FRACTIONAL CALCULUS FOR CONTINUUM MECHANICS - THEORY, APPLICATIONS AND FUTURE TASKS

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1. Introduction

The elegance and the conciseness of fractional operators in describing smoothly the transition between different physical phenomena are attracting an increasing numbers of scientists and researchers. In the frame of Solid Mechanics this mathematical approach has been introduced especially in describing viscous behavior of materials, where the non-locality is taken with respect to the time variable (memory) [Mainardi et al., 2010]. However it has been also used in the stress space through the fractional gradient of plastic potential in order to control magnitude and direction of viscoplastic strain evolution [Sumelka, 2014a], or in the space variable [Lazopoulos, 2006]. In the latter case, the order of the derivative is usually lower than the classical one, a major asset in comparison to gradient theories which provide derivatives of the displacement of higher orders thus needing extra boundary conditions to be solved, whose physical meaning is still unclear [Carpinteri et al., 2009].

Herein the review comments on the existing generalisations of the classical continuum mechanics utilising fractional calculus for space variable will be presented. Both theory, applications and future tasks will be defined. Special attention will be addressed to the formulation presented in [Sumelka, 2014b, Sumelka, 2014c].

2. Fractional continua

The proposed generalisation of classical continuum mechanics using fractional calculus exists in the literature (cf. [Vazquez, 2004, Lazopoulos, 2006, Di Paola et al., 2009, Atanackovic and Stankovic, 2009, Carpinteri et al., 2011, Drapaca and Sivaloganathan, 2012]). However the description presented in papers [Sumelka, 2014b, Sumelka, 2014c] has at least the following important original features:

(i) the proposed new formulation has clear physical interpretation and is developed by analogy to general framework of classical continuum mechanics;

(ii) we deal with finite deformations (in contrast to [Atanackovic and Stankovic, 2009, Carpinteri et al., 2011] where small deformations are considered only);

(iii) contrary to previous works e.g. [Atanackovic and Stankovic, 2009, Carpinteri et al., 2011, Drapaca and Sivaloganathan, 2012] the generalised fractional measures of the deformation e.g. fractional deformation gradients or fractional strains have the same physical dimensions as classical one (thus their classical interpretation remain unchanged);
characteristic length scale of the particular material is defined explicitly (an in classical non-local models);
objectivity requirements are proved;
and finally, the discussed concept bases on the fractional material and spatial line elements in contrast to [Drapaca and Sivaloganathan, 2012] where the whole formulation bases on fractional motion.

3. Numerical examples

During the talk the application of fractional continuum mechanics for linear elasticity and rate independent plasticity will be discussed. The role of order of fractional continua and length scale will be pointed out.

4. Conclusions

Fractional continua can be introduced utilizing different concepts and it belongs to future task to judge if there exist unique approach. Nevertheless, one can say, that Fractional Continuum Mechanics causes that we are getting closer and closer to fulfil Leibnitz's prophecy about the fractional calculus, perspectives: “It will lead to a paradox, from which one day useful consequences will be drawn”.

References