SMALL-SCALE EFFECT IN THE FRAMEWORK OF FRACTIONAL AND ERINGEN NON-LOCAL MODELS

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ABSTRACT

In this paper we present the comparison of two non-local formulations for small-scale effect modelling, namely fractional and Eringen models. The problem is illustrated by the analysis of 1D axial vibration of a nanorod. The discussion considers natural frequencies as a function of body dimensions and chosen length scale. For fractional model the influence of the order of fractional derivative is also discussed.

1. Introduction

Nowadays micro or nanostructures have increasing technological applications. Micro- or nanoelectromechanical systems (MEMS or NEMS), incorporating beams or plates in the micro or nano length scale, have application in different fields of mechanics, as well as in biotechnology and biomedical fields. Due to the dimensions of these structures, which are small and comparable to molecular distances, size effects are significant in their mechanical behaviour. Classical Continuum Mechanics (CCM), because of being length–scale independent, cannot handle size effect, thus non-local models are needed. In this paper two approaches will be contrasted, namely the Eringen non-local elastic model [1] and the Fractional Continuum Mechanics (FCM) with elastic behaviour [2].

2. Problem formulation

We consider a bar of constant cross section area, length $L$, density $\rho$ and Young modulus $E$, in absence of external forces. The problem of 1D linear elasticity under FCM is governed by the following equation

$$E l_f^{\alpha-1} \frac{\Gamma(2-\alpha)}{2} \frac{\partial}{\partial x} \left( x - \frac{C}{l_f} D_x^\alpha u - \frac{C}{l_f} D_{x+l_f}^\alpha u \right) = \rho \frac{\partial^2 u}{\partial t^2}$$

where $x$ denotes space, $t$ is time, $u$ is displacement, $l_f$ is a length scale in the fractional model, $\Gamma$ is the Gamma function, $^{\alpha}D_x^{\alpha}$ is the Riesz-Caputo differential operator and $\alpha$ is the order of the fractional differentiation. Alternatively this problem may be stated, under the hypotheses of the Eringen elastic non-local model, according to the following equation [3]:

$$E \frac{\partial^2 u}{\partial x^2} = \left( 1 - (\epsilon_0 \alpha)^2 \frac{\partial^2}{\partial x^2} \right) \rho \frac{\partial^2 u}{\partial t^2}$$

$l_e = \epsilon_0 \alpha$ being the length scale for Eringen model. Assuming clamped-clamped boundary conditions, we may obtain the non-local eigenfrequencies of the rod $\omega_{NL}$. In the case of FCM, the determination of natural frequencies requires solving an algebraic eigenproblem, due to the approximation of fractional spatial derivatives at discrete positions, whereas for the Eringen model the frequencies can be obtained in closed form (see Aydogdu [3]).
3. Numerical examples and discussion

In Fig. 1 the eigenfrequencies corresponding to the first vibration mode (k=1), $\omega_{NL}$ of both FCM and Eringen models are compared with those obtained with CCM (local) model $\omega_L$, for different orders of fractional derivative $\alpha$, and four different values of the non-dimensional length-scale parameter $\bar{T} = \{0.1, 0.5, 1, 2\}$ ($\bar{T} = 2l/L$ for FCM, $\bar{T} = l_{nf}/L$ for Eringen).

As shown $\omega_{NL}$ is dependent on length-scale parameter for both models, decreasing when $\bar{T}$ increases, which strengthens the relevance of the length over which non-local effects are taken into account. Likewise, the solution of the FFC model depends on the value of the derivative order. Therefore $\alpha$ can be considered as an additional parameter in a non-local approach, offering more possibilities for reproducing the behavior of nano-structures. Moreover, using different types of fractional derivatives permit to obtain a closer approximation to the experimental observations. Certainly, this would also require additional results for a proper calibration.

4. References