Fracture criteria for dynamically loaded concrete and spalling in the structure*

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Introduction

The concrete structures are often subjected to untypical loadings (1). For example the blast wave as well as impact belong to this type. This presentation considers only the first class of loadings and describes the dynamic failure criterion for quasi-brittle material like concrete. In dynamic numerical analysis the criterion for very fast loading should take into account strain rate sensitivity of the material. We present the cumulative failure criterion because during fast nonlinear processes the time delay appear and the tensile and compressive strength is much higher than quasi-static strength. The authors present the general consideration of dynamic behavior of concrete and some numerical examples.

Failure criterion

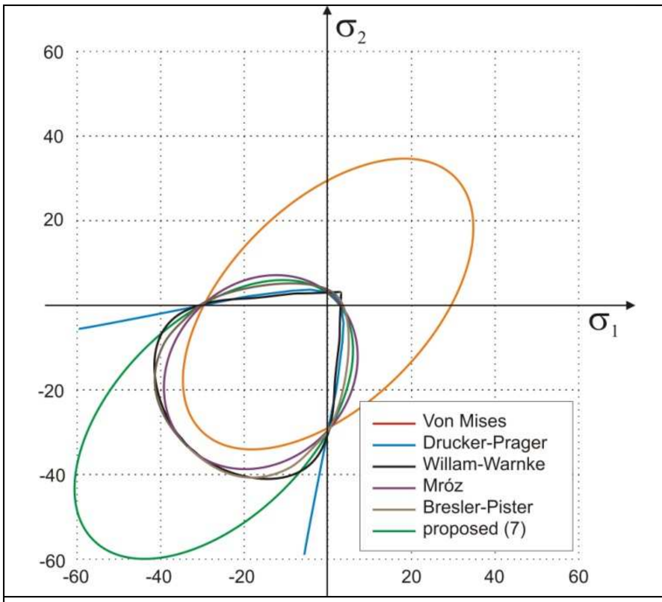
Cumulative Fracture Criterion (CFC) introduced (2) (3) and discussed before (4) (5) (6)is used and has been added to Abaqus/Explicit environment by VUMAT procedure. This criterion in integral form is as following:

$$t_{c0} = \int_0^{tc} \left(\frac{\sigma_F^{eq}(t)}{\sigma_{F0}^{eq}}\right)^{\alpha(T)} dt \qquad \text{if} \qquad \sigma_F^{eq}(t) > \sigma_{F0}^{eq},$$

where t_{c0} is the longest critical time, $\alpha(T)$ is the parameter connected with energy activated during the separation process and σ_{F0}^{eq} is quasi-static equivalent tensile strength of concrete. This measure is introduced to describe better the deformation in advanced triaxial state of stress (5):

$$\sigma_F^{eq} = \frac{k-1}{2k(1-v)} I_1 + \frac{1}{2k}\sqrt{\left(\frac{k-1}{1-2v} I_1\right)^2 + \frac{6k}{(1-v)^2} J_2}$$

where $I_1 = \sigma_1 + \sigma_2 + \sigma_3$ and $J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2]$. σ_{F0}^{eq} is the generalization of the von Mises equivalent stresses (7). $I_1$and J_2 are the first and the second invariants of the stress tensor and deviatoric part of the stress tensor, respectively. The k influences the shape of the critical failure surface in the space of principal stresses. The criterion describes generally the time up to failure under the stress impulse. Equivalent stresses σ_F^{eq} was generally used in the past during quasi-static analysis to predict limit stresses for concrete. Other limit stresses surface can be also used. Von Mises equivalent stress is acceptable for steel (2). Authors present the comparison of several limit surfaces (8) which can be used as description of σ_F^{eq}in CFC. The parameters of the failure surface in the Fig. 1 are identified for tensile strength equals 3 MPa, compressive strength equals 30 MPa and biaxial compressive strength equals 33.6 MPa dependent on the number of parameters. We present only selected one, two and three parameters failure surfaces.



Von Mises failure criterion:

$$f(J_2) = \sqrt{J_2} - A^2 = 0$$

Drucker-Prager failure criterion:

$$f(I_1, J_2) = AI_1 + \sqrt{J_2} - B^2 = 0$$

Willam-Wernke failure criterion:

$$f(I_1, J_2, \theta) = AI_1 + B(\theta)\sqrt{\tfrac{2}{5}J_2} - C = 0$$

Mróz failure criterion:

$$f(I_1, J_2) = (I_1 - A)^2 + BJ_2^2 - C = 0$$

Bresler-Pister failure criterion:

$$f(I_1, J_2) = AI_1 + BI_1^2 - \sqrt{\tfrac{2}{3}J_2} - C^2 = 0$$

Fig. 1 Comparison of failure criteria (8)

*The work is dedicated to the memory of Prof. J.R. Klepaczko

Tab.1 Parameters of concrete

E	35e9 [Pa]
ν	0.2
ρ	2395 [kg/m3]
k	10
t_{c0}	0.000049 [s]
$\alpha(T)$	0.95
σ_{F0}^{eq}	4.2e6 [Pa]

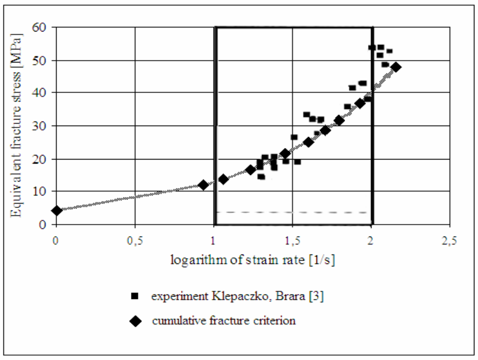


Fig. 2 Equivalent fracture stress as a function of strain rate logarithm

Numerical examples

The results of two cases of the specimens tested in Hopkinson pressure bar with the velocity of projectile 7m/s and 12m/s are presented in Figs 3, 4. Both analyses are continued up to final failure of the specimens. On the plots on the right hand sides by dots we have indicated the actual state of stresses accompanies the following situations.

The fracture of concrete specimens is a results of tension (one dominant crack appears), although at first the whole specimen is compressed. The compressive wave is caused by the early wave interaction in the striker and Hopkinson bar. Figs 3, 4 present the sequences of the distribution of longitudinal stresses. Colored figure represent the results of computations obtained in the environment of ABAQUS/Explicit. For initial velocity 7m/s only one dominant crack is placed 55mm from the surface hit by projectile. In the second considered case (12 m/s) two cracks appeared. First crack about 80mm from the hit surface and the second about 55mm. Qualitatively the results of computations and experiments observed in laboratory are in good agreement what confirms the validity of constitutive assumptions and FE mesh quality.

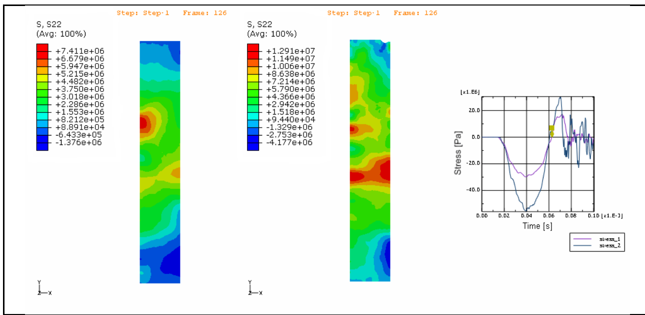


Fig. 3 Distribution of axial stress before failure

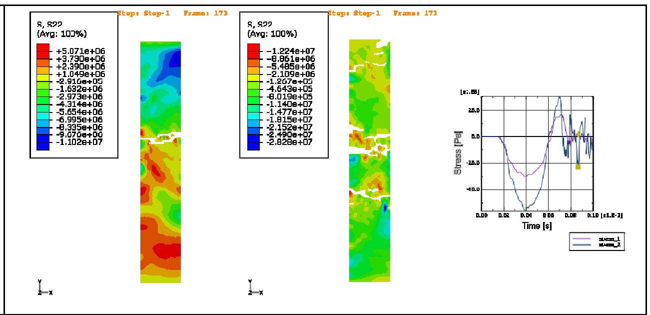


Fig. 4 Distribution of axial stress after failure

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