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<table>
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<tr>
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5. Comparison of results

Table (1) collects all obtained results of $K_I$. The values of $K_I$ in case of 2D LEFM are very close to analytical solution. Also the model 3D gives compared results of stress intensity factor, but only on the edges of the model. The average values of 3D LEFM and 3D XFEM analysis are very close.

6. Conclusions

The performed studies confirm, that 2D numerical model gives sufficiently accurate results in fracture mechanics analysis. Since the value of $K_I$ obtained in 2D analysis is higher than in 3D model (which is more in line with reality), the design of structure based on 2D model is reliable. The XFEM analysis gives also very accurate values and what’s more provides new possibilities in fracture mechanics numerical methods. The preparation of model for XFEM analysis takes much less time. XFEM carries the opportunity of application of fracture criterion proposed by user. This opportunity will be used in authors’ future research. Succeeding work will also be concentrated on establishment and application of substitute model for glass edge, that will enable to avoid necessity of time consuming accurate modelling of geometry of machined edges of elements.

References