Generalized dynamic failure criteria in dynamic plasticity

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1. DYNAMIC FRACTURE MECHANICS AND DYNAMIC FAILURE MECHANICS

The *Dynamic Fracture Mechanics* is well established area of research with many technical and technological achievements. In that case theoretical analyses or experimental studies are involved in order to define the critical conditions when a stationary crack or a network of cracks already existing in a solid will begin to move. In the next stage of fracture the cracks can propagate with high speeds until a complete disintegration of construction. Concerning dynamical processes of fracturing the loading time may be relatively long or very short. The loading times in milliseconds or in microseconds leads to very high strain rates in the process zone of material separation near the crack tip. Those very high strain rates are associated with a very high rate sensitivity coupled with an adiabatic increase of temperature. In addition, a delay accumulation of defects, for example micro-cavities, and inertia in the process zone can play an important role. The critical conditions are defined by different fracture criteria. A more detailed discussion of *Dynamic Fracture Mechanics* has been offered elsewhere, [1].

The *Dynamic Failure Mechanics* (DFM) differs because no initial cracks is assumed in a solid under fast or impact loading. In that case a short time loading can be caused by mechanical waves or explosions. Typical examples are the tests of cylindrical specimens in tension or compression or different test configurations in shear tests, for example thin tubular specimens. Another specific case is spalling of soft and hard materials. Several stages may occur in DFM. In the beginning an elastic field is developed due to elastic wave propagation with many wave interferences. This first stage occurs due to existence of free surfaces causing wave reflections. In the next stage a plastic field appears, or in a more brittle or non-homogenous materials a field of damage. But after a short time interval, local stress concentrations and plastic strain fields appear causing local instabilities.

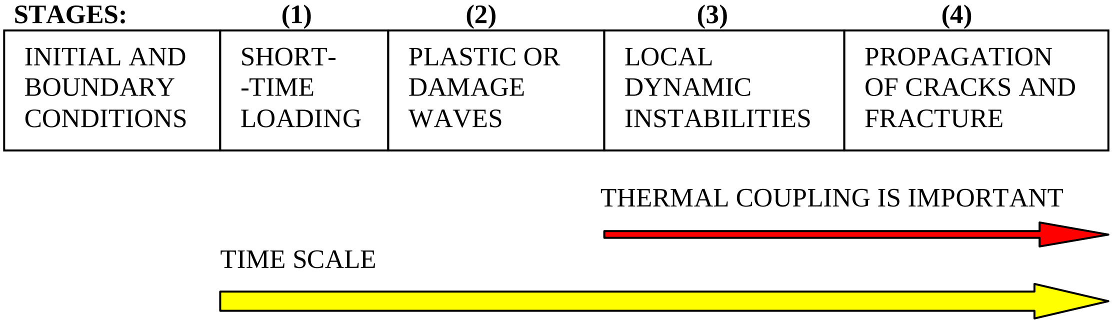

Fig. 1 *Four stages of failure process in dynamic loading of structures.*

Because those processes are fast the local strain fields produce increased temperature fields due to conversion of plastic work into heat. Finally, the last stage is propagation of cracks and complete separation. Several stages mentioned above constitutes the process that can be called the failure. A scheme of the failure process is shown in Fig.1.

In order to solve and analyze the failure process in the framework of DFM two fundamental factors must be taken into consideration. The first one is definition of the constitutive relations in elasticity and thermo-visco-plasticity. The second factor is the failure criteria which must be introduced into analyses. In recent decades many constitutive relations have been proposed which relate stress, strain, strain rate and temperature. They are mostly based on the concept “mechanical equation of state” where the final stress depends on the current variables: strain, strain rate and temperature. Such approach causes many problems because plastic strain cannot be assumed in constitutive modeling as the physical variable. The concept of the mechanical equation of state causes many false conclusions when dynamic instabilities and failures are studied. The most optimal solution in dynamic plasticity is application of constitutive relations with internal state variables, for example [2]. A correct combination of constitutive relation with adequate failure criterion is the most important in DFM.

Up to now the failure criteria are proposed and evaluated for quasi-static lading. However, at high loading rates the failure criteria proposed so far lack of generality and usually are limited to one mode of deformation, for example shear or one-dimensional strain. Because of high strain gradients in dynamics, for example plastic waves, failure criteria should be defined locally in elementary volumes.

2. A COMBINED FAILURE CRITERION

More detailed analyses of fracture surfaces after plastic deformation lead to the conclusion that almost every time the mode of fracture is mixed. Part of the volume near fracture zone is fractured by material separation in tension and another part in shear. A typical example is the so-called cup-cone fracture, which is a frequent result of a tension test. Therefore, a combination of opening and shear mode is taken into account in this new criterion. It is assumed that the proportions between opening and shear depends on statistics of the triaxiality factor.

3. EFFECTS OF STRESS TRIAXIALITY ON FAILURE

It is known for a long time that an increase of pressure enhances ductility. However the hydrostatic pressure may be positive (tension) or negative (compression), therefore the effects of pressure may introduce variety of micro-mechanisms in material separation. In spite of many attempts, majority of quasi-static criteria defining ductile separation are far to be complete. In dynamics a progress is still unsatisfactory. The main reason is a lack of systematic experimental data. The most advanced area is the phenomenon of spalling with many available criteria with different levels of approximation. Of course in spalling, where one-dimensional strain dominates, the hydrostatic component of stress tensor after reflection of compressive plane wave, or the mean stress, is positive, $\sigma_m = \sigma_{kk}/3$. In elastic solid under one-dimensional strain the mean pressure is defined by $\sigma_m = (\sigma_{11} + 2\sigma_{22})/3$ with $\sigma_{22} = \sigma_{33}$. The explicit formula for the mean stress in on-dimensional strain is given by Eq.(9) where λ and μ are Lamè's constants and Θ is the dilatation, α_t is the coefficient of thermal expansion and ΔT_A is the increment/decrement of temperature in adiabatic conditions. Since in one-dimensional strain $\Theta = \varepsilon_{11}$ then Eq.(9) is transformed into the simplified version on the right side

$$\sigma_m = \frac{2}{3}\mu\varepsilon_{11} + \lambda(\Theta - 3\alpha_t \Delta T_A) \quad \text{or} \quad \sigma_m = \frac{2}{3}\mu\varepsilon_{11} + \lambda(\varepsilon_{11} - 3\alpha_t \Delta T_A) \tag{9}$$

In order to estimate a possible range of tension pressures in spalling the range of expected ultimate strain is assumed: $0.001 < \varepsilon_u < 0.01$. Those values are typical strains when incipient spall occurs. In the next stage of spalling, development of micro-cavities relax the mean stress. In order to show the range of maximum pressures Eq.(9) was quantitatively analyzed assuming steel: $\mu = 81$ GPa, $\lambda = 112$ GPa, $\alpha_t = 10^{-5}$ K^{-1}, then the mean pressure varies within the limits: 0.166 GPa $< \sigma_m <$ 1.66 GPa. Because adiabatic changes of temperature in metals in elastic state are relatively low, typically up to 2.0 K, this contribution to the mean pressure can be in general neglected. For example the critical stress of spall for a spheroidized 1045 steel is ~2.6 GPa. Thus the critical stress is usually higher than the estimated values of the mean stress. This is due to the cumulative and rate-dependent damage which occurs in short-time loading.

A more general definition of the pressure level, called the stress triaxiality $\alpha = \sigma_m/\sigma_{eq}$, where σ_{eq} is the equivalent stress is a very useful variable in failure mechanics. In one-dimensional strain the stress triaxiality is obtained as

$$\alpha = \frac{\sigma_{11} + 2\sigma_{22}}{\sigma_{11} - \sigma_{22}} \quad \text{or} \quad \alpha = \frac{1}{3} + \frac{\lambda}{2\mu}\left(1 - \frac{3\alpha_t}{\varepsilon_{11}}\Delta T_A\right) \tag{10}$$

For the limits of strain assumed previously the limits of stress triaxiality varies from $\alpha_{max} = 1.0204$ to $\alpha_{min} = 0.9831$. Therefore the stress triaxiality in spalling is around 1.0. This is the largest triaxiality possible in mechanical testing.

5. FAILURE CRITERION FOR IMPACT SHEARING

In general, all phases leading to failure by shear deformation follow stages shown in Fig.1. Because in shear the cross section of deformed area remains constant, therefore the effect of geometrical instability does not occur in shearing and the only mechanism leading to instability and failure is the material softening due to adiabatic heating. In isothermal conditions of plastic shearing, the main factor in material separation (shear fracture) is the critical shear strain. This is a local failure criterion which can be verified by microscopic observations of deformation gradients.

References

[1] J.R. Klepaczko, Dynamic instabilities and failures in impact tension, compression and shear, J. Phys. IV France, **134** (2006), 857-867.